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General Solution of the Problem of Hydrostatic Equilibrium of the Earth

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Summary

If de Sitter's hydrostatic equations are developed independent of the external potential theory, the hydrostatic geopotential coefficient J_h occurs explicitly on the right-hand side of these equations. Since this J_h has to be treated as an unknown in the solution of the problem, it becomes rather difficult to solve these hydrostatic equations independently, regardless of which of the dynamical parameters associated with the Earth is taken as the initial datum. The solution of these equations is possible, however, with the help of a boundary condition derived from the external potential theory which neither assumes nor discounts the presence of equilibrium conditions in the Earth's interior. If a general solution is constructed on these lines, the three particular solutions, usually quoted in literature, stem from it in the wake of the appropriate assumptions. Of course, out of these the only meaningful solution is that corresponding to the polar moment of inertia as the initial datum. It is essential that the solution be constructed in this way in order to demonstrate clearly the correct structure of the problem of hydrostatic equilibrium.

The anomalous gravity field of the Earth referred to the hydrostatic figure is compared with that referred to the international reference ellipsoid.

Introduction

In a previous paper (Khan 1968) I modified de Sitter's (1924) equations of the classical hydrostatic theory to make their development independent of the external potential theory. It was shown that if these modified equations are solved all by themselves, as advocated by some investigators, the quantity J_h which appears explicitly on the right-hand side of these equations is equated with the satellite-determined non-hydrostatic J and the solution becomes of rather doubtful geophysical significance. If the J_h is to be treated as an unknown, as it should be, it becomes rather difficult to solve these equations for f_h, no matter what parameter (i.e., polar or mean moment of inertia, dynamical flattening H or satellite-determined J) is defined as the initial datum, and one has to look for some additional boundary condition. Now, if hydrostatic equilibrium exists, the actual figure of the earth should be coincident (Caputo 1965) with the equilibrium figure predicted for it from its rate of rotation. This is the definition of the hydrostatic figure and hence con-

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stitutes a natural boundary condition for its solution. In this paper we present a general solution based on de Sitter's development which makes use of this boundary condition and discuss the various possible particular solutions which can be obtained from it. The procedure outlined here is really a restatement of the method of de Sitter (1924) who did indeed employ a relation obtained from the external potential theory to eliminate J_h from his equations (Khan 1968). However, since this elimination was made at an early stage in the development of his equations, it could possibly remain undetected and lead to a confusion of the correct structure of the method, as pointed out earlier by Khan (1968). We will demonstrate the correct structure of the problem of hydrostatic equilibrium clearly and show that the three frequently quoted hydrostatic solutions, namely the hydrostatic flattening f_h corresponding to (1) the polar or mean moment of inertia, (2) the dynamical flattening H, and (3) the difference between the polar and equatorial moment of inertia can be obtained from it simply by defining the appropriate initial datums. Of course, the most meaningful solution from the geophysical point of view is still the one in which the polar or the mean moment of inerita of the hydrostatic model is taken equal to that determined for the real earth (O'Keefe 1960; Henriksen 1960; Jeffreys 1963; Khan 1967). This hydrostatic model is used as a reference to estimate the minimum strength in the Earth's interior required to support the stresses arising from the departure of the real Earth from hydrostatic equilibrium and also, to compute the Earth's anomalous gravity field since such a field would reflect the long wavelength hydrostatic stresses which may exist in the Earth's crust and mantle.

Modified hydrostatic equations

The modified equations are given as

$$q = 1 - \tfrac{2}{3}f_h + \tfrac{2}{3}J_h + \tfrac{4}{9}f_h^2 - \tfrac{2}{5}(1 - \tfrac{2}{3}f_h + \tfrac{4}{9}f_h^2)\frac{\sqrt{(1+\eta_s)}}{1+\lambda_s} \tag{1}$$

and

$$\eta_s f' = 3f' - \tfrac{6}{7}f_h^2 + \tfrac{4}{7}mf_h - J_h(5 + \tfrac{10}{21}f_h + \tfrac{20}{21}m) \tag{2}$$

where

f_h = flattening the Earth would have if it were in hydrostatic equilibrium

$$f' = f_h - \tfrac{5}{42}f_h^2$$

$$q = \tfrac{3}{2}\frac{C}{Ma_e^2} = \frac{J}{H}$$

C = moment of inertia of the Earth about the polar axis

M = mass of the Earth

a_e = equatorial radius of the Earth

$$m = \frac{\omega^2 r_m^3}{GM}$$

ω = angular velocity of the Earth

r_m = the radius of the Earth at which $\sin^2\phi = \tfrac{1}{3}$, where ϕ is latitude

η_s = surface value of the parameter η which depends upon the internal density distribution of the Earth

λ_s = surface value of the parameter λ which is defined as the departure from unity of the average value of a function $F(\eta)$ (occurring in the hydrostatic theory) over the range of integration

With $\lambda_s = 0$, one can eliminate η_s from equations (1) and (2) and get (Khan 1968) an explicit expression for f_h as

$$f_h = \frac{1}{3-\eta_0}\left[(5J_h+\delta_2) + \frac{(5J_h+\delta_2)\,\delta_1}{3-\eta_0} + \frac{25\,A\,J_h^2}{(3-\eta_0)^2}\right] \tag{3}$$

where

$$\left.\begin{aligned} \eta_0 &= \tfrac{25}{4}F^2 q'^2 - 1 \\ q' &= 1-q \\ F &= 1+\lambda_s \\ A &= \tfrac{17}{14} - \tfrac{5}{42}\eta_0 - \tfrac{25}{3}F^2 q q' \\ \delta_1 &= \tfrac{25}{3}F^2 q' J_h - \tfrac{4}{7}m + \tfrac{10}{21}J_h \\ \text{and}\quad \delta_2 &= \tfrac{20}{21}m\,J_h \end{aligned}\right\} \tag{4}$$

The quantity δ_1 is approximately of the order of f_h and δ_2 is of the order of f_h^2.

Considering the potential in free space of an ellipsoid of revolution which has both polar and equatorial symmetry, we can obtain the following equation:

$$J = f - \tfrac{1}{2}m - \tfrac{1}{2}f^2 + \tfrac{1}{7}mf. \tag{5}$$

From this equation one can easily obtain an expression for f, i.e.,

$$f = J + \tfrac{1}{2}m + \tfrac{5}{14}Jm + \tfrac{1}{2}J^2 + \tfrac{3}{56}m^2. \tag{6}$$

Equations (5) and (6) are derived from the external potential theory with no assumptions whatsoever as to the conditions existing in the Earth's interior (Caputo 1965). Hence, they should be valid irrespective of whether or not hydrostatic equilibrium conditions exist inside the Earth. Thus equation (5) or (6) should relate the observed J with real flattening in a non-hydrostatic case and hydrostatic J with hydrostatic flattening in a hydrostatic case. For convenience of discussion, let us write equation (5) or (6) as

$$G(J,f) = 0 \tag{7}$$

for a non-hydrostatic case and as

$$G(J_h,f_h) = 0 \tag{8}$$

for a hydrostatic case.

In case of an Earth in hydrostatic equilibrium, equations (3) and (8) must be satisfied simultaneously. This is equivalent to saying that if we can determine J_h, f_h obtained from equation (3) or (8) should be identical.

General solution of the problem of hydrostatic equilibrium

To construct the general solutions, we have to examine the hydrostatic equation (3) which can be written as

$$F(m, H, J_h, f_h) = 0. \tag{3a}$$

In terms of the basic parameters, the above equation can be written as

$$F(C, A, f_h, \omega, a, M) = 0. \tag{3b}$$

If the hydrostatic equilibrium exists, the figure of the Earth predicted from the external potential theory should be coincident with the hydrostatic figure. This gives us an important boundary condition of the problem, i.e., equations (3) and (5) must match at the outer boundary of the Earth. Hence,

$$F(m, H, J_h, f_h) = G(J_h, f_h)$$

or more specifically,

$$\frac{1}{3-\eta_0}\left[(5J_h+\delta_2)+\frac{(5J_h+\delta_2)\delta_1}{3-\eta_0}+\frac{25AJ_h^2}{(3-\eta_0)^2}\right] = \tfrac{1}{2}J_h^2+\alpha_1 J_h+\alpha_2 \tag{9}$$

where

$$\alpha_1 = 1+\tfrac{5}{14}m$$

$$\alpha_2 = \tfrac{1}{2}m+\tfrac{3}{56}m^2$$

and the other quantities appearing in equation (9) are defined in equation (4).

Equation (9) gives the general solution of the problem of hydrostatic equilibrium. The particular solutions are obtained by examining the left-hand side of equation (9) in the form of equation (3a) or (3b) and by properly defining the basic parameters occurring in that equation. If the rate of rotation ω and the mass M are chosen to be the same for the hydrostatic Earth and the real Earth, there are three possible solutions which correspond to the following boundary conditions

$$\left.\begin{aligned}(\omega, M, a, C) &= \text{constant}\\(\omega, M, a, H) &= \text{constant}\\(\omega, M, a, J) &= \text{constant}\end{aligned}\right\} \tag{10}$$

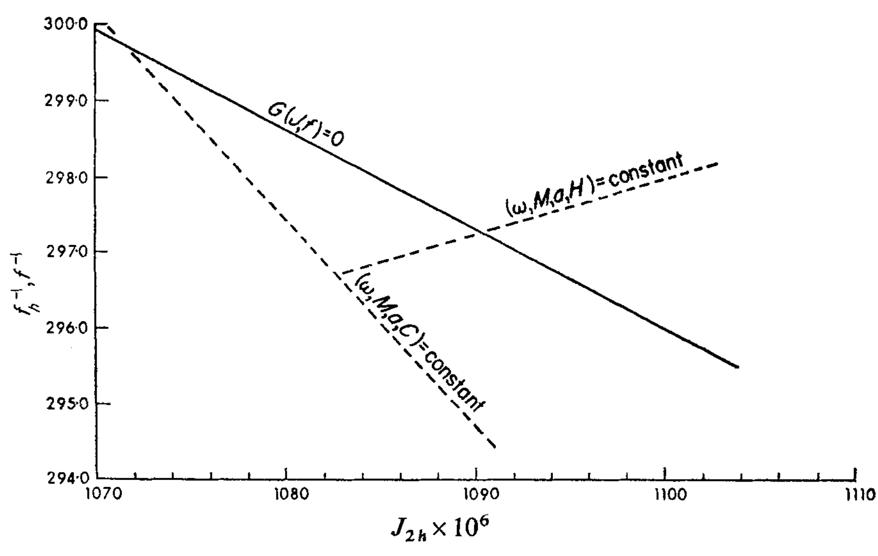

FIG. 1. Graphical solution of equation (9) for $\dfrac{(\omega, M, a, H)}{(\omega, M, a, C)}$ = constant.

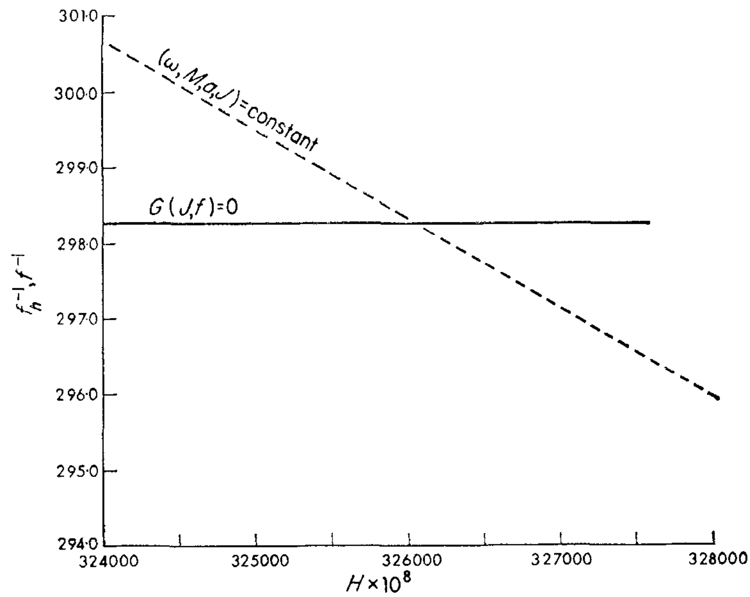

FIG. 2. Graphical solution of equation (9) for (ω, M, a, J) = constant.

Graphical solutions of the general hydrostatic equation (9) are given in Fig. 1 for the first two cases and in Fig. 2 for the last case. The plots of the function $G(J_h, f_h)$ are shown by solid curves while those of $F(m, H, J_h, f_h)$ subject to the set of boundary conditions mentioned in equation (10) are shown by broken-line curves. It is obvious from the shape of these plots that each set has an unique intersection point in the region in which we are interested. Note that in Fig. 1 the abcissa is scaled in terms of J_{2h} where $J_{2h} = \frac{2}{3}J_h$. In all the solutions, λ_s has been taken equal to zero. It was shown previously (Khan 1968) that reasonable variations in values of λ_s do not affect the solution critically. The rate of rotation ω is treated constant via the parameter m in all the solutions given in Figs 1 and 2.

For the case when $(\omega, M, a, C) =$ constant, the hydrostatic flattening is $f_h^{-1} = 299{\cdot}75 \pm 0{\cdot}05$ and the hydrostatic J is $J_h = 1607{\cdot}49 \times 10^{-6}$. This value is about the same as reported by Henriksen (1960), O'Keefe (1960) and in a previous paper by me (Khan 1967). These investigators used de Sitter's hydrostatic equations *per se*. The results obtained from the two procedures are identical because, as pointed out before, de Sitter's specific elimination of J_h (with the help of a relation obtained from the external potential theory) from the hydrostatic equations in order to obtain his equations for hydrostatic theory is, in fact, equivalent to the procedure given in this paper. However, since this elimination is hidden in the development of de Sitter's second order hydrostatic theory (Khan 1968) it can lead to the erroneous impression that, in de Sitter's second order theory, the value of hydrostatic flattening can be obtained from a solution of equation (3), (3a) or (3b) alone. Hence, it is more instructive to state it explicitly. For this model, the hydrostatic flattening is *smaller* than the real flattening.

Ther merit of this solution lies in the fact that the polar moment of inertia of the hydrostatic Earth is equal to that of the real Earth, as obtained from observational data on the geopotential coefficient J and the dynamical flattening H. Consequently, it is possible to avoid certain dynamical complications which arise when the moment of inertia of the hydrostatic model is taken different from that of the real Earth.

For the case when (ω, M, a, H) = constant, the hydrostatic flattening is $f_h = 297{\cdot}29 \pm 0{\cdot}05$ and the corresponding hydrostatic J is $J_h = 1635{\cdot}225 \times 10^{-6}$. This value of the hydrostatic flattening is very near the flattening obtained in pre-satellite times. The pre-satellite method essentially consisted of predicting J from the hydrostatic theory and using it in equation (5) to compute hydrostatic flattening which was then assumed to give the best approximation to the real flattening. Since equation (5) is valid both for hydrostatic or non-hydrostatic equilibrium the pre-satellite method will tend to give similar results as the solution proposed here, if the only known data used in the solution is the dynamical flattening H. However, it is evident that the method proposed here is simpler and more efficient than the pre-satellite method, but one has to assume $\lambda_s = 0$, if one wants to avoid the iterative procedure of pre-satellite times.

However, the polar moment of inertia of this hydrostatic model is greater than the real Earth and this introduces some dynamical complications. It would imply a change in the radial stratification of the Earth, such as would result because of the equatorial bulge of the real Earth being more compressed and consequently the real Earth having a higher density gradient than the hydrostatic state would require. This creates the problem of suggesting some reasonable physical phenomenon responsible for such a process. Some increase in the polar moment of inertia could possibly be accounted for by the fact that when the earth readjusts itself to the equilibrium shape defined by the above model, there will be an increase in the polar moment of inertia because of the expansion of the equatorial bulge to conform to the new figure. Approximate calculations show, however, that this factor can only account for a small fraction of the total variation required by this model. This is the hydrostatic model of pre-satellite times when the geopotential coefficient J for real Earth was not precisely known and hence, the moment of inertia of the real Earth could not be determined.

For the hydrostatic model (ω, M, a, J) = constant, the solution of the equation $G(J_h, f_h) = 0$ will obviously give a constant value of flattening as can be seen from Fig. 2. The hydrostatic flattening for this model is $f_h^{-1} = 298{\cdot}29 \pm 0{\cdot}05$ and the corresponding hydrostatic value of H is $H = 3260{\cdot}50 \times 10^{-6}$. However, the polar moment of inertia of the hydrostatic model is greater than that of the real Earth and the dynamical problems in this case are of a similar nature as those enumerated for the second hydrostatic model.

The results of the above three solutions are summarized in Table 1.

Minimum strength of the Earth

The stress differences arising because of the departure of the Earth from hydrostatic equilibrium for the model (ω, M, a, C) = constant are given by Jeffreys (1963). On the supposition that the stresses are supported by strength (1) down to the core, or (2) down to a depth of 0·1 of the Earth's radius, the strength S needed to support the P_2 inequality is given as follows:

Case 1

$$S = 4{\cdot}3 \times \Delta J_2 \times 10^{12}\ \text{dyne cm}^{-2} = 4{\cdot}7 \times 10^{7}\ \text{dyne cm}^{-2}.$$

Case 2

$$S = 7{\cdot}9 \times \Delta J_2 \times 10^{12}\ \text{dyne cm}^{-2} = 8{\cdot}7 \times 10^{7}\ \text{dyne cm}^{-2}$$

Gravity field referred to the equibrium figure

The equilibrium model (ω, M, a, C) = constant is used as reference for computing the anomalous gravity field of the Earth, and this gravity field is compared with that computed relative to the international reference ellipsoid. The field is an

Table 1

(*a*) Possible hydrostatic figures obtained from the general solution*

Hydrostatic Model	$J_h \times 10^6$	$H \times 10^6$	C/Ma^2	A/Ma^2	f_h^{-1}	$f_h^{-1} - f^{-1}$
(ω, M, a, H)	1635·225	3273·64	0·33300851	0·33191836	297·29 ± 0·05	−0·96
(ω, M, a, C)	1607·49	3240·43	0·33071598	0·32964432	299·75 ± 0·05	+1·50
(ω, M, a, J)	1623·969	3260·50	0·33204876	0·33096611	298·29 ± 0·05	0

(*b*) Data for the real Earth

$J \times 10^6$	$H \times 10^6$	C/Ma^2	A/Ma^2	f^{-1}
1623·969†	3273·64‡	0·33071598	0·32963333	298·25 ± 0·05

* Based on $m = 0{\cdot}00344980$ (Khan 1967; Jeffreys 1963).

† Kozai (1964).

‡ Khan (1967).

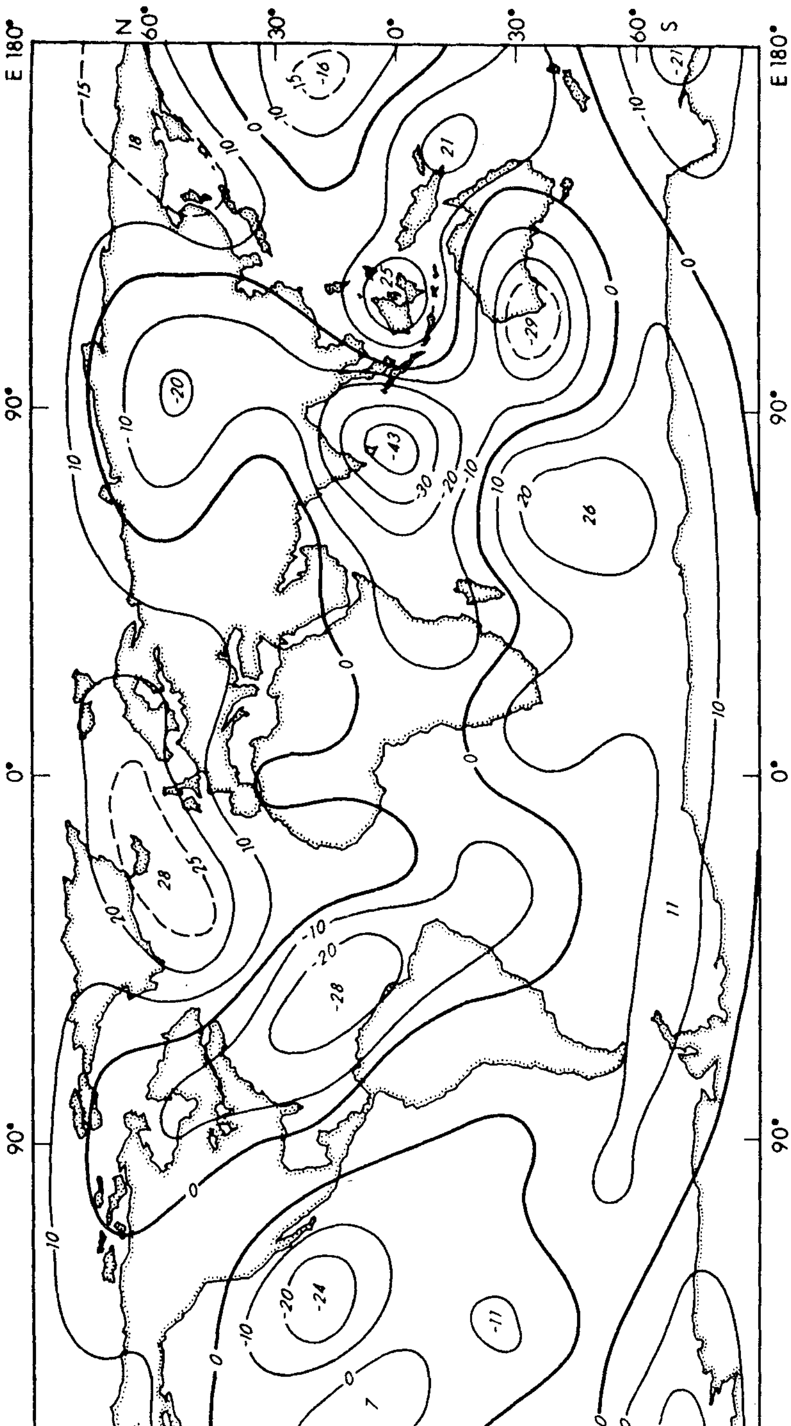

FIG. 3. Gravity anomalies in milligals referred to the international reference ellipsoid.

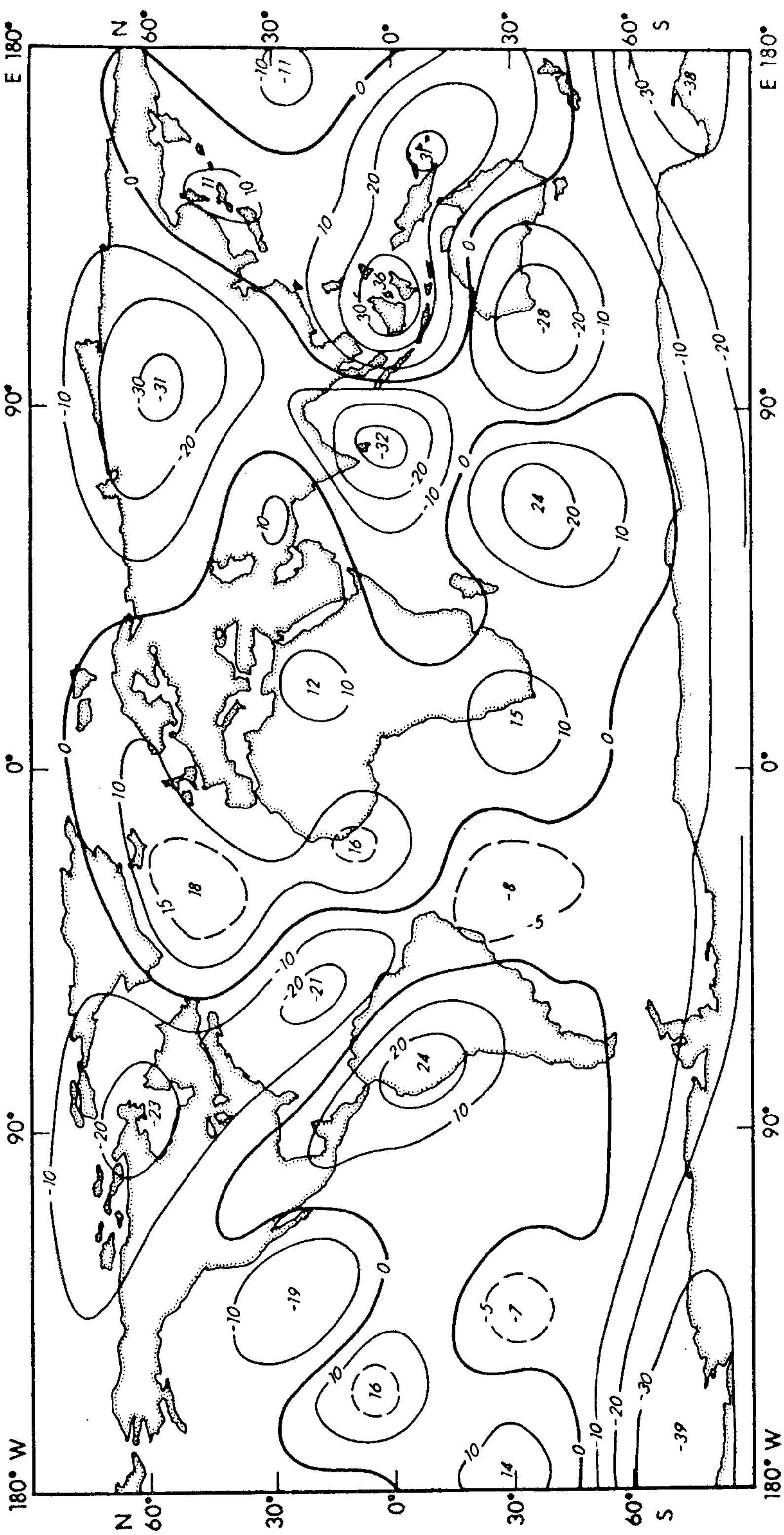

FIG. 4. Gravity anomalies in milligals referred to the equilibrium figure with a flattening of $f = 1/299{\cdot}75$.

8th degree and order spherical harmonic representation using Kozai's (1964) zonal harmonic coefficients and Gaposhkin's (1966) tesseral harmonic coefficients obtained from satellite orbital data. If the anomalous gravity field is to be used for any studies regarding the Earth's crust and mantle, it must be computed with reference to the equilibrium figure (O'Keefe & Kaula 1963; O'Keefe 1965; Fischer 1967) of the Earth, because such a figure is a figure of zero stress and departures from it will, *inter alia*, be indicative of the hydrostatic stresses existing in the Earth's crust and mantle. The gravity anomalies referred to the international reference ellipsoid are shown in Fig. 3. Fig. 4 shows the gravity anomalies referred to an ellipsoid with flattening 1/299·75. It is obvious from a comparison of Fig. 3 with Fig. 4 how the picture of the anomalous gravity field is a function of the ellipsoid adopted as the reference figure. This is also evident from some of the discussions given by O'Keefe & Kaula (1963), and Fischer (1967). In any case it is clear that the satellite-determined gravity anomalies referred to the equilibrium figure do not exceed 43 milligals (roughly) for an 8th harmonic representation of the gravity field, as seen from Fig. 3 or 4. The most pronounced gravity anomaly is negative and occurs in the Indian Ocean just to the south of Ceylon. The magnitude of this anomaly (Fig. 3) reduces by about 11 milligals to 32 milligals (Fig. 4) when the gravity field is referred to the equilibrium figure. However, in that case (Fig. 4) the positive gravity anomaly over the New Guinea and Borneo Islands areas gets accentuated by an almost equal amount. Also the negative gravity anomaly located to the east of Zapadno and Sibirskaya in U.S.S.R. becomes more pronounced by about 11 milligals. The well-pronounced positive anomaly to the south-southwest of Iceland in Fig. 3 fades somewhat in Fig. 4. The two negative anomalies flanking the southern tip of North America are equally observable in both the representations, while the positive anomaly over and around Peru is much more pronounced in Fig. 4. The negative anomaly over the Hudson Bay area is decidedly more pronounced in Fig. 4. Several other contrasting features of interest can be pointed out from a study of the two gravity field representations. It is interesting to note that the variance of the anomalous field is 145 $mgal^2$ when referred to the hydrostatic figure and 136 $mgal^2$ with reference to the international reference ellipsoid.

Review of the previous methods

Below we give a very brief review of the previous methods, examining them in perspective in the light of the general solution.

de Sitter's pre-satellite method

This method is discussed in detail in numerous papers (de Sitter 1924; de Sitter & Brouwer 1938; Bullard 1948; Jeffreys 1952; Message 1955; Khan 1968). H is taken as the initial datum and an attempt is made to find a value of J_h which would be compatible with the selected value of H (both J_h and H being functions of m). This is done by estimating a quantity $q = 3/2\, C/Ma_e^2$ $(J = qH)$ from a knowledge of the internal density distribution of the Earth (de Sitter 1924; Bullard 1948). However, the method is obsolete because the satellite determination of J has made it possible to compute the quantity q for the real earth directly. Even if for some reason, one still desires to compute f_h from H only as the initial datum, it is much simpler to use the solution outlined in this paper because considerable labour can be saved provided one agrees to put $\lambda_s = 0$ which does not make any appreciable difference anyway (Khan 1968).

However, it must be appreciated that de Sitter's whole effort was really directed to devise a method which would give the best approximation to the real flattening of the Earth, not necessarily the hydrostatic flattening.

Previous post-satellite methods

Jeffreys (1963) has given an excellent numerical method based on a simplified density model. He computes the various hydrostatic parameters from the first order theory and evaluates the second order correction terms by the numerical evaluation of the appropriate integrals. The method reported in this paper could be really regarded as a counterpart of Jeffreys' (1963) method with the exception that I have employed de Sitter's development of the hydrostatic theory.

In the previous applications of de Sitter's development to compute the hydrostatic flattening, however, it is sometimes claimed that using satellite-determined J, and dynamical flattening H and hence knowing the polar moment of inertia of the Earth, f_h should be computed from the hydrostatic equations alone without using any controls from the external potential theory. If this is accepted, the use of de Sitter's (1924) equations is automatically ruled out because these equations are derived with the help of external potential theory. Consequently, the modified equations (Khan 1968) should be used, but in that case, the equations cannot be solved because of the explicit appearance of J_h on the right-hand side of these equations and because of the necessity of treating this quantity as an unknown in the solution. If these modified equations are solved with the help of satellite-determined J (i.e., $J = J_h$), one gets the results given in the last section of Table 2. The first part of Table 2 gives the results obtained by different investigators using de Sitter's equations. The important results of Jeffreys (1963) and Ledersteger (1967) on hydrostatic flattening are not included in Table 2 because they did not use de Sitter's equations in a way which is pertinent to the discussion given in this paper.

Summary and conclusions

If the hydrostatic geopotential coefficient J, appearing explicitly on the right-hand side of the modified hydrostatic equations is treated as an unknown in the solution of the hydrostatic equilibrium problem, it becomes rather difficult to solve these equations all by themselves and one has to look for an additional boundary condition. This boundary condition is inherent in the definition of the hydrostatic equilibrium

Table 2

Comparison of hydrostatic flattening values

Post-satellite method

f_h = hydrostatic flattening; $f = 1/298{\cdot}25 \pm 0{\cdot}05$

(*a*) Using de Sitter's equations			
		f_h^{-1}	$f_h^{-1} - f^{-1}$
Henriksen	(1960)	300·0	+1·75
O'Keefe	(1960)*	299·8	+1·55
Khan	(1967)	299·86±0·05	+1·61
(*b*) Using modified hydrostatic equations alone†			
From Khan	(1968)	296·70±0·05‡	−1·55
		297·04±0·05§	−1·21

*Henriksen's calculations.

†See text.

‡Based on $m = 0{\cdot}00344980$ (Khan 1967; Jeffreys 1963), $H = 0{\cdot}00327364$, (Khan 1967) and $J_2 = 0{\cdot}001082645$ (Kozai 1964).

§Based on $m = 0{\cdot}00344992$ (Henriksen 1960), $H = 0{\cdot}00327070$ and $J_2 = 0{\cdot}00108270$.
$J_2 = 0{\cdot}00108270$.

and is derived from the external potential theory which neither assumes nor discounts the existence of hydrostatic equilibrium in the Earth's interior. It requires that the equilibrium figure of the Earth coincide exactly with that predicted from the external potential theory, if hydrostatic equilibrium exists in the Earth's interior and is stated in terms of equation (5). The solution obtained with the help of this boundary condition turns out to be sufficiently general so that the three most frequently mentioned particular solutions in literature can be obtained from this by merely defining the appropriate initial datum. Geophysically, the most meaningful model, of course, remains to be the one whose polar or mean moment of inertia is held equal to that of the real Earth, calculated from the satellite-determined J and the dynamical flattening H computed via the constant of precession of the real Earth. For this hydrostatic model the flattening is $f_h^{-1} = 299{\cdot}75$. The solution is significant in that it demonstrates the correct structure of the problem of hydrostatic equilibrium of the Earth. The anomalous gravity field of the Earth with respect to the international reference ellipsoid and the equilibrium figure is shown in Figs 3 and 4. The equilibrium figure provides the best reference for computing the anomalous field because such a field would also reflect the hydrostatic stresses which become very important in geophysical studies on a regional scale.

Acknowledgments

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