FILOMAT (Niš) **16** (2002), 7–17

# GENERAL CONFORMAL ALMOST SYMPLECTIC N-LINEAR CONNECTIONS IN THE BUNDLE OF ACCELERATIONS

**Monica Purcaru and Nicoleta Aldea**

**Abstract**

The aim of this paper[1] is to find the transformation for the coefficients of an N-linear connection on $E = Osc^2M$, by a transformation of nonlinear connections, to define in the bundle of accelerations the general conformal almost symplectic N-linear connection notion and to determine the set of all general conformal almost symplectic N-linear connections on E. We treat also some special clases of general conformal almost symplectic N-linear connections on E.

## 1 Introduction

The literature on the higher order Lagrange spaces geometry highlights the teoretical and practical importance of these spaces: [4] − [7].

Motivated by concrete problems in variational calculation, higher order Lagrange geometry, based on the k-osculator bundle notion, has witnessed a wide acknowledgment due to the papers: [4] − [7], published by Radu Miron and Gheorghe Atanasiu.

The geometry of k-osculator spaces presents not only a special theoretical interest, but also an applicative one.

Due to its content, the present paper continues a trend of interest with a long tradition in the modern differential geometry, i.e. the study of remarkable geometrical structures.

In the present paper we find the transformations for the coefficients of an N-linear connection on $E = Osc^2M$, by a transformation of nonlinear connections, (§2).

We define the general conformal almost symplectic N-linear connection notion on E, (§3).

[1]2000 *Mathematics Subject Classification.* 53C05.

*Key words and phrases.* 2-osculator bundle, general conformal almost symplectic N-linear connection, conformal almost symplectic d-structure

We determine the set of all these connections and we treat also some special clasess of general conformal almost symplectic N-linear connections on E, (§4).

This paper is a generalization of the papers:[10] − [13]. Concerning the terminology and notations, we use those from: [4], [9], which are essentially based on M.Matsumoto's book: [2].

## 2 The set of the transformations of $N$-linear connections in the 2-osculator bundle

Let $M$ be a real n-dimensional $C^\infty$-manifold and let $(Osc^2M, \pi, M)$ be its 2-osculator bundle, with $E = Osc^2M$ the total space.

The local coordinates on E are denoted by: $(x^i, y^{(1)i}, y^{(2)i})$, briefly: $(x, y^{(1)}, y^{(2)})$.

If N is a nonlinear connection on E, with the coefficients $(N_{(1)}{}^i{}_j, N_{(2)}{}^i{}_j)$, then let D be an N-linear connection on E, with the coefficients $D\Gamma(N) = = (L^i_{jk}, C_{(1)}{}^i_{jk}, C_{(2)}{}^i_{jk})$.

If $\overline{N}$ is another nonlinear connection on E, with the coefficients $\overline{N}_{(1)}{}^i{}_j(x, y^{(1)}, y^{(2)}), \overline{N}_{(2)}{}^i{}_j(x, y^{(1)}, y^{(2)})$, then there exist a uniquely determined tensor fields $A_{(\alpha)}{}^i{}_j \in \tau^1_1(E), \quad (\alpha = 1, 2)$ such that:

$$\overline{N}_{(\alpha)}{}^i{}_j = N_{(\alpha)}{}^i{}_j - A_{(\alpha)}{}^i{}_j, \quad (\alpha = 1, 2). \tag{2.1}$$

Conversely,if $N_{(\alpha)}{}^i{}_j$ and $A_{(\alpha)}{}^i{}_j, (\alpha = 1, 2)$ are given, then $\overline{N}_{(\alpha)}{}^i{}_j, (\alpha = 1, 2)$, given by (2.1) is a nonlinear connection.

Let us suppose that the mapping $N \to \overline{N}$ is given by (2.1).

According to Cap.III, §3.3, [4], we have:

$D_{\frac{\delta}{\delta x^k}} \frac{\delta}{\delta y^{(\alpha)j}} = L^i_{jk} \frac{\delta}{\delta y^{(\alpha)i}}, \ D_{\frac{\delta}{\delta y^{(\beta)k}}} \frac{\delta}{\delta y^{(\alpha)j}} = C_{(\beta)}{}^i_{jk} \frac{\delta}{\delta y^{(\alpha)i}},$
$(\beta = 1, 2;\ \alpha = 0, 1, 2;\ y^{(0)i} = x^i)$ and
$\frac{\overline{\delta}}{\delta x^i} = \frac{\partial}{\partial x^i} - \overline{N}_{(1)}{}^j{}_i \frac{\partial}{\partial y^{(1)j}} - \overline{N}_{(2)}{}^j{}_i \frac{\partial}{\partial y^{(2)j}}, \ \frac{\overline{\delta}}{\delta y^{(1)i}} = \frac{\partial}{\partial y^{(1)i}} - \overline{N}_{(1)}{}^j{}_i \frac{\partial}{\partial y^{(2)j}}, \ \frac{\overline{\delta}}{\delta y^{(2)i}} = \frac{\partial}{\partial y^{(2)i}}.$

It follows first of all that the transformation (2.1) preserve the coefficients $C_{(2)}{}^i_{jk}$.

Taking in account the fact that:

$$\frac{\bar{\delta}}{\delta x^i} = \frac{\delta}{\delta x^i} + A_{(1)}{}^j{}_i \frac{\partial}{\partial y^{(1)j}} + A_{(2)}{}^j{}_i \frac{\partial}{\partial y^{(2)j}}, \ \frac{\bar{\delta}}{\delta y^{(1)i}} = \frac{\delta}{\delta x^{(1)i}} + A_{(1)}{}^j{}_i \frac{\partial}{\partial y^{(2)j}},$$

it follows:

$$D_{\frac{\bar{\delta}}{\delta x^k}} \frac{\bar{\delta}}{\delta y^{(2)j}} = D_{\frac{\bar{\delta}}{\delta x^k}} \frac{\partial}{\partial y^{(2)j}} = \overline{L}^i_{jk} \frac{\partial}{\partial y^{(2)i}} = D_{(\frac{\delta}{\delta x^k} + A_{(1)}{}^l{}_k \frac{\partial}{\partial y^{(1)l}} + A_{(2)}{}^l{}_k \frac{\partial}{\partial y^{(2)l}})} \frac{\partial}{\partial y^{(2)j}} =$$

$$= D_{\frac{\delta}{\delta x^k}} \frac{\partial}{\partial y^{(2)j}} + A_{(1)}{}^l{}_k D_{(\frac{\delta}{\delta y^{(1)l}} + N_{(1)}{}^m{}_l \frac{\partial}{\partial y^{(2)m}})} \frac{\partial}{\partial y^{(2)j}} + A_{(2)}{}^l{}_k D_{\frac{\partial}{\partial y^{(2)l}}} \frac{\partial}{\partial y^{(2)j}} =$$

$$= L^i_{jk} \frac{\partial}{\partial y^{(2)i}} + A_{(1)}{}^l{}_k C_{(1)}{}^i{}_{jl} \frac{\partial}{\partial y^{(2)i}} + A_{(1)}{}^l{}_k N_{(1)}{}^m{}_l C_{(2)}{}^i{}_{jm} \frac{\partial}{\partial y^{(2)i}} + A_{(2)}{}^l{}_k C_{(2)}{}^i{}_{jl} \frac{\partial}{\partial y^{(2)i}} =$$

$$= (L^i_{jk} + A_{(1)}{}^l{}_k C_{(1)}{}^i{}_{jl} + A_{(1)}{}^l{}_k N_{(1)}{}^m{}_l C_{(2)}{}^i{}_{jm} + A_{(2)}{}^l{}_k C_{(2)}{}^i{}_{jl}) \frac{\partial}{\partial y^{(2)i}}.$$

$$D_{\frac{\bar{\delta}}{\delta y^{(1)k}}} \frac{\bar{\delta}}{\delta y^{(2)j}} = D_{\frac{\bar{\delta}}{\delta y^{(1)k}}} \frac{\partial}{\partial y^{(2)j}} = \overline{C}_{(1)}{}^i{}_{jk} \frac{\partial}{\partial y^{(2)i}} = D_{(\frac{\delta}{\delta y^{(1)k}} + A_{(1)}{}^l{}_k \frac{\partial}{\partial y^{(2)l}})} \frac{\partial}{\partial y^{(2)j}} =$$

$$= D_{\frac{\delta}{\delta y^{(1)k}}} \frac{\partial}{\partial y^{(2)j}} + A_{(1)}{}^l{}_k D_{\frac{\partial}{\partial y^{(2)l}}} \frac{\partial}{\partial y^{(2)j}} = C_{(1)}{}^i{}_{jk} \frac{\partial}{\partial y^{(2)i}} + A_{(1)}{}^l{}_k C_{(2)}{}^i{}_{jl} \frac{\partial}{\partial y^{(2)i}} =$$

$$= (C_{(1)}{}^i{}_{jk} + A_{(1)}{}^l{}_k C_{(2)}{}^i{}_{jl}) \frac{\partial}{\partial y^{(2)i}}.$$

Therefore the change we are looking for is:

$$(2.2) \quad \begin{cases} \overline{L}^i_{jk} = L^i_{jk} + A_{(1)}{}^l{}_k C_{(1)}{}^i{}_{jl} + A_{(1)}{}^l{}_k N_{(1)}{}^m{}_l C_{(2)}{}^i{}_{jm} + A_{(2)}{}^l{}_k C_{(2)}{}^i{}_{jl} \\ \overline{C}_{(1)}{}^i{}_{jk} = C_{(1)}{}^i{}_{jk} + A_{(1)}{}^l{}_k C_{(2)}{}^i{}_{jl}, \\ \overline{C}_{(2)}{}^i{}_{jk} = C_{(2)}{}^i{}_{jk}. \end{cases}$$

So, we have proved:

**Proposition 2.1** *The transformation (2.1) of nonlinear connections imply the transformations (2.2) for the coefficients* $D\Gamma(N) = (L^i_{jk}, C_{(1)}{}^i{}_{jk}, C_{(2)}{}^i{}_{jk})$ *of the N-linear connection D.*

Now, we can prove:

**Theorem 2.1** *Let N and* $\bar{N}$ *be two nonlinear connections on E, with the coefficients* $(N_{(1)}{}^i{}_j,\ N_{(2)}{}^i{}_j)$, $(\overline{N}_{(1)}{}^i{}_j,\ \overline{N}_{(2)}{}^i{}_j)$*-respectively. If* $D\Gamma(N) = (L^i_{jk}, C_{(1)}{}^i{}_{jk}, C_{(2)}{}^i{}_{jk})$ *and* $D\overline{\Gamma}(\overline{N}) = (\overline{L}^i_{jk}, \overline{C}_{(1)}{}^i{}_{jk}, \overline{C}_{(2)}{}^i{}_{jk})$ *are two N-, respectively* $\bar{N}$*-linear connections on the differentiable manifold E, then there exists only one quintet of tensor fields* $(A_{(1)}{}^i{}_j, A_{(2)}{}^i{}_j, B^i_{jk}, D_{(1)}{}^i{}_{jk}, D_{(2)}{}^i{}_{jk})$ *such that:*

$$(2.3)\quad \begin{cases} \overline{N}_{(\alpha)}{}^{i}{}_{j} = N_{(\alpha)}{}^{i}{}_{j} - A_{(\alpha)}{}^{i}{}_{j}, \quad (\alpha = 1,2), \\ \overline{L}^{i}_{jk} = L^{i}_{jk} + A_{(1)}{}^{l}{}_{k} C_{(1)}{}^{i}_{jl} + A_{(1)}{}^{l}{}_{k} N_{(1)}{}^{m}{}_{l} C_{(2)}{}^{i}_{jm} + A_{(2)}{}^{l}{}_{k} C_{(2)}{}^{i}_{jl} - B^{i}_{jk}, \\ \overline{C}_{(1)}{}^{i}_{jk} = C_{(1)}{}^{i}_{jk} + A_{(1)}{}^{l}{}_{k} C_{(2)}{}^{i}_{jl} - D_{(1)}{}^{i}_{jk}, \\ \overline{C}_{(2)}{}^{i}_{jk} = C_{(2)}{}^{i}_{jk} - D_{(2)}{}^{i}_{jk}. \end{cases}$$

**Proof.** *The first equality (2.3) determines uniquely the tensor fields* $A_{(\alpha)}{}^{i}{}_{j}$, $(\alpha = 1,2)$, [3]. *Since* $C_{(\alpha)}{}^{i}_{jk}$, $(\alpha = 1,2)$ *are tensor fields, the second equation (2.3) determines uniquely the tensor field* $B^{i}_{jk}$. *Similary the third and the fourth equation (2.3) determine the tensor fields* $D_{(1)}{}^{i}_{jk}$ *and* $D_{(2)}{}^{i}_{jk}$ *respectively.*

We have immediately:

**Theorem 2.2** *If* $D\Gamma(N) = (L^{i}_{jk}, C_{(1)}{}^{i}_{jk}, C_{(2)}{}^{i}_{jk})$ *are the coefficients of an N-linear connection D on E and* $(A_{(1)}{}^{i}{}_{j}, A_{(2)}{}^{i}{}_{j}, B^{i}_{jk}, D_{(1)}{}^{i}_{jk}, D_{(2)}{}^{i}_{jk})$ *is a quintet of tensor fields on E, then:* $D\overline{\Gamma}(\overline{N}) = (\overline{L}^{i}_{jk}, \overline{C}_{(1)}{}^{i}_{jk}, \overline{C}_{(2)}{}^{i}_{jk})$ *given by (2.3) are the coefficients of an* $\overline{N}$ *-linear connection* $\bar{D}$ *on E.*

The tensor fields $(A_{(1)}{}^{i}{}_{j}, A_{(2)}{}^{i}{}_{j}, B^{i}_{jk}, D_{(1)}{}^{i}_{jk}, D_{(2)}{}^{i}_{jk})$ are called the difference tensor fields of $D\Gamma(N)$ to $D\overline{\Gamma}(\overline{N})$ and the mapping $D\Gamma(N) \to D\overline{\Gamma}(\overline{N})$ given by (2.3) is called a transformation of $N$-linear connection to $\overline{N}$-linear connection, [2].

# 3 The notion of general conformal almost symplectic N-linear connection in the bundle of accelerations

Let M be a real $n = 2n'$-dimensional $C^{\infty}$-manifold and let $(Osc^2M, \pi, M)$ be its 2-osculator bundle. The local coordinates on the total space $E = Osc^2M$ are denoted by $(x^i, y^{(1)i}, y^{(2)i})$.

We consider on E an almost symplectic $d$-structure, defined by a $d$-tensor field of the type $(0,2)$, let us say $a_{ij}(x^i, y^{(1)i}, y^{(2)i})$, alternate:

$$(3.1)\quad a_{ij}(x, y^{(1)}, y^{(2)}) = -a_{ji}(x, y^{(1)}, y^{(2)}),$$

and nondegenerate:

(3.2) $det \left\| a_{ij}(x, y^{(1)}, y^{(2)}) \right\| \neq 0,\ \forall y^{(1)} \neq 0,\ \forall y^{(2)} \neq 0.$

We associate to this $d$-structure Obata's operators:

(3.3) $\Phi^{ir}_{sj} = \frac{1}{2}(\delta^i_s \delta^r_j - a_{sj}a^{ir}), \Phi^{*ir}_{sj} = \frac{1}{2}(\delta^i_s \delta^r_j + a_{sj}a^{ir}),$

where $(a^{ij})$ is the inverse matrix of $(a_{ij})$:

(3.4) $a_{ij}a^{jk} = \delta^k_i.$

Obata's operators have the same properties as the ones associated with the metrical d-structure on E, [8].

Let $\mathcal{A}_2(E)$ be the set of all alternate d-tensor fields of the type $(0,2)$ on E. As is easily shown, the relation for $b_{ij}, c_{ij} \in \mathcal{A}_2(E)$ defined by:

(3.5) $b_{ij} \sim c_{ij} \Longleftrightarrow \{\exists \rho(x, y^{(1)}, y^{(2)}) \in \mathcal{F}(E) | b_{ij} = e^{2\rho} c_{ij}\}$

is an equivalent relation on $\mathcal{A}_2(E)$.

**Definition 3.1** *The equivalent class: $\hat{a}$ of $\mathcal{A}_2(E)_{/\sim}$, to which the almost symplectic d-structure $a_{ij}$ belongs, is called a conformal almost symplectic d-structure on $E = Osc^2M$.*

*Every $a'_{ij} \in \hat{a}$ is a d-tensor field alternate and nondegenerate, expresed by:*

(3.6) $a'_{ij} = e^{2\rho} a_{ij}.$

*Obata's operators are defined for $a'_{ij} \in \hat{a}$ by putting $(a'^{ij}) = (a'_{ij})^{-1}$. Since equation (3.6) is equivalent to:*

(3.7) $(a'^{ij}) = e^{-2\rho} a^{ij},$

we have

**Proposition 3.1** *Obata's operators depend on the conformal almost symplectic d-structure $\hat{a}$, and do not depend on its representative $a'_{ij} \in \hat{a}$.*

Let N be a nonlinear connection on E with the coefficients $({N_{(1)}}^i{}_j, {N_{(2)}}^i{}_j)$ and let D be an N-linear connection on E with the coefficients in the adapted basis $\{\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, \frac{\partial}{\partial y^{(2)i}}\} : D\Gamma(N) = (L^i_{jk}, C_{(1)}{}^i_{jk}, C_{(2)}{}^i_{jk}).$

**Definition 3.2** *An N-linear connection D on E, is said to be a general conformal almost symplectic N-linear connection on E, if it verifies the following relations:*

$$(3.8)\quad a_{ij|k} = K_{ijk},\ a_{ij} \overset{(\alpha)}{|}_{k} = Q_{(\alpha)ijk},\ (\alpha = 1, 2),$$

*where* $K_{ijk},\ Q_{(\alpha)ijk},\ (\alpha = 1, 2)$, *are tensor fields of the type* $(0, 3)$, *having the properties of antisymmetry in the first two indices:*

$$(3.9)\quad K_{ijk} = -K_{jik},\ Q_{(\alpha)ijk} = -Q_{(\alpha)jik},\ (\alpha = 1, 2),$$

$|$, $\overset{(\alpha)}{|}$, *denote the h-and* $v_\alpha$*-covariant derivatives,* $(\alpha = 1, 2)$, *with respect to* $D\Gamma(N)$.

Particularly, we can give:

**Definition 3.3** *An N-linear connection D on E, for which there exists a 1-form* $\omega$ *in* $\mathcal{X}^*(Osc^2M), (\omega = \tilde{\omega}_i dx^i + \dot{\omega}_{(1)i}\ \delta y^{(1)i} + \dot{\omega}_{(2)i}\ \delta y^{(2)i})$ *such that:*

$$(3.10)\quad a_{ij|k} = 2\tilde{\omega}_k a_{ij},\ a_{ij} \overset{(\alpha)}{|}_{k} = 2\ \dot{\omega}_{(\alpha)k}\ a_{ij},\ (\alpha = 1, 2),$$

*where* $|$ *and* $\overset{(\alpha)}{|}$ *denote the h-and* $v_\alpha$*-covariant derivatives* $(\alpha = 1, 2)$ *with respect to* $D\Gamma(N)$, *is said to be compatible with the conformal almost symplectic structure* $\hat{a}$, *or a conformal almost symplectic N-linear connection on E with respect to the conformal almost symplectic structure* $\hat{a}$, *corresponding to the 1-form* $\omega$, *and is denoted by:* $D\Gamma(\omega)$.

For any representative $a'_{ij} \in \hat{a}$ we have:

**Theorem 3.1** *For* $a'_{ij} = e^{2\rho} a_{ij}$, *a conformal almost symplectic N-linear connection with respect to* $\hat{a}$, *corresponding to the 1-form* $\omega$, $D\Gamma(\omega)$ *satisfies:*

$$(3.11)\quad a'_{ij|k} = 2\tilde{\omega}'_k a_{ij},\ a'_{ij} \overset{(\alpha)}{|}_{k} = 2\ \dot{\omega}'_{(\alpha)}\ a_{ij},\ (\alpha = 1, 2),$$

*where* $\omega' = \omega + d\rho$.

Since in Theorem 3.1. $\omega' = 0$ is equivalent to $\omega = d(-\rho)$, we have:

**Theorem 3.2** *A conformal almost symplectic N-linear connection with respect to $\hat{a}$, corresponding to the 1-form $\omega$, $D\Gamma(\omega)$, is an almost symplectic N-linear connection with respect to some $a'_{ij} \in \hat{a}$, (i.e. $a'_{ij|k} = 0, a'_{ij} \overset{(\alpha)}{|}_{k} = 0$, $(\alpha = 1, 2)$), if and only if $\omega$ is exact.*

# 4 The set of all general conformal almost symplectic N-linear connections in the bundle of accelerations

Let $\overset{0}{N}$ and N be two nonlinear connections on $E = Osc^2M$, with the coefficients $(\overset{0}{N}_{(1)}{}^{i}{}_{j}, \overset{0}{N}_{(2)}{}^{i}{}_{j})$ and $(N_{(1)}{}^{i}{}_{j}, N_{(2)}{}^{i}{}_{j})$ respectively.

Let $D\overset{0}{\Gamma}(\overset{0}{N}) = (\overset{0}{L}{}^{i}_{jk}, \overset{0}{C}_{(1)jk}^{\ \ i}, \overset{0}{C}_{(2)jk}^{\ \ i})$, be the coefficients of an arbitrary fixed $\overset{0}{N}$-linear connection on $E$. Then any $N$-linear connection on E, with the coefficients $D\Gamma(N) = (L^{i}_{jk}, C_{(1)jk}^{\ \ i}, C_{(2)jk}^{\ \ i})$, can be expressed in the form (2.3), taking $D\Gamma(N)$ for $D\bar{\Gamma}(\bar{N})$ and $D\overset{0}{\Gamma}(\overset{0}{N})$ for $D\Gamma(N)$, where $(A_{(1)}{}^{i}{}_{j}$, $A_{(2)}{}^{i}{}_{j}, B^{i}_{jk}, D_{(1)jk}^{\ \ i}, D_{(2)jk}^{\ \ i})$ is the difference tensor fields of $D\overset{0}{\Gamma}(\overset{0}{N})$ to $D\Gamma(N)$.

In order that $D\Gamma(N)$ is a general conformal almost symplectic N-linear connection on E, that is (3.8) holds for $D\Gamma(N)$, it is necessary and sufficient that $B^{i}_{jk}, D_{(1)jk}^{\ \ i}, D_{(2)jk}^{\ \ i}$ satisfy:

$$
(4.1)\quad \begin{cases}
\Phi^{*ir}_{sj} B^{s}_{rk} = -\frac{1}{2} a^{im} [a_{mj\overset{0}{|}k} + A_{(1)}{}^{l}{}_{k} a_{mj} \overset{(1)}{\overset{0}{|}}_{l} \\
\qquad + (A_{(2)}{}^{l}{}_{k} + A_{(1)}{}^{r}{}_{k} \overset{0}{N}_{(1)}{}^{l}{}_{r}) a_{mj} \overset{(2)}{\overset{0}{|}}_{l} - K_{mjk}], \\
\Phi^{*ir}_{sj} D_{(1)rk}^{\ \ s} = -\frac{1}{2} a^{im} (a_{mj} \overset{(1)}{\overset{0}{|}}_{k} + A_{(1)}{}^{l}{}_{k} a_{mj} \overset{(1)}{\overset{0}{|}}_{l} - Q_{(1)mjk}), \\
\Phi^{*ir}_{sj} D_{(2)rk}^{\ \ s} = -\frac{1}{2} a^{im} (a_{mj} \overset{(1)}{\overset{0}{|}}_{k} - Q_{(2)mjk}),
\end{cases}
$$

where $\overset{0}{|}$ and $\overset{(\alpha)}{\overset{0}{|}}$, $(\alpha = 1, 2)$, denote the $h$-and $v_\alpha$-covariant derivatives,

$(\alpha = 1, 2)$, with respect to $D \overset{0}{\Gamma} (\overset{0}{N})$.

Thus, we have:

**Proposition 4.1** *Let $D \overset{0}{\Gamma} (\overset{0}{N})$ be a fixed $\overset{0}{N}$-linear connection on $E$. Then the set of all general conformal almost symplectic $N$-linear connections, $D\Gamma(N)$ is given by (2.3), where $B^{\ i}_{jk}$, $D_{(\alpha)}{}^{i}_{jk}$, $(\alpha = 1, 2)$, are arbitrary tensor fields satisfying (4.1). Especially, if $D \overset{0}{\Gamma} (\overset{0}{N})$ is a general conformal almost symplectic N-linear connection, then (4.1.) becomes:*

$$
(4.2) \begin{cases}
\Phi^{*ir}_{sj} B^{s}_{rk} = -\frac{1}{2} a^{im} [A_{(1)}{}^{l}_{k} a_{mj} \overset{(1)}{\underset{0}{|}}_{l} + (A_{(2)}{}^{l}_{k} + A_{(1)}{}^{r}_{k} \overset{0}{N}_{(1)}{}^{l}_{r}) a_{mj} \overset{(2)}{\underset{0}{|}}_{l}], \\
\Phi^{*ir}_{sj} D_{(1)}{}^{s}_{rk} = -\frac{1}{2} a^{im} A_{(1)}{}^{l}_{k} a_{mj} \overset{(1)}{\underset{0}{|}}_{l}, \\
\Phi^{*ir}_{sj} D_{(2)}{}^{s}_{rk} = 0,
\end{cases}
$$

From Theorem 5.4.3[4], however, the system (4.1) has solutions in $B^{\ i}_{jk}$, $D_{(\alpha)}{}^{i}_{jk}$,
$(\alpha = 1, 2)$. Substituting in (2.3) from the general solution we have:

**Theorem 4.1** *Let $D \overset{0}{\Gamma} (\overset{0}{N})$ be a fixed $\overset{0}{N}$-linear connection on $E$. The set of all general conformal almost symplectic $N$-linear connections $D\Gamma(N)$ is given by:*

$$(4.3)\begin{cases} L^{i}_{jk} = \overset{0}{L}{}^{i}_{jk} + X_{(1)}{}^{l}{}_{k} C_{(1)}{}^{i}_{jl} + X_{(1)}{}^{l}{}_{k} N_{(1)}{}^{m}{}_{l} C_{(2)}{}^{i}_{jm} + X_{(2)}{}^{l}{}_{k} C_{(2)}{}^{i}_{jl} + \\ \quad + \frac{1}{2} a^{im} [a_{mj\overset{0}{|}k} + X_{(1)}{}^{l}{}_{k} a_{mj} \overset{(1)}{\overset{0}{|}}{}_{l} + (X_{(2)}{}^{l}{}_{k} + X_{(1)}{}^{r}{}_{k} \overset{0}{N}_{(1)}{}^{l}{}_{r}) a_{mj} \overset{(2)}{\overset{0}{|}}{}_{l} - \\ \quad - K_{mjk}] + \Phi^{ir}_{sj} X^{s}_{rk}, \\ C_{(1)}{}^{i}_{jk} = \overset{0}{C}_{(1)}{}^{i}_{jk} + \overset{0}{C}_{(2)}{}^{i}_{jl} X_{(1)}{}^{l}{}_{k} + \frac{1}{2} a^{im} (a_{mj} \overset{(1)}{\overset{0}{|}}{}_{k} + X_{(1)}{}^{l}{}_{k} a_{mj} \overset{(2)}{\overset{0}{|}}{}_{l} - \\ \quad - Q_{(1)mjk}) + \Phi^{ir}_{sj} Y_{(1)}{}^{s}_{rk}, \\ C_{(2)}{}^{i}_{jk} = \overset{0}{C}_{(2)}{}^{i}_{jk} + \frac{1}{2} a^{im} (a_{mj} \overset{(2)}{\overset{0}{|}}{}_{k} - Q_{(2)mjk}) + \Phi^{ir}_{sj} Y_{(2)}{}^{s}_{rk}, \end{cases}$$

*where* $N_{(\alpha)}{}^{i}{}_{j} = \overset{0}{N}_{(\alpha)}{}^{i}{}_{j} - X_{(\alpha)}{}^{i}{}_{j}, X_{(\alpha)}{}^{i}{}_{j}, X^{i}_{jk}, \ Y_{(\alpha)}{}^{i}_{jk}, \ (\alpha = 1, 2)$ *are arbitrary tensor fields, and* $\overset{0}{|}, \ \overset{(\alpha)}{\overset{0}{|}}$, *denote the h-and* $v_\alpha$*-covariant derivatives,* $(\alpha = 1, 2)$, *with respect to* $D \overset{0}{\Gamma} (\overset{0}{N})$.

If we take a general conformal almost symplectic N-linear connection as $D \overset{0}{\Gamma} (\overset{0}{N})$, in Theorem 4.1, then (4.3) becomes:

$$(4.4)\begin{cases} L^{i}_{jk} = \overset{0}{L}{}^{i}_{jk} + X_{(1)}{}^{l}{}_{k} C_{(1)}{}^{i}_{jl} + X_{(1)}{}^{l}{}_{k} N_{(1)}{}^{m}{}_{l} C_{(2)}{}^{i}_{jm} + X_{(2)}{}^{l}{}_{k} C_{(2)}{}^{i}_{jl} + \\ \quad + \frac{1}{2} a^{im} [X_{(1)}{}^{l}{}_{k} Q_{(1)mjl} + (X_{(2)}{}^{l}{}_{k} + X_{(1)}{}^{r}{}_{k} \overset{0}{N}_{(1)}{}^{l}{}_{r}) Q_{(2)mjk}] + \Phi^{ir}_{sj} X^{s}_{rk}, \\ C_{(1)}{}^{i}_{jk} = \overset{0}{C}_{(1)}{}^{i}_{jk} + \overset{0}{C}_{(2)}{}^{i}_{jl} X_{(1)}{}^{l}{}_{k} + \frac{1}{2} a^{im} Q_{(2)mjl} X_{(1)}{}^{l}{}_{k} + \Phi^{ir}_{sj} Y_{(1)}{}^{s}_{rk}, \\ C_{(2)}{}^{i}_{jk} = \overset{0}{C}_{(2)}{}^{i}_{jk} + + \Phi^{ir}_{sj} Y_{(2)}{}^{s}_{rk}, \end{cases}$$

where $N_{(\alpha)}{}^{i}{}_{j} = \overset{0}{N}_{(\alpha)}{}^{i}{}_{j} - X_{(\alpha)}{}^{i}{}_{j}, X_{(\alpha)}{}^{i}{}_{j}, X^{i}_{jk}, \ Y_{(\alpha)}{}^{i}_{jk}, \ (\alpha = 1, 2)$ are arbitrary tensor fields and $\overset{0}{|}, \ \overset{(\alpha)}{\overset{0}{|}}$, denote the $h$-and $v_\alpha$-covariant derivatives, $(\alpha = 1, 2)$, with respect to $D \overset{0}{\Gamma} (\overset{0}{N})$.

**Observations 4.1.**

(i) If we consider ${X_{(\alpha)}}^{i}{}_{j} = X^{i}_{jk} = {Y_{(\alpha)}}^{i}{}_{jk} = 0,\ (\alpha = 1, 2)$, then from (4.3) we obtain the set of all general conformal almost symplectic N-linear connections, corresponding to the same nonlinear connection N, [13].

(ii) If we take $K_{ijk} = 2a_{ij}\tilde{\omega}_k, Q_{(\alpha)ijk} = 2a_{ij}\ \dot{\omega}_{(\alpha)k}, (\alpha = 1, 2)$, such that $\omega = \tilde{\omega}_i dx^i + \dot{\omega}_{(1)i}\ \delta y^{(1)i} + \dot{\omega}_{(2)i}\ \delta y^{(2)i}$ is a 1-form in $\mathcal{X}^*(Osc^2 M)$, and if we preserve the nonlinear connection N, $(i.e. N = \overset{0}{N})$, then from (4.3) we obtain the set of all conformal almost symplectic N-linear connections, corresponding to the same nonlinear connection N, [12].

(iii) If we consider $K_{ijk} = 0, Q_{(\alpha)ijk} = 0, (\alpha = 1, 2)$, and if we preserve the nonlinear connection N, $(i.e. N = \overset{0}{N})$, then from (4.3) we obtain the set of all almost symplectic N-linear connections, corresponding to the same nonlinear connection N, [11].

(iv) Finally, if we preserve the nonlinear connection N, $(i.e. N = \overset{0}{N})$ from (4.4), we obtain the transformations of general conformal almost symplectic N-linear connections, corresponding to the same nonlinear connection N, [13].

# References

[1] Atanasiu Gh., Ghinea I.*Connexions Finsleriennes Générales Presque Symplectiques*, An. Şti.Univ. "Al.I.Cuza" Iaşi, Secţ I a Mat. 25 (Supl.), 1979, 11-15.

[2] Matsumoto M.,*The theory of Finsler connections,*Publ.Study Group Geom.5, Depart.Math., Okayama Univ., 1970.

[3] Miron R.,*Introduction to the Theory of Finsler Spaces,*The Proc.of Nat.Sem. on Finsler Spaces, Braşov, 1980, 131-183.

[4] Miron R.,*The Geometry of Higher-order Lagrange spaces. Applications in Mechanics and Physics,*Kluwer Academic Publishers, FTPH82, 1997.

[5] Miron R. and Atanasiu Gh.,*Lagrange Geometry of Second Order, Math. Comput. Modelling,*Pergamon, 20, 4/5(1994), 41-56.

[6] Miron R. and Atanasiu Gh.,*Prolongation of Riemannian, Finslerian and Lagrangian structures,* Rev.Roumaine Math.Pures Appl., 41,3/4(1996), 237-249.

[7] Miron R. and Atanasiu Gh.,*Higher-order Lagrange spaces,* Rev. Roumaine Math.Pures Appl., 41,3/4(1996), 251-262.

[8] Miron R. and Hashiguchi M.,*Metrical Finsler Connections,* Fac.Sci., Kagoshima Univ.(Math., Phys.& Chem.), 12(1979), 21-35.

[9] Miron R. and Hashiguchi M., *Conformal Finsler Connections,* Rev.Roumaine Math.pures Appl., 26, 6(1981), 861-878.

[10] Purcaru M., Şofroniciu I., Păun M., *The Almost Symplectic Structures in the Bundle Space of the Accelerations,*Proc. of the $23^{rd}$ Conf. on Geometry and Topology, 1993,141-146.

[11] Purcaru M.,*Almost Symplectic N-Linear Connection in the Bundle of Accelerations,*Novi Sad J.Math., vol.29, no.3(1999), 281-289.

[12] Purcaru M., Stoica E., *Conformal Almost Symplectic N-Linear Connections in the Bundle of Accelerations,*Proc. of the Fourth International Workshop on Differential Geometry and Its Applications, Braşov, 16-22 sept. 1999, 236-242.

[13] Purcaru M., Stoica E., Târnoveanu M., *Nondegenerate Antisymmetrical Structures in the Bundle of Accelerations,*Bull.Malaysian Math.Sc.Soc. (Second Series), 25(2002), 149-156.

"Transilvania" University of Braşov, Department of Geometry, Iuliu Maniu 50, 2200 Braşov, Romania
*E-mail*: mpurcaru@unitbv.ro