

London Mathematical Society Lecture Note Series: 213

General Theory of  
Lie Groupoids and Lie Algebroids

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**CAMBRIDGE**  
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