

# Generalised array manifold model for wireless communication channels with local scattering

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**Abstract:** The authors propose the use of a generalised array manifold for parameterised spatial signature estimation in wireless communication channels with local scattering. The array manifold commonly used for point sources is generalised to include linear combinations of the nominal array response vectors and their derivatives. The motivation behind this idea is to obtain better estimates of the spatial signatures for direction of arrival (DOA) based signal waveform estimation. The estimators proposed exploit the orthogonality between the so-called noise and signal subspaces, leading to a separable solution for the derivative coefficients. As a result, a search is required for the DOAs only. For uniform linear arrays, the spatial signatures are shown to be approximately Vandermonde vectors with damped modes, and a closed-form estimator such as ESPRIT may be used in this case. Simulation examples are included to compare the signal estimation performance obtained using the proposed generalised manifold and the conventional array manifold.

## 1 Introduction

The use of antenna arrays as a tool for improving coverage, reducing interference, and increasing capacity in wireless communication systems has recently attracted significant interest [1]. For the uplink (remote to base) portion of the system, signals can be separated at the array based on knowledge of their spatial and/or temporal signatures. The array can also be used on the downlink (base to remote) channel to transmit energy towards one user and not at another. Such systems can reduce radiated power requirements, allow for multiple cochannel users, and reduce signal contamination from adjacent cells.

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In this work we study uplink signal separation using direction of arrival (DOA) based algorithms for situations where the multipath propagation is due to local scatterers in the vicinity of the sources. The time dispersion introduced by the multipath propagation is assumed to be small in comparison with the reciprocal of the signals' bandwidth. The channel between the source and the array may then be modelled as a single vector, referred to here as the spatial signature. Even small errors in the signatures may cause substantial degradation in interference rejection, especially in scenarios with large power differences between the sources.

Fast fading is usually attributed to scattering local to the user, and in [2–5] local scattering models have been used to derive channel models for flat Rayleigh fading that include the spatial dimension. By assuming a spatial distribution for the multipath components in terms of a nominal direction and angular spread it is possible to determine the fading correlation between the elements of an array. The fading correlation may be used to determine the spacing between antennas to get sufficient spatial diversity [2] and for examining the system capacity enhancement with base station antenna arrays [5]. Local scattering is not uncommon in many cellular radio systems, since the base station antennas are typically mounted on a tower away from potential multipath reflectors.

Traditional DOA estimation techniques rely on the fact that the spatial signature is a known function of the DOA. To determine the spatial signature it then suffices to estimate the DOA. Due to the multipath propagation considered here, the spatial signature will not belong to the array manifold parameterised by DOA alone. If however the scatterers are local to the mobile and the base station is some distance away, the scattered signals from a given user will be coherent and confined to a relatively small angular region. Thus, under the assumption of local scattering, use of the DOA may still make sense for determining the spatial signatures, since they will be close to some vector from the conventional planewave manifold.

The problem of estimating the parameters of the angular distribution of multipaths from multiple realisations of the spatial signature has been addressed in [4]. Here, a deterministic approach is taken instead, using a generalised array manifold model consisting of a linear combination of the nominal steering vector and its gradient [6]. For the uplink, the actual realisation of the spatial signature is of interest. If the parameters of the distribution are determined, the spatial signature

still remains to be estimated. For the downlink in a frequency division duplex (FDD) system, the opposite is true. The parameters of the distribution are needed since the uplink and downlink channels are uncorrelated when the duplex distance is larger than the coherence bandwidth.

In essence, we propose adding degrees of freedom to the conventional model. The resulting model is still only an approximation, suitable for scenarios with multipath propagation from local scatterers. It is not likely that the model exactly describes the observed data in a multipath environment. In the general case, the deviation of the spatial signature from the closest array manifold vector may be due to many different phenomena. In the presence of other errors, the dominating error source should be compensated for.

The use of information about the derivative of the array manifold is not new. Derivative constraints have been used to improve the robustness of linearly constrained minimum variance beamformers (LCMV) [7–9]. By using derivative constraints, the response of the beamformer is flattened near the steered direction. As a result, the sensitivity to steering angle errors is reduced. Another way to correct for steering errors is presented in [10], where a Taylor expansion of the steering vector is used to iteratively tune the beamformer. In contrast to the problem studied in this paper, these methods assume that the true spatial signature belongs to the nominal array manifold.

The autocalibration methods considered in [11–13] assume a parametric perturbation to the array manifold, and require derivatives of the steering vectors with respect to the perturbation parameters. In these papers, a maximum *a posteriori* approach is taken, in which the perturbation parameters are assumed to have a known *a priori* gaussian distribution. The local scattering model proposed here could also be addressed by such a bayesian framework, if the gradient term is viewed as a perturbation with some prior probability distribution. This would however require knowledge of the statistics of the gradient coefficient. As mentioned, a deterministic approach is instead taken in this paper, where the gradient coefficient is regarded as a deterministic parameter.

## 2 Data model

A scenario with  $d$  mobile sources emitting narrowband signals is considered. The scenario is assumed to be time invariant during the observation period, and the time dispersion introduced by the multipath propagation is assumed to be small in comparison with the reciprocal of the bandwidth of the emitted signals. The signal received by the  $m$  element antenna array  $\mathbf{x}(t)$  is therefore assumed to obey the following model:

$$\mathbf{x}(t) = \sum_{i=1}^d \mathbf{v}_i s_i(t) + \mathbf{n}(t) = \mathbf{V}\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where

$$\mathbf{V} = [\mathbf{v}_1 \quad \cdots \quad \mathbf{v}_d], \quad \mathbf{s}(t) = [s_1(t) \quad \cdots \quad s_d(t)]^T \quad (2)$$

The  $i$ th column of  $\mathbf{V}$ , denoted  $\mathbf{v}_i$  is the spatial signature, or channel, associated with the  $i$ th source, and  $s_i(t)$  is the transmitted signal. Additive noise is represented by  $\mathbf{n}(t)$ . The noise is assumed to be spatially white (although this assumption may be relaxed), and inde-

pendent of the transmitted signals. Under these assumptions, the covariance matrix of the signal received by the array is given by

$$\mathbf{R} = \mathbb{E}\{\mathbf{x}(t)\mathbf{x}^*(t)\} = \mathbf{V}\mathbf{S}\mathbf{V}^* + \sigma^2\mathbf{I} \quad (3)$$

where  $\sigma^2$  is the noise power and  $\mathbf{S}$  is the covariance matrix of the signals,  $\mathbf{S} = \mathbb{E}\{\mathbf{s}(t)\mathbf{s}^*(t)\}$ . It is assumed that the signals transmitted by each source are noncoherent in the sense that  $\mathbf{S}$  is full rank.

We consider the problem of estimating  $\mathbf{V}$  using a parametric model for the spatial signatures. Clearly, if the spatial parameterisation is not valid owing to the propagation environment, or the array is uncalibrated, one must resort to other approaches, for example using temporal properties of the signals  $s(t)$ .

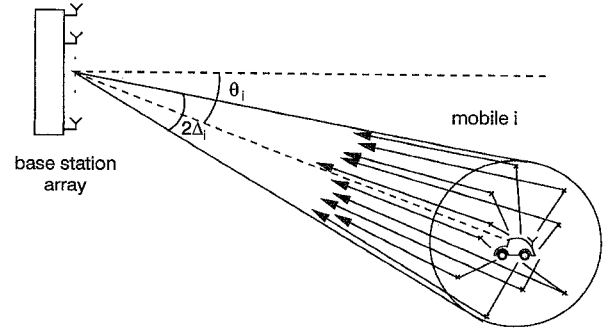


Fig. 1 Local scattering

### 2.1 Local scattering

Let  $s_i(t)$  be the signal emitted by the  $i$ th source. Due to multipath propagation, its contribution to the output of the array is modelled as a superposition of  $N_i$  scattered signals

$$\sum_{k=1}^{N_i} \beta_{ik} \mathbf{a}(\theta_i + \tilde{\theta}_{ik}) s_i(t - \tau_{ik}) \quad (4)$$

Here  $\beta_{ik}$  is the (complex) amplitude of the  $k$ th scattered signal from the  $i$ th source, and  $\mathbf{a}(\theta)$  is the  $m \times 1$  nominal steering vector representing the array response due to a plane wave with DOA  $\theta$ . The quantities  $\theta_i$  and  $\theta_i + \tilde{\theta}_{ik}$  represent the nominal DOA of the  $i$ th signal, and the arrival angle of the  $k$ th scattered signal, respectively. The delay associated with the  $k$ th scattered signal is denoted by  $\tau_{ik}$ . The situation is illustrated in Fig. 1. Without loss of generality, assume that the time delay associated with the first ray is zero. Assume that the time dispersion introduced by the multipath propagation is small compared with the reciprocal of the signal's bandwidth so the time delay may be approximated as a phase shift  $s_i(t - \tau_{ik}) \approx e^{-j2\pi f_c \tau_{ik}} s_i(t)$ , where  $f_c$  is the carrier frequency. Let  $\alpha_{ik} = \beta_{ik} e^{-j2\pi f_c \tau_{ik}}$ . The contribution from the  $i$ th signal in eqn. 4 may then be approximated as

$$\sum_{k=1}^{N_i} \beta_{ik} \mathbf{a}(\theta_i + \tilde{\theta}_{ik}) s_i(t - \tau_{ik}) \approx \left( \sum_{k=1}^{N_i} \alpha_{ik} \mathbf{a}(\theta_i + \tilde{\theta}_{ik}) \right) s_i(t) \quad (5)$$

This agrees with the model in eqn. 1 if  $\mathbf{v}_i$  is defined to be

$$\mathbf{v}_i = \sum_{k=1}^{N_i} \alpha_{ik} \mathbf{a}(\theta_i + \tilde{\theta}_{ik}) \quad (6)$$

## 2.2 Approximating the spatial signature

Determining the DOAs and amplitudes of all the  $D = \sum_{i=1}^d N_i$  rays incident on the array is a formidable task and is typically not possible since  $D$  is very large. Instead, we take another approach and approximate the spatial signature. The assumption of local scattering near each user and a correspondingly small angular spread means that  $\Delta_i$  in Fig. 1 is small. The angles  $\tilde{\theta}_{ik}$  will thus also be relatively small, and a first-order Taylor expansion of eqn. 6 may be used to approximate  $\mathbf{v}_i$

$$\begin{aligned} \mathbf{v}_i &\simeq \sum_{k=1}^{N_i} \alpha_{ik} (\mathbf{a}(\theta_i) + \tilde{\theta}_{ik} \mathbf{d}(\theta_i)) \\ &= \left( \sum_{k=1}^{N_i} \alpha_{ik} \right) \mathbf{a}(\theta_i) + \left( \sum_{k=1}^{N_i} \alpha_{ik} \tilde{\theta}_{ik} \right) \mathbf{d}(\theta_i) \\ &= \mathbf{a}(\theta_i) + \phi_i \mathbf{d}(\theta_i) \end{aligned} \quad (7)$$

where

$$\phi_i = \sum_{k=1}^{N_i} \alpha_{ik} \tilde{\theta}_{ik} \quad (8)$$

$$\mathbf{d}(\theta) = \frac{\partial \mathbf{a}(\theta)}{\partial \theta} \quad (9)$$

and the spatial signature is assumed to be scaled so that  $\sum_k \alpha_{ik} = 1$ . Substituting eqn. 7 into  $\mathbf{V}$  leads to the following compact matrix notation:

$$\mathbf{V} \simeq \mathbf{A}(\underline{\theta}, \underline{\phi}) = \mathbf{A}(\underline{\theta}) + \mathbf{D}(\underline{\theta}) \Phi(\underline{\phi}) \quad (10)$$

where

$$\begin{aligned} \mathbf{A}(\underline{\theta}) &= [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_d)] \\ \mathbf{D}(\underline{\theta}) &= [\mathbf{d}(\theta_1), \dots, \mathbf{d}(\theta_d)] \\ \Phi(\underline{\phi}) &= \text{diag}\{\phi_1, \dots, \phi_d\} \\ \underline{\theta} &= [\theta_1, \dots, \theta_d]^T \\ \underline{\phi} &= [\phi_1, \dots, \phi_d]^T \end{aligned} \quad (11)$$

This measurement model is referred to as the generalised array manifold (GAM) model. Both  $\mathbf{a}(\theta)$  and the gradient  $\mathbf{d}(\theta)$  are assumed to be known (calibrated) functions of  $\theta$ , and the problem addressed in this paper is the estimation of the spatial signatures in terms of  $\underline{\theta}$  and  $\underline{\phi}$  given  $N$  observations of the array output.

## 2.3 Unique parameter estimates

Subspace methods are considered for estimating the parameters of the approximate GAM model. A natural question is then under what conditions unique parameter estimates may be determined. This may also be reformulated as follows. Assume that the GAM model is exact, i.e. eqn. 10 holds with equality. The question is then under what conditions the parameters  $\underline{\theta}$  and  $\underline{\phi}$  are identifiable from the column span of  $\mathbf{A}(\underline{\theta}, \underline{\phi})$ . For this to be the case, the spatial signatures have to be rank- $d$  unambiguous, i.e. no linear combination of  $d$  spatial signatures can produce another spatial signature. This holds if an  $m \times (d+1)$  matrix  $\mathbf{A}(\underline{\theta}, \underline{\phi})$  has full rank for any collection of distinct parameters  $\theta_1, \dots, \theta_{d+1}$  and arbitrary  $\phi_1, \dots, \phi_{d+1}$ . In [14], sufficient conditions are derived for the GAM model. Consider the  $m \times 2(d+1)$  matrix  $[\mathbf{A}(\underline{\theta}) \mathbf{D}(\underline{\theta})]$ . If this matrix has full rank for all  $\underline{\theta}$  with distinct elements, then, except for a set of zero measure, the parameters can be uniquely determined if  $d \leq m - 2$ . However, this discussion is concerned with determining the parameters from the

column span of  $\mathbf{A}(\underline{\theta}, \underline{\phi})$ , which is the case for subspace based methods. For other estimation methods and models, other conditions may arise. Another way to handle the uniqueness problem may be to use an *a priori* distribution for the coefficients of the Taylor expansion similar to [12]. As a final remark, in the derivation of the model, higher-order terms were neglected, and the DOAs determined with the GAM model may therefore differ from the true underlying nominal DOAs for larger spreads. This is also illustrated by a numerical example in Section 6. However, the model is primarily intended as a measurement model for spatial signatures, and the result gives an upper bound on the number of signals that may be handled.

## 2.4 Spatial reference point

It has been observed that the choice of the spatial phase reference point affects the performance of linearly constrained minimum variance (LCMV) beamformers using derivative constraints [8, 9]. In what follows, it is shown that this choice does not affect the proposed model. Assume that the steering vector  $\mathbf{a}(\theta)$  has the form

$$\mathbf{a}(\theta) = \begin{bmatrix} g_1(\theta) e^{-j2\pi f_c \tau_1(\theta)} \\ \vdots \\ g_m(\theta) e^{-j2\pi f_c \tau_m(\theta)} \end{bmatrix} \quad (13)$$

where  $g_i(\theta)$  is the complex response of the  $i$ th element and  $\tau_i(\theta)$  represents the time delay relative to some spatial reference point. Suppose that another reference point is chosen and let  $\bar{\mathbf{a}}(\theta)$  represent the corresponding response vector. As pointed out in [9], the two steering vectors are related through

$$\bar{\mathbf{a}}(\theta) = \mathbf{a}(\theta) e^{-j2\pi f_c \Delta_\tau(\theta)} \quad (14)$$

where  $\Delta_\tau(\theta)$  represents the time delay associated with the propagation between the two different reference points. The derivative is then given by

$$\begin{aligned} \bar{\mathbf{d}}(\theta) &= \frac{\partial \bar{\mathbf{a}}(\theta)}{\partial \theta} \\ &= e^{-j2\pi f_c \Delta_\tau(\theta)} \mathbf{d}(\theta) \\ &\quad - j2\pi f_c \frac{\partial \Delta_\tau(\theta)}{\partial \theta} e^{-j2\pi f_c \Delta_\tau(\theta)} \mathbf{a}(\theta) \end{aligned}$$

Using some simple algebra it can be shown that any linear combination of the form  $\bar{\rho}_1 \bar{\mathbf{a}}(\theta) + \bar{\rho}_2 \bar{\mathbf{d}}(\theta)$  may be written as

$$\bar{\rho}_1 \bar{\mathbf{a}}(\theta) + \bar{\rho}_2 \bar{\mathbf{d}}(\theta) = \rho_1 \mathbf{a}(\theta) + \rho_2 \mathbf{d}(\theta)$$

provided that

$$\begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} = e^{-j2\pi f_c \Delta_\tau(\theta)} \begin{bmatrix} 1 & -j2\pi f_c \frac{\partial \Delta_\tau(\theta)}{\partial \theta} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{\rho}_1 \\ \bar{\rho}_2 \end{bmatrix} \quad (15)$$

Thus, the range of the measurement model is not affected by the choice of spatial reference point since there exists a one-to-one mapping between  $\rho_i$  and  $\bar{\rho}_i$ .

## 3 Parameterised spatial signature estimation

In this Section two algorithms for estimating  $\underline{\theta}$  and  $\underline{\phi}$  are proposed that take advantage of the special structure of the GAM model in eqn. 10. The basic idea behind the algorithms comes from similar GAMs that arise in situations involving diversely polarised antenna

arrays [14–16]. The key advantage is that a search is required only for the DOA parameters; the gradient coefficients are separable and solved for explicitly given the resulting DOA estimates.

The algorithms exploit the orthogonality between the noise subspace and the signal subspace. The eigenvectors associated with the  $m - d$  smallest eigenvalues of the sample covariance matrix of the  $N$  observations are used as an estimated basis of the noise subspace. The collection of estimated noise subspace eigenvectors is denoted  $\hat{\mathbf{E}}_n$ .

### 3.1 MUSIC-based approach

In the standard MUSIC algorithm [16], the DOAs are estimated by searching one by one for values of  $\theta$  that make  $\mathbf{a}(\theta)$  nearly orthogonal to  $\hat{\mathbf{E}}_n$ . The measure of orthogonality for MUSIC is defined to be

$$V_{MU}(\theta) = \frac{\mathbf{a}^*(\theta)\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\mathbf{a}(\theta)}{\mathbf{a}^*(\theta)\mathbf{a}(\theta)} \quad (16)$$

and the  $d$  smallest minima of  $V_{MU}(\theta)$  are taken to be the estimates of the DOAs. With the GAM model,  $\mathbf{a}(\theta)$  must be replaced with

$$\mathbf{a}(\theta) + \phi\mathbf{d}(\theta) = \bar{\mathbf{A}}(\theta)\bar{\phi} \quad (17)$$

where  $\bar{\mathbf{A}}(\theta) = [\mathbf{a}(\theta) \ \mathbf{d}(\theta)]$  and  $\bar{\phi} = [1 \ \phi]^T$ . For this case the MUSIC cost function becomes

$$V_{MU}(\theta, \phi) = \frac{\bar{\phi}^*\bar{\mathbf{A}}^*(\theta)\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\bar{\mathbf{A}}(\theta)\bar{\phi}}{\bar{\phi}^*\bar{\mathbf{A}}^*(\theta)\bar{\mathbf{A}}(\theta)\bar{\phi}} \quad (18)$$

The MUSIC criterion is seen to be a ratio of quadratic forms in  $\bar{\phi}$ , and thus minimising  $V_{MU}(\theta, \phi)$  with respect to  $\phi$  is equivalent to finding, as a function of  $\theta$ , the following minimum generalised eigenvalue and eigenvector:

$$\bar{\mathbf{A}}^*(\theta)\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\bar{\mathbf{A}}(\theta)\mathbf{z}_{\min} = \lambda_{\min}\bar{\mathbf{A}}^*(\theta)\bar{\mathbf{A}}(\theta)\mathbf{z}_{\min} \quad (19)$$

As proposed in [16], the DOA estimates can then be found by viewing  $\lambda_{\min}$  as a function of  $\theta$ , and searching for its minima. The gradient coefficient  $\hat{\phi}_i$  can be determined from the eigenvector associated with  $\lambda_{\min}(\hat{\theta}_i)$ .

### 3.2 Noise subspace fitting

As an alternative, consider the noise subspace fitting (NSF) approach outlined in [17, 18]. Under the GAM model, the NSF algorithm estimates  $\underline{\theta}$  and  $\underline{\phi}$  as the minimising arguments of the following cost function:

$$V_{\text{NSF}}(\underline{\theta}, \underline{\phi}) = \text{Trace}(\mathbf{A}^*(\underline{\theta}, \underline{\phi})\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\mathbf{A}(\underline{\theta}, \underline{\phi})\mathbf{W}) \quad (20)$$

where  $\mathbf{W} = \mathbf{W}^* > 0$  is a  $d \times d$  weighting matrix. Using arguments similar to those in [14], the cost function may be written as

$$V_{\text{NSF}}(\underline{\theta}, \underline{\phi}) = \Phi^*M(\underline{\theta})\Phi \quad (21)$$

where

$$\Phi = \begin{bmatrix} \mathbf{e} \\ \underline{\phi} \end{bmatrix} \quad (22)$$

and  $\mathbf{e}$  is a column vector composed of  $d$  ones and

$$\begin{aligned} \mathbf{M}(\underline{\theta}) &= \begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{ad} \\ \mathbf{M}_{da} & \mathbf{M}_{dd} \end{bmatrix} \\ &= \begin{bmatrix} (\mathbf{A}^*\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\mathbf{A}) \odot \mathbf{W}^T & (\mathbf{A}^*\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\mathbf{D}) \odot \mathbf{W}^T \\ (\mathbf{D}^*\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\mathbf{A}) \odot \mathbf{W}^T & (\mathbf{D}^*\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\mathbf{D}) \odot \mathbf{W}^T \end{bmatrix} \end{aligned} \quad (23)$$

Here,  $\odot$  denotes the element-by-element product. The cost function may be rewritten as

$$V_{\text{NSF}} = (\underline{\phi} + \mathbf{M}_{dd}^{-1}\mathbf{M}_{da}\mathbf{e})^*\mathbf{M}_{dd}(\underline{\phi} + \mathbf{M}_{dd}^{-1}\mathbf{M}_{da}\mathbf{e}) + \mathbf{e}^T(\mathbf{M}_{aa} - \mathbf{M}_{ad}\mathbf{M}_{dd}^{-1}\mathbf{M}_{da})\mathbf{e} \quad (24)$$

From this it follows that the estimate of  $\underline{\phi}$  is separable from that of the DOAs, and is given by

$$\hat{\underline{\phi}} = -\mathbf{M}_{dd}^{-1}\mathbf{M}_{da}\mathbf{e} \quad (25)$$

The concentrated cost function is then given by

$$V_{\text{NSF}}(\underline{\theta}) = \mathbf{e}^T(\mathbf{M}_{aa} - \mathbf{M}_{ad}\mathbf{M}_{dd}^{-1}\mathbf{M}_{da})\mathbf{e} \quad (26)$$

Thus,  $V_{\text{NSF}}(\underline{\theta})$  is the sum of the elements of the Schur complement of  $\mathbf{M}(\underline{\theta})$ . The algorithm is implemented as follows:

— Estimate  $\hat{\underline{\theta}}$  as the argument that minimises  $V_{\text{NSF}}(\underline{\theta})$  in eqn. 26.

— Solve for  $\hat{\underline{\phi}}$  by using  $\hat{\underline{\theta}}$  in eqn. 25.

Recall that the GAM model was derived as an approximation to the spatial signatures of the sources. If, however, the model of eqn. 10 is valid, the weighting matrix  $\mathbf{W}$  can be chosen so that the NSF method yields asymptotically efficient parameter estimates (i.e. the asymptotic variance of the estimates attains the Cramér–Rao bound). This follows directly from the results of [14, 19]. The optimal  $\mathbf{W}$  is parameter dependent, so the NSF approach must be preceded by a step where  $\underline{\theta}$  and  $\underline{\phi}$  are estimated consistently (e.g. using the MUSIC approach described earlier). It is well known that using a consistent estimate of the optimal  $\mathbf{W}$  has no effect on the asymptotic properties of the estimates.

As a final comment on the approach, for small angular spreads it may be reasonable to neglect the scattering when estimating the DOAs. The spatial signature may then be approximated by using the estimated DOAs in eqn. 25 to solve for  $\hat{\underline{\phi}}$ . Simulations indicate that such an approach performs well for small angular spreads.

## 4 Uniform linear arrays

The special structure of uniform linear arrays (ULAs) with omnidirectional elements may be utilised to obtain a ‘sub-optimal’ but computationally efficient solution. For a ULA with  $m$  elements separated by  $\delta$  wavelengths, the array response vector and its derivative are given by

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{j2\pi\delta \sin \theta} \\ \vdots \\ e^{j2\pi\delta(m-1) \sin \theta} \end{bmatrix} \quad \mathbf{d}(\theta) = \begin{bmatrix} 0 \\ j2\pi\delta \cos \theta e^{j2\pi\delta \sin \theta} \\ \vdots \\ j2\pi\delta(m-1) \cos \theta e^{j2\pi\delta(m-1) \sin \theta} \end{bmatrix} \quad (27)$$

where the first sensor is used as spatial phase reference point. Row  $(k + 1)$  of eqn. 7 may be approximated as

$$\begin{aligned} (\mathbf{a}(\theta_i) + \phi_i\mathbf{d}(\theta_i))_{k+1} &= e^{j2\pi\delta k \sin \theta_i} (1 + \phi_i j2\pi\delta k \cos \theta_i) \\ &\simeq e^{j2\pi\delta k \sin \theta_i} e^{j\phi_i 2\pi\delta k \cos \theta_i} \end{aligned} \quad (28)$$

In eqn. 28, the approximation is based on the fact that for small apertures and small angular spreads,  $\delta k$  and

$|\phi_i|$  are small, resulting in  $|\phi_j 2\pi\delta k \cos \theta_i| \ll 1$ . Using eqn. 28,

$$\mathbf{v}_i \simeq \mathbf{v}(\omega_i) = \begin{bmatrix} 1 \\ e^{j\omega_i} \\ \vdots \\ e^{j(m-1)\omega_i} \end{bmatrix} \quad (29)$$

where

$$\omega_i = 2\pi\delta(\sin \theta_i + \phi_i \cos \theta_i) \quad (30)$$

Note that  $\phi_i$  and  $\omega_i$  are complex scalars. If the angular spread is zero,  $\phi_i = 0$ , then eqn. 29 reduces to the conventional plane wave model:  $\mathbf{v}(\omega_i) = \mathbf{a}(\theta_i)$  as defined by eqn. 27. The condition that the elements be omnidirectional may be relaxed to requiring identical elements with approximately flat responses over the angular sector of interest. Different spatial reference points in eqn. 27 will lead to different scalings of the spatial signature in eqn. 29.

This suggests the use of a Vandermonde matrix, defined as  $\mathbf{V}(\underline{\omega}) = [\mathbf{v}(\omega_1) \dots \mathbf{v}(\omega_d)]$ , as measurement model for the spatial signatures. As outlined in [20], the ESPRIT algorithm [21] may be used to estimate the damped exponentials of this Vandermonde model. This may be viewed as an approximation of the GAM model described in the previous Section, applicable to ULAs. Whereas the GAM model uses three real parameters, this Vandermonde model uses only two real parameters, namely the real and imaginary parts of  $\omega_i$ . Consequently, we require  $d \leq m - 1$  for the parameters  $\{\omega_i\}$  to be uniquely determined. Note that this uniqueness result differs from the result in Section 2.3, as a more restricted Vandermonde model is assumed.

## 5 Minimum variance beamforming

The array manifold derivative is commonly used in linearly constrained minimum variance (LCMV) beamformers to make them more robust to steering angle errors [7–9]. However, as explained subsequently, this is not equivalent to the approach taken herein. A weight vector  $\mathbf{w}_i$  is used to form a linear estimate of the  $i$ th signal as  $\hat{s}_i(t) = \mathbf{w}_i^* \mathbf{x}(t)$  subject to certain constraints. The vector  $\mathbf{w}_i$  is chosen to minimise the output power

$$\mathbf{w}_i = \arg \min_{\mathbf{w}} E |\mathbf{w}^* \mathbf{x}(t)|^2 = \arg \min_{\mathbf{w}} \mathbf{w}^* \mathbf{R} \mathbf{w} \quad (31)$$

subject to the linear constraints

$$\mathbf{C}^* \mathbf{w}_i = \mathbf{f} \quad (32)$$

The most commonly used constraint is  $\mathbf{a}^*(\theta_i) \mathbf{w}_i = 1$ , which gives unit power in the look direction  $\theta_i$ . The resulting beamformer, also known as the minimum variance distortionless response (MVDR) beamformer, is sensitive to steering errors in  $\theta_i$ . To flatten the response in the angular domain and make the beamformer more robust to steering errors, a derivative constraint such as  $\mathbf{d}^*(\theta_i) \mathbf{w}_i = 0$  may be used. For this case, the weight vector for the  $i$ th signal is given by

$$\mathbf{w}_i = \mathbf{R}^{-1} \mathbf{C} (\mathbf{C}^* \mathbf{R}^{-1} \mathbf{C})^{-1} \mathbf{f} \\ \mathbf{C} = [\mathbf{a}(\theta_i) \quad \mathbf{d}(\theta_i)] \quad \mathbf{f} = [1 \quad 0]^T \quad (33)$$

As noted in [8, 9], the resulting beam pattern depends on the location of the phase reference, a property that is clearly not desirable. This motivated the work in [9], which considered derivatives of the output power  $F(\theta) = |\mathbf{w}_i^* \mathbf{a}(\theta)|^2$  to avoid constraining the phase of the beamformer. However, for the scenarios studied in the

following Section, this phase-independent method performed worse and its results are therefore not included. The poor performance may be explained by the fact that the phase-independent approach only uses a constraint on the real part of  $\mathbf{w}_i^* \mathbf{d}(\theta_i)$ . As a result, part of the gradient term is not blocked and this causes the performance degradation.

The LCMV beamformer is constrained to block the gradient term of the spatial signature, whereas the proposed GAM model aims at coherently combining it with the nominal steering vector. In the local scattering scenarios studied below, the GAM model approach performs better than the derivative constrained LCMV beamformer.

## 6 Numerical examples

To make a meaningful performance analysis of the proposed GAM model, a statistical model for the spatial signatures should be used. However, such an analysis is complicated by the dependence between the spatial signatures and their estimates. Instead, we investigate the performance through simulations. In the simulations, the spatial signatures are generated by drawing 30 local scatterers with random phase from a uniform angular distribution of width  $2\Delta$ . This agrees with the model of eqn. 6. The signatures are then normalised so that  $\mathbf{v}_i^* \mathbf{v}_i = m$ .

For a ULA, three different measurement models may be used, the conventional model,  $\mathbf{A}(\underline{\theta})$ , the Vandermonde model  $\mathbf{V}(\underline{\omega})$  described in Section 4, and the GAM model  $\mathbf{A}(\underline{\theta}, \underline{\phi})$ . In the simulations, the DOAs of the conventional model are estimated with the standard MUSIC algorithm, and the damped modes of the Vandermonde model are estimated with ESPRIT. The parameters of the GAM model are estimated with both the MUSIC approach of Section 3.1 and with the NSF approach of Section 3.2.

With estimates of the spatial signatures,  $\hat{\mathbf{V}}$ , several different linear signal waveform estimators may be constructed [22]. In the first two examples, the so-called deterministic signal copy vectors are used. The estimated signals are given as

$$\hat{\mathbf{s}}(t) = (\hat{\mathbf{V}}^* \hat{\mathbf{V}})^{-1} \hat{\mathbf{V}}^* \mathbf{x}(t) \quad (34)$$

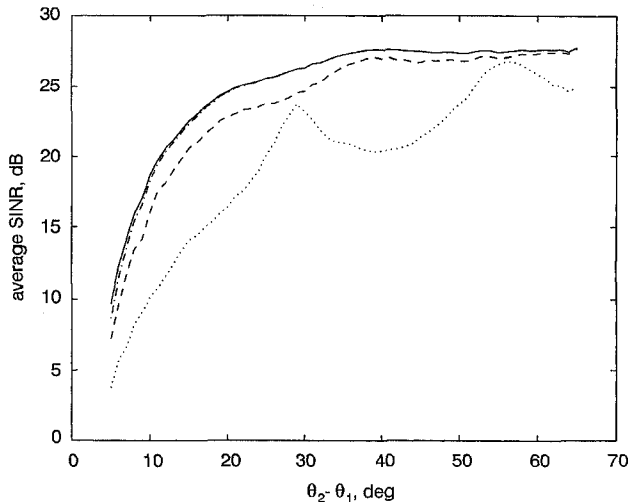
In contrast with the LCMV beamformer, this method uses the estimates of the spatial signatures of all signals.

In the first example, a ULA with six elements separated by half a wavelength is used. In each trial 100 snapshots are collected. The signals are estimated and the signal to interference plus noise ratio (SINR) is averaged over 2000 trials. The standard ESPRIT algorithm is used for determining initial estimates for MUSIC and NSF. Two sources with 20 and 40dB signal to noise ratio (SNR) are present. The angular width of both sources as seen from the array is  $2\Delta = 4^\circ$ . In Fig. 2, the average SINR for the weaker estimated signal is shown for different angular separations.

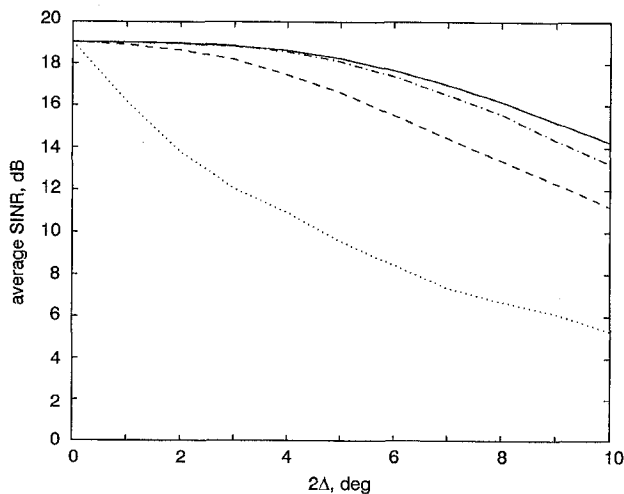
In the second example, a ULA with eight elements is used and three well separated signals with nominal DOAs  $-30^\circ$ ,  $0^\circ$  and  $30^\circ$  and SNR 30, 10 and 30dB are present. For each trial, 500 snapshots are collected, and the results are averaged over 2000 trials. In Fig. 3, the average SINR in the estimated weaker signal is shown for different angular spreads.

The advantage of using the GAM model varies with the scenario. More interestingly, it varies with the

actual angles of the signals. This may be explained as follows. If the nominal DOAs of the signals are such that the inner product of the corresponding steering vectors is large, the signal waveform estimator will create a wide zero in the angular domain. If the steering vectors from the nominal array manifold are nearly orthogonal, the waveform estimator will essentially be a standard beamformer matched to the steering vector. Due to the angular spreading, interference from other sources will then leak through. Thus, the largest performance gains will occur for scenarios with DOAs giving nearly orthogonal steering vectors. The simulations also indicate that the NSF algorithm performs slightly worse than the MUSIC approach. This may be explained by the approximate nature of the model (eqn. 7).



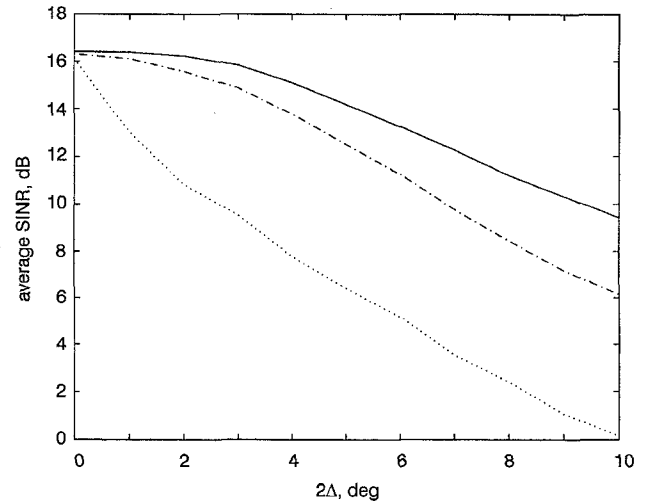
**Fig. 2** Different angular separations  $2\Delta = 4^\circ$   
 — GAM model MUSIC  
 - - - GAM model NSF  
 - · - Vandermonde model ESPRIT  
 ····· conventional model MUSIC



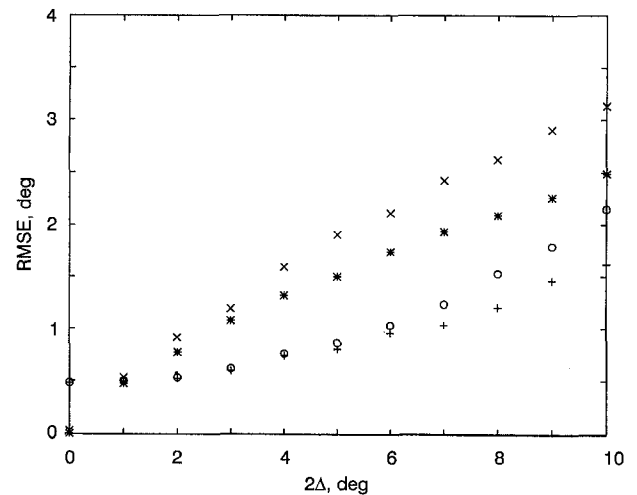
**Fig. 3** Different angular spreads  
 — GAM model MUSIC  
 - - - GAM model NSF  
 - · - Vandermonde model ESPRIT  
 ····· conventional model MUSIC

We also consider using the GAM model together with LCMV beamformers. With DOAs estimated with the conventional model and the standard MUSIC algorithm, the MVDR beamformer,  $\mathbf{w}_i = \hat{\mathbf{R}}^{-1}\mathbf{a}(\hat{\theta}_i)$ , and the LCMV beamformer with a first order derivative constraint as given by eqn. 33 are constructed. The phase

reference is chosen to coincide with the centre of the array. Finally, an MVDR beamformer  $\mathbf{w}_i = \hat{\mathbf{R}}^{-1}(\mathbf{a}(\hat{\theta}_i) + \hat{\phi}_i \mathbf{d}(\hat{\theta}_i))$  with  $\hat{\theta}_i$  and  $\hat{\phi}_i$  estimated using the GAM model and MUSIC is also considered. In each trial, 500 snapshots are collected for the scenario considered in the second example. In Fig. 4 the average SINR of the weaker signal estimate is shown for different angular spreads. This example demonstrates that the proposed GAM model also offers a significant performance gain when used in the form of the LCMV solution.



**Fig. 4** LCMV beamformers for different angular spreads  
 — MVDR, GAM model  
 - - - LCMV, derivative const.  
 ····· MVDR, conventional model



**Fig. 5** Root mean square error of nominal DOA estimate  
 × conventional model, MUSIC  
 \* ESPRIT  
 o GAM model, NSF  
 + GAM model, MUSIC

For the last example, the root mean square error (RMSE) of the DOA estimate of the weaker signal is plotted in Fig. 5 for the different models and estimators. Note that the approximate nature of the GAM model leads to higher RMSE for larger spreads also when the GAM model is used. Our experience is that the standard ESPRIT algorithm in general gives lower RMSE as compared to the conventional MUSIC algorithm. The estimates calculated with the NSF approach have higher RMSE than the estimates calculated with the MUSIC approach for the GAM model, owing to the approximate nature of the model.

## 7 Summary

For channels with local scattering we have proposed the use of a generalised array manifold (GAM) model for DOA-based spatial signature estimation and signal waveform estimation. Two procedures for estimating the parameterised spatial signatures were proposed, a MUSIC-like estimator and one based on noise subspace fitting. In addition, a computationally efficient solution based on ESPRIT was proposed for ULAs. To demonstrate the advantages of the proposed model, numerical examples of signal waveform estimation in the presence of strong interference were given. They indicate a significant performance gain over using the conventional array manifold. One drawback of the method is that the array needs to be well calibrated. In addition to the conventional manifold, the gradient of the steering vectors must be known as well.

## 8 Acknowledgment

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