General Optimized Lower and Upper Bounds for Discrete and Continuous Arithmetic Asian Options

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Abstract

We develop accurate analytical pricing formulae for discretely and continuously monitored arithmetic Asian options under general stochastic asset models, including exponential Lévy models, stochastic volatility models, and the constant elasticity of variance diffusion. The payoff of the arithmetic Asian option depends on the arithmetic average price of the underlying asset monitored over a pre-specified period. Unluckily, the pricing of arithmetic Asian options does not admit true analytical solutions, even under the lognormal model, as the distribution law of the arithmetic average is not known analytically. For more than two decades, much effort has been put into the research on efficient methodologies for computing the price of this option or, in general, expected values of functionals of the average value, under different model assumptions for the underlying. Developing such methods is of considerable practical importance as arithmetic averages see wide application in many fields of finance. The main objective of this research is to present a simple, accurate and fast pricing formula in the form of a lower bound for arithmetic Asian options allowing flexible modelling of the underlying asset price dynamics, filling this way an important long-standing gap in the literature.

Our proposed method is distinguished from other pricing methodologies for Asian options due to a number of appealing features. First, it can be applied flexibly to a wide range of non-Gaussian models, such as pure jump Lévy models, Merton's normal and Cai and Kou's generalized hyperexponential jump diffusions, models with/out jumps in the asset price/volatility dynamics, and the CEV diffusion, without restricting to models admitting time changed Brownian (Lévy) representations which may not be always common or straightforward to use. Second, we provide interesting theoretical findings related to the pricing of Asian options in the CEV

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diffusion model, an important asset price model which flexibly allows for different implied volatility shapes by varying the elasticity parameter and leverage effect, and which requires special treatment in our analysis due to its distinct distributional properties. Third, in the absence of symmetry relations between fixed and floating strike Asian options beyond the exponential Lévy asset price model (see Eberlein and Papapantoleon 2005), by a change of numéraire we are able to switch from fixed to floating strike option price results. Moreover, for first time in the literature, we provide a formulation which applies also to continuous Asian options under general model assumptions. The final line of research that we contribute to in this work is concerned with deriving a theoretical upper bound to the error made by our lower bound price approximation that can be calculated numerically.

Our extensive numerical experiments highlight the notable performance and robustness of our pricing formula for different test cases.

References

Eberlein, E., A. Papapantoleon. 2005. Equivalence of floating and fixed strike Asian and lookback options. *Stochastic Processes and their Applications* **115**(1) 31–40.