# General-rank Transmit Beamforming for Multi-group Multicasting Networks Using OSTBC

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***Abstract*—This paper addresses adaptive beamforming in multi-group multicasting networks where groups of users subscribe to independent services that are simultaneously served by the base station. Beamformers are designed to maximize the minimum signal-to-interference-plus-noise ratio (SINR) of the users in all groups subject to a total transmit power constraint. By combining multi-group multicast beamforming with Alamouti space-time block coding, the degrees of freedom in the beamformer design is doubled resulting in drastically improved beamforming performance. In our paper we extend recent approaches in [1] and [2] for rank-two beamforming, originally devised for single-group multicasting networks that are free of multi-user interference, to multi-group multicasting networks, where multi-user interference represents a major challenge. Simulation results demonstrate that the proposed approach significantly outperforms the existing approaches.**

***Index Terms*—Adaptive beamforming, multicasting, orthogonal space-time block coding (OSTBC), semi-definite relaxation (SDR), inner approximation**

## I. Introduction

In order to fulfill the growing demand for data rates and spectral efficiency in future wireless multimedia broadcast and multicast services [3], transmit beamforming techniques have been extensively studied in recent years [4], [5]. As compared to the single-group multicast transmit beamforming where a single group of users receives the same information symbols [6], [7], the spectral efficiency in multi-group multicast transmit beamforming can be further improved by serving several groups of cochannel users simultaneously [8]-[15].

The seminal work on multi-group multicast beamforming [8] deals with two quality-of-service (QoS) based problems: the problem of minimizing the total transmit power while satisfying the minimum SINR requirements of all receivers; and a max-min fair problem of maximizing the minimum SINR of all users in different groups subject to a total transmit power constraint. The latter problem is investigated in this paper. The QoS based beamforming problems are proven to be NP-hard and a semidefinite relaxation (SDR) based approach is developed in [8] to address the problems. Recently, an iterative inner approximation approach involving sequential convex optimization has been proposed in [15] to solve the latter problem more efficiently. As in the single-group multicasting case, when the number of users is large, the flexibility of designing spatially selective beamformers in the conventional adaptive beamforming approaches in [8] and [15] can be rather limited, and new techniques for improving the beamforming performance are of great practical importance.

In this paper, we apply the general-rank beamforming approach to solve the problem by combining multi-group multicast beamforming with orthogonal space-time block coding (OSTBC). Similar as in conventional beamforming we assume that the channel state information (CSI) of all users is available at the transmitter side. This approach follows the general idea of [1] and [2], originally proposed for single-group multicasting networks where multiuser interference is absent. As compared to the rank-two beamforming approaches in [1] and [2], we consider the multi-group network where multi-user interference is dominant. In this approach, transmit beamforming is used jointly with Alamouti OSTBC to serve the users [16], hence the users of each group are generally served with up to two beamformers over two consecutive time slots using the Alamout code. Due to the orthogonality of the code, the decoding complexity at the receivers is not increased and symbol by symbol detection can be performed. The use of two beamformers per group doubles the degrees of freedom in the beamformer design and offers improved beamforming performance. Interestingly our QoS based max-min fair beamforming design results in identical SDR formulations as in the conventional beamforming approach. However, unlike in the conventional approach where only rank-one solutions are optimal, here SDR solutions involving a rank smaller or equal to two are proven to be optimal for the original problem. Furthermore, following the approach of [15], in this paper we propose an iterative inner approximation technique for general rank-beamforming that is more computationally efficient as compared to the SDR based outer approximation technique. Simulation results show that the proposed approach significantly outperforms the existing approaches.

## II. Conventional Multi-group Multicasting

Consider a wireless communication system where a base station equipped with an antenna array of $N$ elements simultaneously transmits information to $M$ single-antenna users. We consider the case that there are $1 \leq G \leq M$ user groups in total, $\{g_1, \ldots, g_G\}$, where $g_k$ is the index set of the users intended to receive the multicasting stream for the $k$th group, and $k \in \mathcal{K}$ where $\mathcal{K}= \{1, \ldots, G\}$. We assume that each user belongs to only one group and decodes the corresponding single data stream. Thus, we have $g_k \cap g_l = \phi$ for any $l \neq k$, and $\cup_k g_k = \{1, \ldots, M\}$, treating the symbols of the remaining groups as noise.

In conventional multi-group multicast beamforming, a single weight vector is designed to transmit information intended for each group, thus there are $G$ beamformers in total [8]. Let us denote $\mathbf{w}_k^*$ and $s_k$ as the $N \times 1$ weight vector that is steered towards the $k$th group and the zero-mean mutually statistically independent signal with unit power intended for the $k$th group, respectively. The $N \times 1$ transmit signal vector is $\sum_{k=1}^{G} s_k \mathbf{w}_k^*$ and the total transmit power equals $\sum_{k=1}^{G} \|\mathbf{w}_k\|^2$. Then, the signal received by the $i$th user in the $k$th group is given by [8]

$$y_i = \underbrace{s_k \mathbf{w}_k^H \mathbf{h}_i}_{\text{signal}} + \underbrace{\sum_{l \neq k} s_l \mathbf{w}_l^H \mathbf{h}_i}_{\text{interference}} + \underbrace{n_i}_{\text{noise}} \tag{1}$$

where $\mathbf{h}_i$ and $n_i$ denote the $N \times 1$ downlink channel vector and the additive white receiver noise with variance $\sigma_i^2$ at the $i$th user in the $k$th group, respectively. Based on (1), the SINR of the $i$th user can be derived as $\frac{|\mathbf{w}_k^H \mathbf{h}_i|^2}{\sum_{l \neq k} |\mathbf{w}_l^H \mathbf{h}_i|^2 + \sigma_i^2}$. The problem of finding the weight vectors that maximize the minimum SINR of all users subject to the power constraint $\mathrm{P}_{\max}$ can be formulated as [8]

$$\begin{aligned} &\max_{\mathbf{w}_k, t} \; t \\ &\text{s.t.} \; \frac{|\mathbf{w}_k^H \mathbf{h}_i|^2}{\sum_{l \neq k} |\mathbf{w}_l^H \mathbf{h}_i|^2 + \sigma_i^2} \geq t, \; \forall i \in g_k \; \forall k, l \in \mathcal{K} \\ &\quad \sum_{k=1}^{G} \|\mathbf{w}_k\|^2 \leq \mathrm{P}_{\max}. \end{aligned} \tag{2}$$

Problem (2) is a quadratically constrained quadratic programming (QCQP) problem and it is NP-hard which inditates that in general the problem has no closed-form solution [8]. In [8], the SDR framework is employed to approximate problem (2) by a semidefinite programming (SDP) problem. The Gaussian randomization technique along with the power control involving linear programming in each randomization instance is then applied on the SDR solution to obtain rank-one beamforming solutions. As an alternative to the generic SDR technique, a computationally efficient iterative inner approximation technique has been proposed in [15], which in each iteration involves first order Taylor approximation of the originally non-convex constrains around the feasible solution obtained from the previous iteration.

## III. Proposed Approach

The central idea of the general-rank beamforming approach proposed in this work is to combine multi-group multicast beamforming with the concept of OSTBC based symbol transmission. In this contribution, we apply the Alamouti code which achieves full rate transmission. In correspondance to the $2 \times 2$ Alamouti code matrix that is applied at the transmitter, a pair of weight vectors instead of a single one is used to transmit the data streams to the designated multicasting groups over two consecutive time slots. We remark that the approach proposed in this paper can also be extended to combine with higher-order OSTBC, however, this extension is associated with a reduced transmission rate as full rate full diversity OSTBC only exist for two transmit antennas [17].

### A. Rank-two Beamforming Model

Denote $\mathbf{s}_k = [s_{k1}, s_{k2}]^T$ as the symbol vector for the $k$th group. In the Alamouti OSTBC, two symbols are transmitted within two time slots, and its code matrix for $\mathbf{s}_k$ is given by [16]

$$\mathcal{X}(\mathbf{s}_k) = \begin{bmatrix} s_{k1} & s_{k2} \\ -s_{k2}^* & s_{k1}^* \end{bmatrix}. \tag{3}$$

Unlike conventional Alamouti transmission schemes where the code matrix in (3) is transmitted from two transmit antennas over two consecutive time slots, here the code is transmitted from all $N$ transmit antennas at the base station using two different beamformers, i.e., $\mathbf{w}_{k1}$ and $\mathbf{w}_{k2}$, which form two virtual antennas over which the code is transmitted. Defining the beamforming matrix $\mathbf{W}_k = [\mathbf{w}_{k1}, \mathbf{w}_{k2}]$, the transmitted signal in each time block is given by $\sum_{k=1}^{G} \mathcal{X}(\mathbf{s}_k) \mathbf{W}_k^H$.

Assuming the block fading channel model, the received signal vector of the $i$th user in the $k$th group in two consecutive time slots of one transmission block is given by

$$\mathbf{y}_i = \underbrace{\mathcal{X}(\mathbf{s}_k) \mathbf{W}_k^H \mathbf{h}_i}_{\text{signal}} + \underbrace{\sum_{l \neq k} \mathcal{X}(\mathbf{s}_l) \mathbf{W}_l^H \mathbf{h}_i}_{\text{interference}} + \underbrace{\mathbf{n}_i}_{\text{noise}} \tag{4}$$

where $\mathbf{y}_i = [y_{i1}, y_{i2}]^T$, $\mathbf{n}_i = [n_{i1}, n_{i2}]^T$, and $y_{ij}$ and $n_{ij}$ denotes the received signal and the additive white noise of the $i$th user at the $j$th time slot, respectively. It is clear that (4) has a similar structure as (1). Using the equivalent channel representation for OSTBC [17], equation (4) can be equivalently written as

$$\tilde{\mathbf{y}}_i = \underbrace{\mathcal{X}(\mathbf{W}_k^H \mathbf{h}_i) \mathbf{s}_k}_{\text{signal}} + \underbrace{\sum_{l \neq k} \mathcal{X}(\mathbf{W}_l^H \mathbf{h}_i) \mathbf{s}_l}_{\text{interference}} + \underbrace{\tilde{\mathbf{n}}_i}_{\text{noise}} \tag{5}$$

where $\tilde{\mathbf{y}}_i = [y_{i1}, -y_{i2}^*]^T$ and $\tilde{\mathbf{n}}_i = [n_{i1}, -n_{i2}^*]^T$. As a common approach to decode the received symbols in OSTBC, the Maximum-Likelihood decoding problem for detecting the symbols of the $i$th user can be formulated as

$$\begin{aligned} &\min_{\mathbf{s}_k \in \mathcal{A}_k} \left\| \tilde{\mathbf{y}}_i - \mathcal{X}(\mathbf{W}_k^H \mathbf{h}_i) \mathbf{s}_k \right\|^2 \\ &= \min_{\mathbf{s}_k \in \mathcal{A}_k} \left\| \frac{1}{\alpha_i} \mathcal{X}(\mathbf{W}_k^H \mathbf{h}_i)^H \tilde{\mathbf{y}}_i - \mathbf{s}_k \right\|^2 \end{aligned} \tag{6}$$

where $\alpha_i = \mathbf{h}_i^H \mathbf{W}_k \mathbf{W}_k^H \mathbf{h}_i$, and $\mathcal{A}_k$ is the vector constellation of $\mathbf{s}_k$. By left-multiplying the decoding matrix $\frac{\mathcal{X}(\mathbf{W}_k^H \mathbf{h}_i)^H}{\alpha_i}$ on both sides of (5), we have

$$\begin{aligned} \hat{\mathbf{s}}_k &= \frac{1}{\alpha_i} \mathcal{X}(\mathbf{W}_k^H \mathbf{h}_i)^H \tilde{\mathbf{y}}_i \\ &= \mathbf{s}_k + \underbrace{\frac{1}{\alpha_i} \mathcal{X}(\mathbf{W}_k^H \mathbf{h}_i)^H \Big(\sum_{l \neq k} \mathcal{X}(\mathbf{W}_l^H \mathbf{h}_i) \mathbf{s}_l\Big)}_{\hat{\mathbf{s}}_k^{(I)}} + \underbrace{\frac{1}{\alpha_i} \mathcal{X}(\mathbf{W}_k^H \mathbf{h}_i)^H \tilde{\mathbf{n}}_i}_{\hat{\mathbf{s}}_k^{(N)}}. \end{aligned} \tag{7}$$

Taking into account that the symbols in $\mathbf{s}_k$ are assumed to be statistically independent and making use of the orthogonality property of $\mathcal{X}(\mathbf{W}_k^H\mathbf{h}_i)$, we have

$$E\{\hat{\mathbf{s}}_k\hat{\mathbf{s}}_k^H\} = \mathbf{I} + \frac{\sum_{l\neq k}\mathbf{h}_i^H\mathbf{W}_l\mathbf{W}_l^H\mathbf{h}_i}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I} + \frac{\sigma_i^2}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I}$$
$$= \frac{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i + \sum_{l\neq k}\mathbf{h}_i^H\mathbf{W}_l\mathbf{W}_l^H\mathbf{h}_i + \sigma_i^2}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I}. \quad (8)$$

where $E\{\cdot\}$ denotes the statistical expectation and $\mathbf{I}$ is the $2\times 2$ identity matrix. From the diagonal structure of (8), we observe that there exists no inter symbol interference, thus $\mathbf{s}_k$ can be decoded using a simple linear symbol-wise decoder. According to (7), the covariance of the desired signal at the $i$th user is

$$E\{\mathbf{s}_k\mathbf{s}_k^H\} = \mathbf{I}, \quad (9)$$

the covariance of the interference at the $i$th user is

$$E\{\hat{\mathbf{s}}_k^{(I)}\hat{\mathbf{s}}_k^{(I)H}\} = \frac{\sum_{l\neq k}\mathbf{h}_i^H\mathbf{W}_l\mathbf{W}_l^H\mathbf{h}_i}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I}, \quad (10)$$

and the covariance of the noise at the $i$th user is

$$E\{\hat{\mathbf{s}}_k^{(N)}\hat{\mathbf{s}}_k^{(N)H}\} = \frac{\sigma_i^2}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I}. \quad (11)$$

Based on (9), (10) and (11), the SINR of the $i$th user can be written as

$$\mathrm{SINR}_i = \frac{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}{\sum_{l\neq k}\mathbf{h}_i^H\mathbf{W}_l\mathbf{W}_l^H\mathbf{h}_i + \sigma_i^2}. \quad (12)$$

The total transmit power in the $j$th time slot in each block is given by

$$\mathrm{P}_j = E\{(\sum_{k=1}^{G}[\mathcal{X}(\mathbf{s}_k)]_j\mathbf{W}_k^H)(\sum_{k=1}^{G}[\mathcal{X}(\mathbf{s}_k)]_j\mathbf{W}_k^H)^H)\}$$
$$= \sum_{k=1}^{G}\mathrm{tr}(\mathbf{W}_k\mathbf{W}_k^H) \quad (13)$$

where $[\mathcal{X}(\mathbf{s}_k)]_j$ denotes the $j$th row of the code matrix $\mathcal{X}(\mathbf{s}_k)$ in (3), $\mathrm{tr}\{\cdot\}$ denotes the trace of a matrix, and where we make use of the statistical independence of the transmitted symbols among users and the orthogonality of the code matrix.

### *B. Beamformer Optimization*

We consider a QoS based max-min fair beamforming approach in which the minimum SINR of all users is maximized subject to the constraint of total transmit power per time slot [8]. We remark that it is fair to consider the constraint of the total transmit power per time slot here because the power constraint in (2) in the conventional problem is the power per time slot as well. Using (12) and (13), the beamforming optimization problem can be presented as

$$\max_{\mathbf{W}_k, t}\ t$$
$$\text{s.t.}\ \frac{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}{\sum_{l\neq k}\mathbf{h}_i^H\mathbf{W}_l\mathbf{W}_l^H\mathbf{h}_i + \sigma_i^2} \geq t,\ \forall i \in g_k\ \forall k, l \in \mathcal{K}$$
$$\sum_{k=1}^{G}\mathrm{tr}(\mathbf{W}_k\mathbf{W}_k^H) \leq \mathrm{P}_{\max}. \quad (14)$$

Following the SDR approach, let

$$\mathbf{X}_k = \mathbf{W}_k\mathbf{W}_k^H = \sum_{j=1}^{2}\mathbf{w}_{kj}\mathbf{w}_{kj}^H, \forall k \in \mathcal{K}. \quad (15)$$

Then problem (14) can be rewritten as

$$\max_{\mathbf{X}_k, t}\ t$$
$$\text{s.t.}\ \frac{\mathbf{h}_i^H\mathbf{X}_k\mathbf{h}_i}{\sum_{l\neq k}\mathbf{h}_i^H\mathbf{X}_l\mathbf{h}_i + \sigma_i^2} \geq t,\ \forall i \in g_k\ \forall k, l \in \mathcal{K}$$
$$\sum_{k=1}^{G}\mathrm{tr}(\mathbf{X}_k) \leq \mathrm{P}_{\max},$$
$$\mathbf{X}_k \succeq 0,\ \mathrm{rank}\{\mathbf{X}_k\} \leq 2,\ \forall k \in \mathcal{K}. \quad (16)$$

where $\mathbf{X}_k \succeq 0$ constrains $\mathbf{X}_k$ to lie in the set of positive semidefinite Hermitian matrices. Substituting the rank-one matrix $\mathbf{X}_k = \mathbf{w}_k\mathbf{w}_k^H$ in the conventional beamforming problem (2) and comparing the resulting problem with (16), we observe that both problems are identical up to the non-convex rank constraints, i.e., the rank-one constraints in the reformulation of (2) and the rank-two constraint in (16). As the set of rank-two matrices includes the set of rank-one matrices, we observe that the general-rank beamforming solutions of (16) generally yield improved QoS as compared to the rank-one beamforming solutions of (2). It follows from the discussion above that the SDR technique applied to both (2) and (16) results in the same convex optimization problem given by

$$\max_{\mathbf{X}_k, t}\ t$$
$$\text{s.t.}\ \frac{\mathbf{h}_i^H\mathbf{X}_k\mathbf{h}_i}{\sum_{l\neq k}\mathbf{h}_i^H\mathbf{X}_l\mathbf{h}_i + \sigma_i^2} \geq t,\ \forall i \in g_k\ \forall k, l \in \mathcal{K}$$
$$\sum_{k=1}^{G}\mathrm{tr}(\mathbf{X}_k) \leq \mathrm{P}_{\max}, \mathbf{X}_k \succeq 0,\ \forall k \in \mathcal{K} \quad (17)$$

which can be solved efficiently by performing a one-dimensional bisection search over $t$ as in [8] with the aid of currently available convex optimization tools such as `CVX` [18]. The computational complexity of the semidefinite programming procedure is $\mathcal{O}(G^3N^6 + MGN^2)$ in each bisection search step, which is the same as in the conventional SDR approach in [8]. We remark that due to the difference in the rank constraints, generally the SDR of (16) is tighter than that of (2).

Denote $\{\mathbf{X}_k^\star\}_{k=1}^G$ as the optimal solution to (17), the optimal value associated with it can serve as the upper bound to the original problem (14) and it is used to evaluate the approximation quality of the proposed approach as shown in the simulation. When $\mathrm{rank}(\mathbf{X}_k^\star)\leq 2, \forall k$, the optimal weight vector solutions to the problem (14) can be obtained by computing the principal components of $\{\mathbf{X}_k^\star\}_{k=1}^G$ straightforwardly. However, if there exists at least one $\mathbf{X}_k^\star$ with $\mathrm{rank}(\mathbf{X}_k^\star)>2$, and if $M\leq G+7$, rank reduction techniques proposed in [19] can be applied to obtain optimal rank-two solutions to the problem (14). The Proof of this statement is omitted here for brevity. However, the proof follows the line of arguments in [2] where similar conditions have been derived for the single-group multicasting problem. Conversely, if $M>G+7$, a modified randomization technique is proposed in this paper for the rank-two case to compute the final solutions, which follows the general procedure of the randomization techniques proposed in [8] and [1].

Let us denote $\mathbf{w}_{k1}^{(r)}$ and $\mathbf{w}_{k2}^{(r)}$ as the candidate weight vectors for $\mathbf{w}_{k1}$ and $\mathbf{w}_{k2}$ in the $r$-th randomization instance, respectively. If $\mathrm{rank}(\mathbf{X}_k^\star)\leq 2$, $\mathbf{w}_{k1}^{(r)}$ and $\mathbf{w}_{k2}^{(r)}$ are computed as the principal components of $\mathbf{X}_k^\star$; conversely, if $\mathrm{rank}(\mathbf{X}_k^\star)>2$, we first perform an eigen-decomposition on $\mathbf{X}_k^\star$ as $\mathbf{X}_k^\star = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{U}_k^H$, then choose $\mathbf{w}_{k1}^{(r)} = \mathbf{U}_k\mathbf{\Sigma}_k^{1/2}\mathbf{e}_r$, and $\mathbf{w}_{k2}^{(r)} = \mathbf{U}_k\mathbf{\Sigma}_k^{1/2}\mathbf{f}_r$ where $\mathbf{e}_r$ and $\mathbf{f}_r$ are $N\times 1$ vectors containing the realizations of i.i.d. complex circular Gaussian distributed random variables with zero mean and unit variance. Then the global power control procedure over all groups involving bisection search and linear programming is performed to compute a candidate set of weight vector solutions. Let $p_{k1}$ and $p_{k2}$ denote the power scaling factors corresponding to $\mathbf{w}_{k1}^{(r)}$ and $\mathbf{w}_{k2}^{(r)}$, respectively. The power control problem can be stated as

$$\begin{aligned}\max_{p_{kj}\geq 0,t}\ & t\\ \text{s.t.}\ & \frac{p_{k1}\beta_{k1i}^{(r)} + p_{k2}\beta_{k2i}^{(r)}}{\sum_{l\neq k}(p_{l1}\beta_{l1i}^{(r)} + p_{l2}\beta_{l2i}^{(r)}) + \sigma_i^2} \geq t, \forall i\in g_k \forall k,l\in\mathcal{K}\\ & \sum_{k=1}^{G}(p_{k1}\alpha_{k1}^{(r)} + p_{k2}\alpha_{k2}^{(r)}) \leq \mathrm{P}_{\max},\ \forall\ j=1,2 \end{aligned} \quad (18)$$

where $\alpha_{kj}^{(r)} := \left\|\mathbf{w}_{kj}^{(r)}\right\|^2$ and $\beta_{kji}^{(r)} := \left|\mathbf{w}_{kj}^{(r)H}\mathbf{h}_i\right|^2$, $\forall j = 1,2$. Among all sets of candidate solutions obtained, the set with the largest SINR value is selected as the final solution.

As an alternative to the SDR based approach, we propose a computationally more efficient approach to obtain approximate solutions iteratively by performing sequential convex optimization, similar as in [15]. Towards this aim, let us consider problem (14), which can be written as

$$\begin{aligned}\max_{\mathbf{w}_{kj},t}\ & t\\ \text{s.t.}\ & -\left|\mathbf{w}_{k1}^H\mathbf{h}_i\right|^2 - \left|\mathbf{w}_{k2}^H\mathbf{h}_i\right|^2 + t\sum_{l\neq k}(\left|\mathbf{w}_{l1}^H\mathbf{h}_i\right|^2 + \left|\mathbf{w}_{l2}^H\mathbf{h}_i\right|^2)\\ & + t\sigma_i^2 \leq 0,\ \forall i\in g_k\ \forall k,l\in\mathcal{K}\\ & \sum_{k=1}^{G}(\|\mathbf{w}_{k1}\|^2 + \|\mathbf{w}_{k2}\|^2) \leq \mathrm{P}_{\max}. \end{aligned} \quad (19)$$

In order to solve the non-convex problem in (19), the general idea is to introduce an iterative procedure in which in the $(p+1)$-th iteration, $\mathbf{w}_{kj}$ and $t$ are replaced by $\mathbf{w}_{kj}^{(p)} + \Delta\mathbf{w}_{kj}$ and $t^{(p)}+\Delta t$, $\forall k\in\mathcal{K}$,$\forall j\in\{1,2\}$, where $\mathbf{w}_{kj}^{(p)}$ and $t^{(p)}$ are the beamforming weight vector and the SINR level obtained from the $p$-th iteration, respectively. By neglecting the non-convex terms $-(\left|\Delta\mathbf{w}_{k1}^H\mathbf{h}_i\right|^2 + \left|\Delta\mathbf{w}_{k2}^H\mathbf{h}_i\right|^2)$ and $\Delta t\sum_{l\neq k}(\left|\Delta\mathbf{w}_{l1}^H\mathbf{h}_i\right|^2 + \left|\Delta\mathbf{w}_{l2}^H\mathbf{h}_i\right|^2 - 2\Re\{\Delta\mathbf{w}_{l1}^H\mathbf{h}_i\mathbf{h}_i^H\mathbf{w}_{l1}^{(p)} + \Delta\mathbf{w}_{l2}^H\mathbf{h}_i\mathbf{h}_i^H\mathbf{w}_{l2}^{(p)}\})$ in the SINR constraint in (19), the problem in the $(p+1)$-th iteration can be approximated as the following convex problem

$$\begin{aligned}\max_{\Delta\mathbf{w}_{kj},\Delta t}\ & \Delta t\\ \text{s.t.}\ & -\left|\mathbf{w}_{k1}^{(p)H}\mathbf{h}_i\right|^2 - \left|\mathbf{w}_{k2}^{(p)H}\mathbf{h}_i\right|^2 + t^{(p)}\sigma_i^2\\ & + \Delta t\sum_{l\neq k}(\left|\mathbf{w}_{l1}^{(p)H}\mathbf{h}_i\right|^2 + \left|\mathbf{w}_{l2}^{(p)H}\mathbf{h}_i\right|^2) + \Delta t\sigma_i^2\\ & + t^{(p)}\sum_{l\neq k}(\left|(\mathbf{w}_{l1}^{(p)}+\Delta\mathbf{w}_{l1})^H\mathbf{h}_i\right|^2 + \left|(\mathbf{w}_{l2}^{(p)}+\Delta\mathbf{w}_{l2})^H\mathbf{h}_i\right|^2)\\ & \leq 0, \forall i\in g_k\ \forall k,l\in\mathcal{K}\\ & \sum_{k=1}^{G}(\left\|\mathbf{w}_{k1}^{(p)}+\Delta\mathbf{w}_{k2})\right\|^2 + \left\|\mathbf{w}_{k2}^{(p)}+\Delta\mathbf{w}_{k2})\right\|^2) \leq \mathrm{P}_{\max}. \end{aligned} \quad (20)$$

Problem (20) can be classified as an inner convex approximation problem. Following from the inner approximation property, this iterative procedure results in a sequence of non-decreasing minimum SINR values. The proposed iterative approximation scheme is initialized with randomly generated weight vectors. With the increase of the iteration $p$, as soon as the increment of the obtained SINR between two consecutive iterations is below a certain threshold, i.e., $t^{(p+1)} - t^{(p)} < \epsilon$, we terminate the iteration. We remark that it can be proven that the iteration algorithm is always feasible and convergent, but not necessarily to an optimal point [15]. The complexity of the rank-two inner approximation procedure is $\mathcal{O}((M+1)^{1/2}(M+2GN+2)(2GN+1)^2)$, while in the rank-one case as shown in [15] the complexity is $\mathcal{O}((M+1)^{1/2}(M+GN+2)(GN+1)^2)$.

## IV. Simulation Results

We assume Rayleigh fading channels with i.i.d. channel coefficients of unit-variance. The noise variance $\sigma_i^2 = 1$ for all $i = 1,\ldots,M$. We consider the case that $N = 4$, $G = 2$ and $M = 30$ with 15 users in each group. Gray-coded QPSK are transmitted to each group of users.

In our simulation example, we compare the proposed rank-two beamforming approaches with the state-of-the-art approach proposed in [15]. In Fig. 1, the worst SINR among all users for different prescribed transmit powers is displayed. All results are averaged over 500 Monto-Carlo runs. Five curves are depicted in Fig. 1, where the curve labeled 'SDR upper bound' stands for the upper bound on the SINR provided by the SDR solutions, 'Method of [15]' refers to the inner convex approximation approach for the rank-one beamforming problem with random initialization as proposed in [15], 'Method of [15] with SDR' stands for the rank-one beamforming approach in which SDR is employed in the initialization step and the inner approximation method in [15] is applied only if optimal rank-one solutions are not obtained in the initialization step, 'Proposed (SDR+Randomization)' refers to the proposed SDR based rank-two beamforming approach with 100 randomization instances in each run, and 'Proposed (Inner approx.)' stands for the proposed rank-two beamforming approach with iterative inner approximation. We set the threshold value for iteration termination to $\epsilon = 10^{-4}$. As shown in Fig. 1, 'Proposed (Inner approx.)' achieves slightly improved performance as compared to 'Proposed (SDR+Randomization)', and both curves are very close to 'SDR upper bound' and achieve better performance than all the rank-one approaches. This result can further be observed from Fig. 2 in which the histogram of the obtained rank of the solution $\{\mathbf{X}_k^\star\}_{k=1}^G$ of problem (17) is displayed versus the total transmit power. As shown in Fig. 2, rank-two solutions are obtained in most of the considered transmit power values. 'Proposed (SDR+Randomization)' can achieve optimal solutions for both rank-one and rank-two cases, and 'Proposed (Inner approx.)' performs well due to its rank-two approximation. Fig. 3 compares the convergence rates of 'Proposed (Inner approx.)' and 'Method of [15]' when $\mathrm{P}_{\max} = 10$dB. We observe that both rates are almost the same, but 'Proposed (Inner approx.)' achieves a better SINR value after the first iteration as compared to 'Method of [15]'.

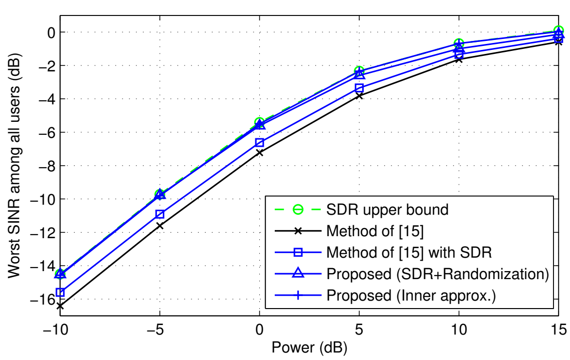


Fig. 1. Worst SINR vs. total transmit power.

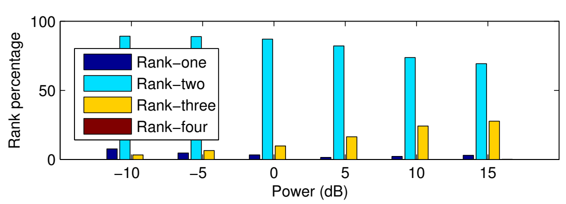


Fig. 2. Rank percentage of $\{\mathbf{X}_k^\star\}_{k=1}^G$ vs. total transmit power.

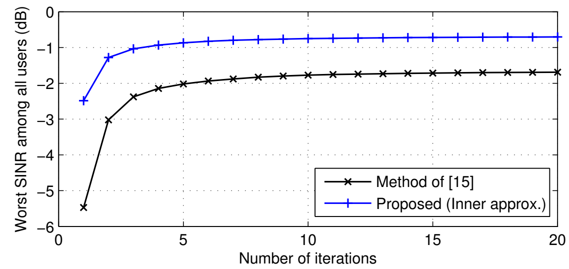


Fig. 3. Convergence rates of iterative inner approximations.

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