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ABSTRACT

We argue that the use of a quadratic approximation to calculate the value of waiting in case of uncertain and irreversible investment shows that the net present value rule and the Dixit-Pindyck approach are only limiting cases of trigger values justifying immediate investment.

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Introduction

Recent literature on investment theory stresses that irreversibility and uncertainty, to some extent inherent in every investment project, give rise to an opportunity cost - the value of waiting (McDonald and Siegel [1986]) - associated with immediate investment when the decision to invest can be postponed for a certain period of time.

This paper concentrates on the evaluation of an investment project with fully irreversible expenditures and uncertain future cash flows. We approximate the present value that justifies immediate investment - i.e. the 'trigger' - using the valuation method of Barone-Adesi and Whaley [1987].

Assuming alternative expiration dates of the option to invest implies that the net present value rule makes abstraction of the flexibility to postpone, a well known result and one of the major objections made by real option analysis. However, the trigger value most often mentioned in literature, as derived by Dixit and Pindyck [1994] and applied by many others (see e.g. Pindyck [1988], Dixit [1992], Dixit [1993] or Mauer and Triantis [1994]), is only correct in cases of perpetual investment opportunities.

Since most actual investment options are finite-lived, deriving the trigger value within a infinite-horizon model implies that investment would be postponed for too long. All else equal, the value of waiting is overestimated. Additionally, this paper shows that, although so often criticised, the net present value rule can also be justified from a real option perspective. Like the Dixit-Pindyck trigger, it simply is a limiting case in a more general and flexible framework.

The paper is organized as follows. We first specify the project's characteristics and draw the analogy with option pricing theory (Section I). In Section II we introduce the general approximation method used to quantify the opportunity cost of immediate investment and derive closed form solutions for two limiting cases in Section III. We provide a graphical illustration of our argument in Section IV and Section V presents some sensitivity results. A short summary ends the paper.

I. Real Option-to-Defer

In this paper, we concentrate on the evaluation of an investment project with fully irreversible expenditures. This implies that only the initial opportunity of postponement enters the investment decision through option valuation[1].

The present value (V) of such a project is generally assumed to follow a geometric Brownian motion, expressing the uncertainty about future cash flows. During a period of possible postponement, the firm can claim the right on this present value by paying an exercise price equal to the present value of all investment expenditures (I), but is not obliged to do so. However, postponing the decision to invest in order to evaluate or attain additional information is costly since the firm foregoes earnings at a continuous rate called the convenience yield (δ), similar to the dividend yield in case of financial options. The value of waiting drops to zero whenever the investor starts the project.

According to option pricing theory, this scenario translates into the valuation of an American call option on a dividend-paying asset. A standard argument in financial option theory states that a positive convenience yield ($\delta > 0$) is necessary for an American call option on a dividend-paying asset to be possibly excercised before maturity. Should there be no foregone earnings in case of postponement, waiting would always prove to be the optimal solution, given the uncertainty about future cash flows. This is due to the fact that option contracts represent rights, as opposed to obligations.

We discuss the valuation of the opportunity cost associated with immediate investment in the next Section.

II. Opportunity Cost Valuation

Given the above scenario, investment will be started up immediately if and only if the net present value of the project (V-I) exceeds the opportunity cost, i.e. the option value (F), instead of zero. Valuation of the opportunity cost is not trivial, however.

Barone-Adesi and Whaley [1987] suggest a quadratic approximation procedure to value American calls option on dividend-paying assets. In short, Barone-Adesi and Whaley approximate the American call option (F) as the sum of its European

[1] Should the expenditures only be partially irreversible, we would also have to take into account a put option at the end of the project's economic life time [Abel, Dixit, Eberly and Pindyck, 1995].

counterpart (f) and a component which accounts for the early exercise premium. This approach proves particularly interesting with respect to our application.

We will apply their methodology, using some additional notation: r is the risk-free interest rate and σ is the standard deviation of V. The decision to invest can be postponed for at most T-t time periods, the option's life time[2].

The present value of the project that triggers off immediate investment, denoted by V^*, is solved iteratively from [Barone-Adesi and Whaley, 1987, p.307]

$$V^* - I = f(V^*,t) + \frac{V^*}{\vartheta}\left\{ 1 - e^{-\delta(T-t)} N\left[d_1(V^*)\right] \right\} \qquad (1)$$

with

$$f(V,t) = Ve^{-\delta(T-t)}N(d_1) - Ie^{-r(T-t)}N(d_2) \qquad (2)$$

$$d_1 = \frac{\ln\left(\frac{V}{I}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t} \qquad (3)$$

$$\vartheta = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2\left[1-e^{-r(T-t)}\right]}} \qquad (4)$$

where N[•] denotes the cumulative distribution function for a standardized normal variable and f(•) is the value of an European option [Black and Scholes, 1973] with equal characteristics as the American option we aim to value.

The firm will decide to invest immediately if the present value of future cash flows at the moment of valuation exceeds the trigger value V^*. If not, investment is postponed, if possible.

Our comment with respect to the trigger value put forth by, amongst others, Dixit and Pindyck, deals with the relation between V^* and the period of possible postponement, T-t. To illustrate our argument, we focus on some limiting cases below.

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[2] As noted by Avinash Dixit, it would be more interesting if the option's life time could be determined endogenously through e.g. competitive entry of other firms or products, instead of imposing *(T-t)* exogeneously. What we actually want to show in this paper however, is that both the net present value rule and the Dixit-Pindyck approach can be captured by simply allowing for alternative expiration dates and valuing the option accordingly. Considering V^* as a function of *(T-t)* then allows to anticipate in which way such an endogenization would alter their results.

III. Limiting Cases with respect to the Possibility of Postponement

We derive the trigger value V^* in case there is no (T-t = 0) or an infinite (T-t = ∞) period of possible postponement. The resulting investment criterion will prove to simplify to the net present value rule and the 'Dixit and Pindyck' trigger value, respectively.

A. No Possibility of Postponement: T-t = 0

In case the decision to invest has to be taken immediately, equation (1) simplifies to

$$V^* - I = \lim_{T-t\to 0}\left[f(V^*,t) + \frac{V^*}{\vartheta}\left\{ 1 - e^{-\delta(T-t)} N\left[d_1(V^*)\right] \right\}\right] \quad (5)$$

Trivially, when T-t = 0 the correction for the early exercise premium of the American option is non-existent. Furthermore, the European option $f(V^*,t=T)$ will only be worth its intrinsic value since the firm considering the investment will not start the project in case it has a negative net present value, V-I < 0. As a result, the right hand side of equation (5) is, in the limiting case of T-t = 0, equal to:

$$V^* - I = Max\left[\ 0, V^* - I\ \right] \quad (6)$$

The lowest value for V^* that satisfies (6) is I, the present value of the expenditures. This implies that current real option literature is incorrect in that the net present value rule *ignores* the opportunity cost of immediate investment. The value of waiting merely *equals zero* under the assumption imposed by the investment criterion, which of course is questionable in most situations.

B. Perpetual Investment Opportunity: T-t = ∞

What level should the present value of the project exceed before a firm starts investing, assuming that the decision could be postponed infinitely ? To this question we turn next. The answer is that the trigger value should solve equation (7).

$$V^* - I = \lim_{T-t\to \infty}\left[f(V^*,t) + \frac{V^*}{\vartheta}\left\{ 1 - e^{-\delta(T-t)} N\left[d_1(V^*)\right] \right\}\right] \quad (7)$$

First, we note that if the European call option $f(V^*,t)$ can only be exercised at expiration, i.e. is perpetual, the foregone earnings during the period T-t make the investment opportunity worthless (see equation (2)): $\lim_{T-t\to\infty} f\left(V^*,t\right)=0$. Additionally, let

$$\lim_{T-t\to\infty} \vartheta = \frac{1}{2}-\frac{r-\delta}{\sigma^2}+\sqrt{\left[\frac{r-\delta}{\sigma^2}-\frac{1}{2}\right]^2+\frac{2r}{\sigma^2}}=\beta \tag{8}$$

Substituting these results in (7) yields

$$V^*-I=\frac{V^*}{\beta}$$

$$\Leftrightarrow V^*=\frac{\beta}{\beta-1}I>\mathrm{I} \tag{9}$$

Equation (9) represents the basic result of Dixit and Pindyck's modified investment criterion. Writing δ as a function of β, it can easily be seen from (8) that $\delta>0$ implies $\beta>1$ [3]. Hence, from (9) it follows that $V^*>I$, the latter being the trigger value according to the net present value rule. As a result, Dixit and Pindyck are able to conclude that the joint elements of irreversibility and uncertainty unambiguously temper investment.

IV. Illustration

It is somewhat surprising that the quadratic approximation of Barone-Adesi and Whaley has not been mentioned in the context of real options before. However, the fact that American option values on continuous dividend-paying assets can only be *approximated* clearly points out the valuation difficulties[4].

Specifically, a major finding of option pricing theory is that each derivative security (F) on V satisfies the following partial differential equation:

$$\frac{\partial F}{\partial t}+\frac{1}{2}\sigma^2V^2\frac{\partial^2 F}{\partial V^2}+(r-\delta)V\frac{\partial F}{\partial V}=rF \tag{10}$$

Imposing the appropriate boundary conditions, determined by the specifications in the derivative's contract, enables to solve (10) for F.

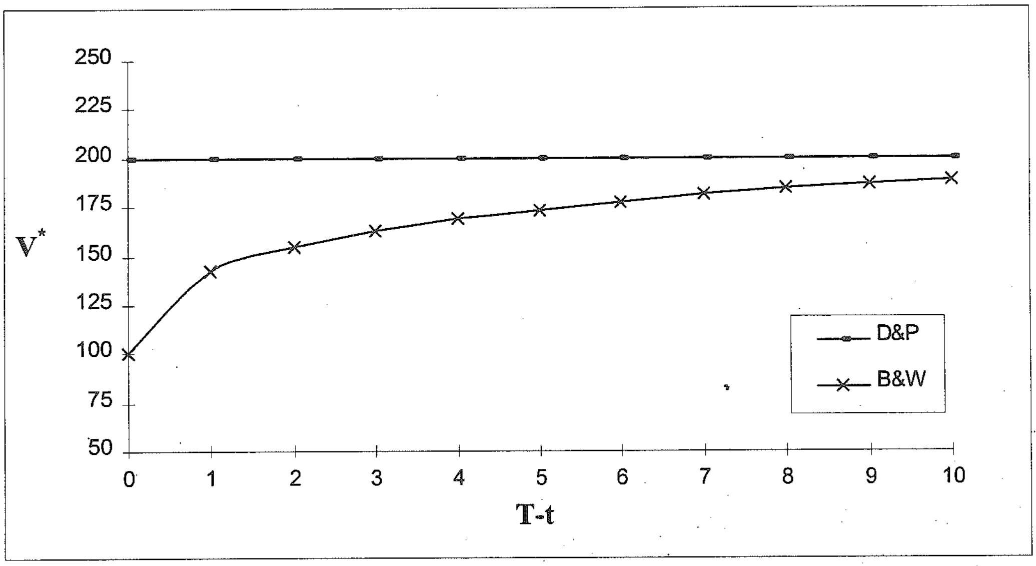
[3] See Appendix.

[4] See e.g. Ho, Stapleton and Subrahmanyam (1994) or Broadie and Glasserman (1997) for alternative methods of determining non-European option premia.

The limiting case which results in the closed form solution mentioned in (9) arises only if the partial derivative of F with respect to t in (10) is set to zero (see e.g. Dixit and Pindyck [1994, p.151]). This restriction is most often imposed implicitely, although it means that the firm either has a monopoly on the competitive advantage it created or can prevent strategic competition for whatever reason. The argument can also be reversed, in that investment apparently does not affect prices or market structure (see Kulatilaka and Perotti [1996] for a game-theoretic analysis).

Figure 1 repeats the argument graphically. The parameter values are taken from an example in Dixit and Pindyck [1994, p.153]: $\delta = 0.04$, $\sigma = 0.2$ and $r = 0.04$, which are annual rates. To accentuate the deviations in the alternative trigger values V^*, we set $I = 100$, instead of I=1 used in Dixit and Pindyck. We calculated the trigger value as a function of T-t according to the quadratic approximation in (7) (=B&W) and according to equation (9) (=D&P), respectively.

Figure 1. Trigger present value as a function of period of possible delay.



With these parameter values, β =2. Dixit and Pindyck's criterion thus requires a present value of at least twice the initial expenditures in order to justify immediate investment. In contrast, the net present value rule indicates that a present value that covers the initial expenditures suffices if the decision has to be made immediately.

Calculations of V^* for intermediate values of T-t show that uncertainty does reduce the incentive to invest in the fully irreversible project as the period of possible postponement increases, but the deviations from Dixit and Pindyck's trigger value are considerable.

V. Sensitivity with respect to the Convenience Yield

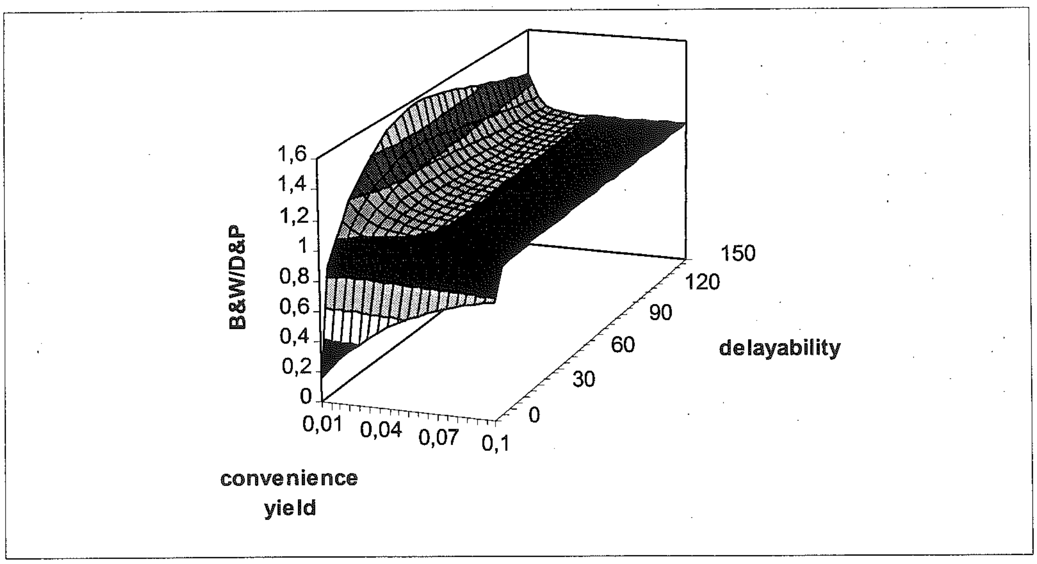
A well known and intuitive result is that the critical present value V^* decreases as the rate of foregone earnings δ increases: It becomes more interesting to invest and hold the project rather than the option. Hence, the curve resulting from the quadratic approximation still starts at I but as δ increase, it will converge to a lower Dixit and Pindyck-trigger in the limit. In case $\delta \rightarrow \infty$, the horizontal line respresenting the Dixit and Pindyck-trigger intersects the Y-axis in figure 1 at exactly $V^* = I$: The net present value result.

In this section, we show that the relationship between the trigger derived using the quadratic approximation (denoted $V^*_{B\&W}$) and the trigger under perpetuality (denoted $V^*_{D\&P}$) depends to some extent on the value of the convenience yield. This is illustrated in figure 2, which also summarizes some previous results.

We calculated the ratio $V^*_{B\&W}/V^*_{D\&P}$ for alternative values for the convenience yield, δ=0.01, 0.015, ..., 0.1 , and plot each as a function of the option's maturity, T-t=0, 5 , ..., 150. Other parameters take the values as in figure 1.

Calculating the ratio $V^*_{B\&W}/V^*_{D\&P}$, rather than the approximated trigger value ensures that the relevant value at T-t approaching infinity is equal to one, independent of the convenience yield. Also, since we know that the trigger resulting from the quadratic approximation equals I for T-t= 0, the ratio points out the extreme effect of assuming perpetuality when the investment opportunity is in fact not delayable.

Figure 2. Influence of the convenience yield, as a function of time.



To interprete the results illustrated in figure 2, first note that the ratio of the two trigger values takes the value 0.5 for *T-t* = *0* and δ = *0.04*. This is because we found that the trigger value assuming perpetualy equals twice the initial expenditures for this parameter configuration (see figure 1). Additionally, we notice that for *T-t=0*, the ratio increases with the convenience yield. Since for T-t=0, $V^*_{B\&W} = I$ at any value of the convenience yield, this indicates the earlier result that the trigger of Dixit and Pindyck increases when δ gets smaller.

Second, the ratio of the trigger values indeed converges towards one as *T-t* grows to infinity, for all δ. This illustrates our main argument, namely that the trigger value of Dixit and Pindyck only originates in a monopoly situation.

Finally, as the ratio exceeds 1 for some levels of the convenience yield, we are held to conclude that convergence of the trigger present value resulting from the quadratic approximation can be mixed at intermediate *T-t*. This should not be to much a problem however. On the one hand, we remark that the ratio is only larger than 1 for δ-values smaller than or slightly larger than the risk free interest rate (the darkest region in figure 2 points to values between 0.8 and 1). A convenience yield which sufficiently exceeds the risk free interest rate is partially what makes an investment attractive. In these cases, no confusion occur. On the other hand, if the ratio crosses the value of 1, this only happens at rather long maturities. For example, for the lowest δ = *0.01*, the associated *T-t* is equal to *10.51* (years). Furthermore, in case it even exists, this 'critical maturity' grows with the level of the convenience yield.

These arguments should suffice to stress that the behaviour illustrated in figure 2 is more a curiosity than a complication. Since, despite these results, the main point of the paper still holds. The limiting cases are maintained since these were not conditional on some specific value for the convenience yield.

Summary

Both the net present value rule and Dixit and Pindyck's valuation offer an exact solution for the present value that triggers off immediate investment in uncertain and irreversible projects. Yet this turns out to be only the case in their own simplified scenario.

Approximating this trigger value for finite expiration dates of the investment opportunity, the net present value proves the correct criterion even from a real option perspective. Just like irreversibility and uncertainty are necessary for the opportunity cost to exist, a strictly positive period of possible postponement is required for the value of waiting to be different from zero. The trigger value most often applied in literature considers the other limiting situation. Investment oppportunities are assumed perpetual, i.e. firms have a monopoly on the competitive advantage they created.

In this paper, we compared the *approximate solutions* for intermediate expiration dates with the trigger values from Dixit and Pindyck's *approximate model*. Since the approximate solutions incorporate finite values for all variables influencing the opportunity cost of immediate investment, this approach offers more flexibility in calculating the value of waiting. Furthermore, it tightens the link between investment theory and option literature and enables us to estimate to what extent relaxing current parameter assumptions would influence the tendency to invest.

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Appendix: $\delta > 0 \Rightarrow V^*_{D\&P} > I$

In this appendix we formally derive the central result of Dixit and Pindyck which states that, assuming perpetuality, a positive convenience yield implies that uncertainty and irreversibility unambiguously temper the tendency to invest.

We first initiate from the result shown in equation (8):

$$\lim_{T-t\to\infty} \vartheta = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} = \beta$$

If we now impose $\beta > 1$, the find that the associated convenience yield is required to be positive:

$$\beta > 1 \Leftrightarrow \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > \frac{r-\delta}{\sigma^2} + \frac{1}{2}$$

$$\Leftrightarrow \left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2} > \left[\frac{r-\delta}{\sigma^2} + \frac{1}{2}\right]^2$$

$$\Leftrightarrow \left[\frac{r-\delta}{\sigma^2}\right]^2 - \frac{r-\delta}{\sigma^2} + \frac{1}{4} + \frac{2r}{\sigma^2} > \left[\frac{r-\delta}{\sigma^2}\right]^2 + \frac{r-\delta}{\sigma^2} + \frac{1}{4}$$

$$\Leftrightarrow \frac{2r}{\sigma^2} > \frac{2(r-\delta)}{\sigma^2}$$

$$\Leftrightarrow \delta > 0$$

Alternatively, we can start from

$$\beta = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}}$$

and rewrite as

$$\delta = \frac{\beta\sigma^2(\beta-1) + 2r(\beta-1)}{2\beta}$$

Note that we can make use of the fact that β is the positive root of the "fundamental quadratic equation" [Dixit and Pindyck, 1994, p.142]. Additionally, both r and σ^2 are positive by definition. Trivially, we then again find that a positive convenience yield implies that $\beta > 1$.