Progress of Theoretical Physics, Vol. 69, No. 3, March 1983

Generalization of the Glauber-Lachs Formula, Charged Particle Distributions and the KNO Scaling at $p\bar{p}$ Collider

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(Received September 20, 1982)

A generalized Glauber-Lachs formula for Bose particle (i.e., unpolarized photon and charged pion) distributions is obtained. It contains a parameter of the number of cells or ensembles ($k\geq 2$) in addition to a ratio of the average multiplicity of Bosons coherently produced and ones incoherently produced ($\gamma=\langle n_c\rangle/\langle n_{in}\rangle$). It holds an analytic KNO scaling function. Comparisons of these formulae with data in pp collision at 200 GeV/c and at $p\bar{p}$ collider at $\sqrt{s}=540$ GeV are made. There are quantitatively good agreements among theoretical values and data.

§ 1. Introduction

Recently charged particle distributions with pseudo-rapidity (η) cutoffs at 200 GeV/c,[1] in ISR region[2] and at $p\bar{p}$ collider[3] have been presented. In particular examinations of the KNO scaling[3)~5] at $p\bar{p}$ collider at $\sqrt{s}=540$ GeV are made in detail. However, in the theoretical point of view, one may say that we have no sufficient physical knowledge on analysing methods of the *charged pion distributions* (i.e., two or more kinds of Bose particle ones). (See Fig. 1). Now we list some physical *problems* related to the description of the charged pion distribution: P1) The charged pion distribution with the pseudo-rapidity cutoffs is equivalent to a sum of two ensembles of negative and positive pions without a charged constraint. P2) We have to take into account the Bose-Einstein statistics only between identical pions, to analyse the charged pion distribution. In other words, the Bose-Einstein statistics between negative and positive pions is not necessary. P3) Recent analyses of pionic interferometry, i.e., "Hanbury-Brown-Twiss" effect in the negative pion ensembles, suggest to us that these pions have a partially coherent property. To understand this point, we have to use the original Glauber-Lachs formula.[6] However, this formula cannot be applied to the charged pion ensemble without the charged constraint, because it is discovered for a description of the polarized photon ensemble. To resolve these problems, it seems to be better to calculate a generalized Glauber-Lachs formula

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**) Partially supported by the Yamada Science Foundation.

containing the number of ensembles (k). The KNO scaling function and moment formulae[7] also must be obtained. Furthermore, comparisons of the new formula with data at 200 GeV/c and at $p\bar{p}$ collider at $\sqrt{s}=540$ GeV will be made, by referring to analyses of the negative pion interferometry.[8)~11)]

In order to analyse mainly pion distributions accumulated, the Poisson, the geometrical,[1] the Planck-Polya[12),13)] and the original Glauber-Lachs formulae,[1),6),7)] have been usually utilized. The explicit expressions are given in Table I. [In Table I we do not list the analytic KNO scaling functions given in Refs. 14)~16).] The physical correspondences among these formulae are shown in Fig. 1. Here it is worth while to pay our attention to the following points: The geometrical distribution fully reflects the Bose-Einstein statistics. On the other hand, the particle ensemble described by the Poisson distribution has fully the c-number property, in spite of the Boson ensemble. The original Glauber-Lachs formula can be applied to one kind Boson ensemble. Therefore it should be again noticed that there is an empty box written by the unknown "Generalized Glauber-Lachs" formula in Fig. 1. As seen in Fig. 1, we do not know a formula related to the Boson ensembles with a finite ratio $\gamma(=\langle n_c\rangle/\langle n_{in}\rangle\neq 0)$ of the average multiplicity of particles produced coherently to one produced incoherently and the number of ensembles larger than two ($k\geq 2$). This unknown formula might be useful to analyse an unpolarized photons (helicity$=\pm 1$) and two kinds of charged pion (π^{charged}) ensemble without a physical constraint in pp collisions, for example, such as,

$$n(ch)=2+n(+)+n(-)=2+2n(-).$$

Second identical negative pion interferometry experiments[8] have shown that the intercepts of its interferometries require a degree of coherence ($\xi^{(-)}$ for the negative pion ensemble) which is not a unit,

Table I. Particle distribution formulae.

Geometrical (-Furry)	$\dfrac{A^n}{(1+A)^{n+1}}$
Planck-Polya	$\dfrac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)}\left(1+\dfrac{\langle n\rangle}{k}\right)^{-k}\left(1+\dfrac{k}{\langle n\rangle}\right)^{-n}$
Glauber-Lachs	$\dfrac{A^n}{(1+A)^{n+1}}\exp\left[-\dfrac{\lvert\zeta\rvert^2}{1+A}\right]L_n\left(-\dfrac{\lvert\zeta\rvert^2}{A(1+A)}\right)$
Poisson	$e^{-\langle n\rangle}\dfrac{\langle n\rangle^n}{n!}$

$A=\langle n_{in}\rangle$ (Bose particles incoherently produced),
k=the number of cells (or ensembles),
$\lvert\zeta\rvert^2=\langle n_c\rangle$ (Bose particles coherently produced),
L_n: the Laguerre polynomial, $\langle n\rangle$: total multiplicity.

	the number of ensembles: $k=1$	$k\geq 2$	∞
$\gamma=\langle n_c\rangle/\langle n_{in}\rangle$ $=0$	Geometrical ← $(k=1)$	Planck-Polya $(k=\infty)$ →	Poisson
	↑ $(\gamma=0)$		
$\gamma=$finite	Glauber-Lachs	[unknown Generalized Glauber-Lachs]	
	↓ $(\gamma=\infty)$		
∞	Poisson		Poisson

Fig. 1. Physical correspondences among the Bose particle distribution formulae.

$$N^{--}/N^{BG}(p_1, p_2)\equiv(\sigma^{-1}d\sigma/dp_1dp_2)/(\sigma^{-1}d\sigma/dp_1)(\sigma^{-1}d\sigma/dp_2), \quad (1a)$$

$$\xrightarrow{(p_1-p_2)^2\to 4m_\pi^2} 1+\xi^{(-)}. \quad (1b)$$

When we use a laser-optical approach related to the original Glauber-Lachs formula, whose availability is pointed out by Fowler and Weiner,[9)~11)] this degree of coherence can be written by

$$\xi^{(-)}=(1+2\gamma^{(-)})/(1+\gamma^{(-)})^2, \quad (2)$$

where the parameter $\gamma^{(-)}$ is the ratio in the negative pion ensemble,

$$\gamma^{(-)}=\langle n_c(-)\rangle/\langle n_{in}(-)\rangle.$$

Combining experimental data of the negative pion interferometries in e^+e^- annihilation at the $E_{cm}=4\sim7$ GeV,[17)] π^+p and K^-p collisions at 16 GeV/c[18)] and π^-p collision at 200 Gev/c,[8)] we obtain the following interval from data in the central region,

$$0.56\leq\xi^{(-)}\leq 0.75. \quad (\text{or } 1\leq\gamma^{(-)}\leq 2) \quad (3)$$

This finite interval suggests to us that it is necessary to take the ratio into consideration (i.e., $\gamma^{(-)}$ (or $\gamma^{(+)}$) and/or γ which is given by)

$$\gamma=\langle n_c(charged)\rangle/\langle n_{in}(charged)\rangle,$$

in particular, in order to analyse charged pion distributions in the central region, because one handles similar pion ensembles. A simple explanation is given as follows: In proper ensembles with pseudo-rapidity cutoffs, we can probably obtain that, due to $\langle n(+)\rangle=\langle n(-)\rangle$ which is independent of the charged constraint *in a sense of an average multiplicity,*

$$\gamma^{(-)}\cong\gamma^{(+)}\cong\gamma\equiv\frac{\langle n_c(+)\rangle+\langle n_c(-)\rangle}{\langle n_{in}(+)\rangle+\langle n_{in}(-)\rangle}.$$

§ 2. Generalized Glauber-Lachs formula

To calculate this formula, we can use a product of the following two kinds of Boson density matrices â la Klauder-Sudarshan,[6),19)]

$$P_2(n_1, n_2)\equiv\langle n_1|\rho_1|n_1\rangle\langle n_2|\rho_2|n_2\rangle$$
$$=\int w(\alpha_1)\frac{|\alpha_1|^{2n_1}}{n_1!}\exp[-|\alpha_1|^2]d\alpha_1\cdot\int w(\alpha_2)\frac{|\alpha_2|^{2n_2}}{n_2!}\exp[-|\alpha_2|^2]d\alpha_2\,. \tag{4}$$

Here $w(\alpha_i)$ with a notation of A_i $(=A/2)$ is given by

$$w(\alpha_i)\equiv\frac{1}{A_i\pi}\exp\left[-\frac{1}{A_i}|\alpha_i-\zeta_i|^2\right]. \tag{5}$$

There are physical correspondences among A_i, $|\zeta_i|^2$, $\langle n_{in}\rangle$ and $\langle n_c\rangle$ such as,

$$\begin{cases}\sum_{i=1}^{2}A_i=A\longleftrightarrow\langle n_{in}\rangle\,,\\ \sum_{i=1}^{2}|\zeta_i|^2=|\zeta|^2\longleftrightarrow\langle n_c\rangle\,.\end{cases} \tag{6}$$

Substituting Eq. (5) into Eq. (4), we get the following expression with the Laguerre polynomials (L_n):

$$P_2(n_1, n_2)=\prod_{i=1}^{2}\frac{A_i^{n_i}}{(1+A_i)^{n_i+1}}\exp\left[-\frac{|\zeta_i|^2}{1+A_i}\right]L_{n_i}\left(-\frac{|\zeta_i|^2}{A_i(1+A_i)}\right). \tag{7}$$

In Eq. (7), *when* a case of the following general k-ensembles is considered,

$$\begin{cases}A'=A/k(\text{the number of ensembles}),\\ \text{and}\quad\sum_{i=1}^{k}n_i=n\,,\end{cases}$$

we obtain the following generalized Glauber-Lachs formula:

$$P_k(n)=\frac{A'^n}{(1+A')^{n+k}}\exp\left[-\frac{\sum_{i=1}^{k}|\zeta_i|^2}{1+A'}\right]L_n^{(k-1)}\left(-\frac{\sum_{i=1}^{k}|\zeta_i|^2}{A'(1+A')}\right), \tag{8}$$

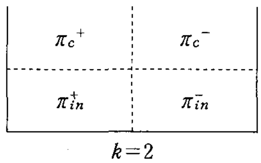
where the following mathematical formula is used:

$$\sum_{n=n_1+n_2+\cdots+n_k}L_{n_1}(x_1)\cdots L_{n_k}(x_k)=L_n^{(k-1)}(x_1+x_2+\cdots+x_k).$$

It should be noticed that in Eq. (8) we simply put $n=n_1+n_2+\cdots+n_k$, because

$n_2 \backslash n_1$	0	1	2	3	
0	0	1	2	3	
1	1	2	3	4	
2	2	3	4	5	

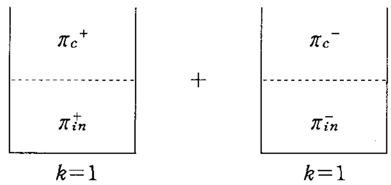
(a)



(b)

$n_2 \backslash n_1$	0	1	2	
0	0+0			
1		1+1		
2			2+2	

(c)



(d)

Fig. 2.(a) Two kinds of Boson ensembles without physical constraints.
(b) Charged pion ensembles by incoherent- and coherent pions.
(c) Two kinds of Boson ensembles with charged constraint.
(d) Separable two charged-pion-ensembles.

there is no physical constraint ($n(+)\neq n(-)$, see Fig. 2(a)). By making use of the ratios such as,

$$\gamma=\langle n_c\rangle/\langle n_{in}\rangle=\sum_{i=1}^{k}|\zeta_i|^2/A\,,\quad \langle n\rangle=\langle n_c\rangle+\langle n_{in}\rangle \quad \text{and} \quad p=1/(1+\gamma),$$

Eq. (8) can be rewritten as follows:

$$P_k(n)=\frac{(p\langle n\rangle/k)^n}{(1+p\langle n\rangle/k)^{n+k}}\exp\left[-\frac{\gamma\langle n\rangle p}{1+p\langle n\rangle/k}\right]L_n^{(k-1)}(-\gamma k/(1+p\langle n\rangle/k)). \tag{9}$$

It should be emphasized that Eq. (9) contains three physical limits,

Eq. (9) —$(k=1)$→ original Glauber-Lachs,
Eq. (9) —$(\gamma=0)$→ Planck-Polya,
Eq. (9) —$(k=1,\ \gamma=\infty)$→ Poisson.
Planck-Polya —$(k=\infty)$→ Poisson.

The generating function is given by

$$Q_k(\lambda)=\sum_{n=0}^{\infty}(1-\lambda)^n P_k(n)=\frac{1}{(1+\lambda A')^k}\exp\left[-\frac{\lambda(\sum_{i=1}^{k}|\zeta_i|^2)}{1+\lambda A'}\right]. \tag{10}$$

The Mueller moments can be calculated by using the following formula:

$$F^{(l)}=\langle n(n-1)\cdots(n-l+1)\rangle=(-1)^l\left[\frac{\partial^l}{\partial\lambda^l}Q(\lambda)\right]\Big|_{\lambda=0}, \tag{11a}$$

$$F^{(1)}=kA'+\sum_{i=1}^{k}|\zeta_i|^2=A+|\zeta|^2, \tag{11b}$$

$$F^{(2)}=(1+1/k)A^2+2(1+1/k)|\zeta|^2A+|\zeta|^4, \tag{11c}$$

$$F^{(l)}\equiv l\,!\,A'^l L_l^{(k-1)}\left(-\frac{|\zeta|^2}{A'}\right). \tag{11d}$$

To get the asymptotic expression for the KNO scaling,[5] we have to modify Eq. (9), by taking into account the following limits, i.e., $\langle n(ch)\rangle$ and $n(ch)\to\infty$ with a finite $z=n(ch)/\langle n(ch)\rangle$ in charged pion ensemble,

$$\lim_{\substack{n(ch),\langle n(ch)\rangle\to\infty\\ z=n(ch)/\langle n(ch)\rangle}}\langle n(ch)\rangle P_k(n(ch))\xrightarrow{\gamma<[\langle n(ch)\rangle/1]}\Psi_k(z,p)$$

$$=\left(\frac{k}{p}\right)^k\left[\frac{z}{\sqrt{z(k/p)^2(1-p)}}\right]^{(k-1)}$$

$$\times\exp\left[-\frac{k(1-p)}{p}-\frac{kz}{p}\right]I_{(k-1)}(2\sqrt{z(k/p)^2(1-p)}), \tag{12}$$

where the condition that $\gamma<[\langle n(ch)\rangle/1]$ is assumed. In Eq. (12) the following formulae are used:

$$\lim_{n\to\infty}n^{-\alpha}L_n^{(\alpha)}(x/n)=x^{-\alpha/2}J_\alpha(2\sqrt{x})\quad\text{and}\quad I_\alpha(x)=e^{-\alpha\pi i/2}J_\alpha(ix),$$

where J_α and I_α denote the Bessel and the modified Bessel functions, respectively. The moments of arbitrary orders are given by the next formula,

$$c_l=\langle n(ch)^l\rangle/\langle n(ch)\rangle^l$$

$$\longrightarrow\int_0^\infty z^l\,\Psi_k(z,p)dz=l\,!\left(\frac{p}{k}\right)^l L_l^{(k-1)}\left(-\frac{k}{p}(1-p)\right). \tag{13}$$

By making use of c_l, we can calculate the following dispersion and moments:

$$\langle n(ch)\rangle/D=1/\sqrt{c_2-1}, \tag{14a}$$

$$M_2=\langle(n(ch)-\langle n(ch)\rangle)^2\rangle/\langle n(ch)\rangle^2=c_2-1, \tag{14b}$$

$$M_3=\langle(n(ch)-\langle n(ch)\rangle)^3\rangle/\langle n(ch)\rangle^3=c_3-2c_2+2, \tag{14c}$$

$$M_4=[\langle(n(ch)-\langle n(ch)\rangle)^4\rangle-3\langle(n(ch)-\langle n(ch)\rangle)^2\rangle^2]/\langle n(ch)\rangle^4$$
$$=c_4-4c_3-3c_2\times c_2+12c_2-6\,. \tag{14d}$$

For pion ensembles with the charged constraint, by making use of Figs. 2(c) and (d), we can easily obtain the following equation, because this case can be decomposed into two same ensembles:

$$\tilde{P}(n(ch))=[P(n(ch)/2)+P(n(ch)/2)]/2\,, \tag{15a}$$

$$\tilde{\Psi}(z)=\lim_{n(ch),\langle n(ch)\rangle\to\infty}\langle n(ch)\rangle\tilde{P}(n(ch))$$
$$=\lim_{n(ch)/2,\langle n(ch)\rangle/2\to\infty}2\langle n(ch)\rangle/2P(n(ch)/2)$$
$$=2\Psi_{k=1}(z). \tag{15b}$$

§ 3. Comparisons of Eqs. (9), (12), (14) and (15b) with data

There are three interesting data given in Refs. 1), 3), 4) and 20). In the following analyses charged particles are assumed to be mainly charged pions.

a) pA(mainly proton)$\to\pi^{ch}$ with N_h(the number of the gray particles)$=0-1$ at 200 GeV/c.[1] The data contains 689 stars. Our theoretical values and data are given in Fig. 3. The least χ^2-values are given in Table II. It should be noticed that there are good agreements among theoretical values and data, and moreover, that the ratio of γ increases as η_L does. [Between η and η_L (laboratory system) there is a relation,

$$\eta=\eta_L-\ln(\gamma_{\rm cm}(1+\beta_{\rm cm})),$$

where $\eta_L=-\ln\tan\theta_L/2$ and $\gamma_{\rm cm}=(1-\beta_{\rm cm}^2)^{-1/2}$ is the Lorentz factor of the C.M.S. for $pp(\bar{p})$ collisions with the proton target at rest.] Furthermore, as seen in Table II, when the original Glauber-Lachs formula is applied for data, bigger values for the γ are obtained. In this case the γ only means a numerical parameter.

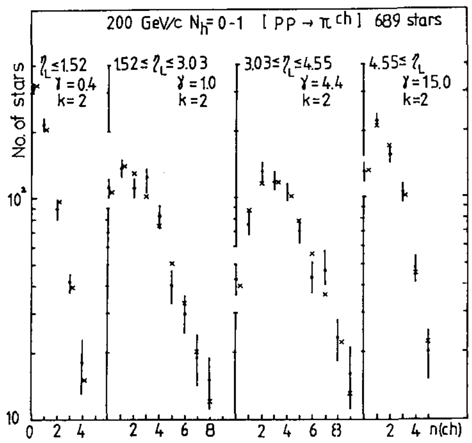


Fig. 3. Comparison of Eq. (9) with data at 200 GeV/c. η_L denotes the pseudorapidity. Theoretical values are given by cross marks.

b) $p\bar{p}\to\pi^{ch}$ at $\sqrt{s}=540$ GeV.[4] The charged particle distributions with $|\eta|<1.5$ and $|\eta|<3.5$ in terms of the KNO scaling variable are shown in Fig. 4. By making use of Eq. (12) the theoretical curves are calculated. By

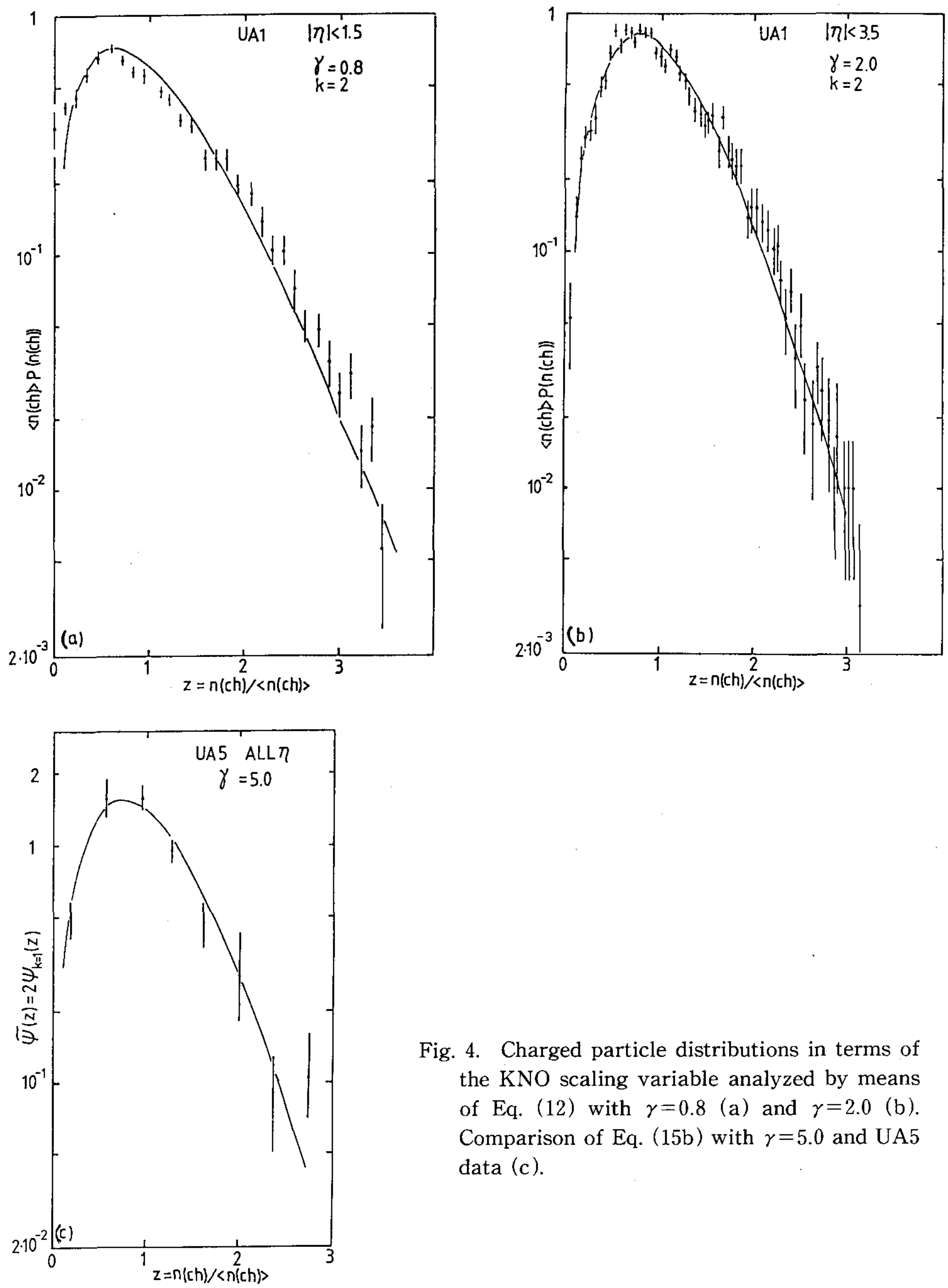


Fig. 4. Charged particle distributions in terms of the KNO scaling variable analyzed by means of Eq. (12) with $\gamma=0.8$ (a) and $\gamma=2.0$ (b). Comparison of Eq. (15b) with $\gamma=5.0$ and UA5 data (c).

using the same values of γ's and Eqs. (13) and (14) we compute the moments given in Table III. There are also good agreements among theoretical values and data.

c) $p\bar{p}\rightarrow\pi^{ch}$ at $\sqrt{s}=540$ GeV with all η: (UA5).[20] In this data the charged

Table II. The least χ^2-values in comparisons of Eq. (9) with data in mainly $pp \to \pi^{ch}$ at 200 GeV/c.

$\eta=\eta_L-3.03$	$\eta_L \le 1.52$	$1.52 \le \eta_L \le 3.03$	$3.03 \le \eta_L \le 4.55$	$4.55 \le \eta_L$
$\langle n(ch)\rangle$	0.91±0.04	2.74±0.09	3.71±0.09	1.70±0.05
χ^2/D.N.F. $k=1$	2.4/7 ($\gamma=3.0$)	13.0/13 (4.1)	14.2/13 (12.7)	6.1/8 (50.0)
$k=2$	2.3/7 ($\gamma=0.4$)	13.7/13 (1.0-1.2)	13.4/13 (4.4-4.6)	5.2/8 (14.8)

Table III. Comparisons of Eq. (14) with data of moments at $\sqrt{s}=540$ GeV, and Eq. (15b) with data of UA5 and ISR at $\sqrt{s}=63$ GeV. Values by means of the de Groot formula[15] are also given.

	$\langle n\rangle$	$\langle n\rangle/D$	M_2	M_3	M_4
UA1 $\lvert\eta\rvert<1.5$	9.9 ±0.7	1.43 0.05	0.463 0.019	0.292 0.031	0.170 0.100
Theory ($\gamma=0.8$)		1.58	0.401	0.292	0.300
UA1 $\lvert\eta\rvert<3.5$	21.5 ±1.5	1.90 0.08	0.279 0.011	0.111 0.007	0.020 0.010
Theory ($\gamma=2.0$)		1.90	0.278	0.130	0.083
UA5 All η	26.8 ±2.1	1.8 0.2	—	—	—
ISR $\sqrt{s}=63$ All η	12.70 ±0.12	1.83 0.03	0.297 0.010	0.125 0.007	0.051 0.006
Theory ($\gamma=5.0$) ($k=1$)		1.81	0.306	0.148	0.097
de Groot[15]		1.82	0.302	0.125	0.035

constraint approximately holds, because only the single diffraction effect is excluded. Comparison of Eq. (15b) with $\gamma=5$ and data is shown in Fig. 4(c) and the dispersion is calculated in Table III. The larger η, the larger γ.

In both cases (a) and (b), it should be emphasized that we can observe that our expectation mentioned above regarding the Bose particle ensembles in the central region holds:

$$\gamma \sim 0.8\text{—}1.0 \sim \gamma^{(-)}. \qquad (16)$$

This relation seems to support that our scheme is not far from a true physical mechanism related to the Bose particle ensembles.

§ 4. Conclusions and discussion

i) The generalized Glauber-Lachs formula which is useful for many kinds of Boson ensembles ($k \geq 2$) is calculated. The analytic KNO scaling function and moment functions are also calculated.

ii) The charged pion distributions and the KNO scaling in pA(mainly proton) collision at 200 GeV/c and at $p\bar{p}$ collider at $\sqrt{s}=540$ GeV can be well reproduced by Eqs. (9) and (12) with almost the same value of γ(0.8—1.0), provided that data in the central region are used. This estimated value is similar to the estimated one (Eq. (3)) in the negative pion interferometries in $\pi^- p$ collision at 200 GeV/c[8] as well as data found in Ref. 18). A discrepancy between the ratio $\gamma^{(-)} \cong 1$ from data in $\pi^- p$ collision at 200 GeV/c[8] obtained in Ref. 9) and the large ratio $\gamma \gtrsim 10$ from the same data analysed in Ref. 7) can be semi-quantitatively understood. Moreover, a difficulty of a negative $k(<0)$ found in Ref. 12), as the Planck-Polya formula is applied to data at low energy, may disappear.

iii) The ratio γ seems to be connected with the pion's energy, as seen in Table II and in Fig. 4,

$$\begin{Bmatrix} \text{small pseudorapidity } \eta \longleftrightarrow \text{small } \gamma \longleftrightarrow \text{geometrical distribution} \\ \text{(large)} \qquad\qquad\qquad \text{(large)} \qquad\qquad \text{(Poisson)} \end{Bmatrix}.$$

When this physical correspondence is confirmed, the ratio γ becomes an *order parameter* in the Bose particle ensembles. Plausible physical considerations for this correspondence are also given in Ref. 21), through observations on the GGLP effect and analyses of the negative pion interferometry.

For the charged pions interferometry at $p\bar{p}$ collider, as the γ is estimated, we can predict the following value by making use of Eq. (11c),

$$N^{ch,ch}/N^{BG} \xrightarrow{(p_1-p_2)^2 \to 4m_\pi^2} 1+\left.\frac{(1+2\gamma)/k}{(1+\gamma)^2}\right|_{\substack{k=2\\ \gamma=0.8}}$$

$$=1+0.4\,.$$

At present we have no experimental data. However, *if* we can use the following experimental data in ISR region (pp collision at $\sqrt{s}=31.5$ GeV[22]),

$$N^{ch,ch}/N^{\rm BG}(|y_1| \leq 0.5 \quad \text{and} \quad (y_1-y_2) \to 0) \sim 1.5\,,$$

we can say that this value seems to support our prediction mentioned above, since we know that there is a quantitatively good coincidence between data in the ISR region[2] and $p\bar{p}$ collider.[3),4)]

A KNO scaling in e^+e^- annihilation given in Ref. 23) is also well reproduced in terms of Eq. (15b) with $\gamma=12.0$. The dispersion is that $\langle n(ch)\rangle/D=2.6$ which is compatible with an experimental value 2.8 ± 0.1.

Comparison of Eq. (12) with the de Groot formula[15] as well as effects of a contamination of ($\bar{K}K$, $\bar{\Lambda}\Lambda$ and $\bar{p}p$) and a production dynamics will be given elsewhere.

Furthermore, it must be stressed that Eq. (9) can be applied to the unpolarized photon distributions.

Acknowledgements

The author would like to thank Professor R. Weiner, Professor G. Fowler and Professor E. Friedländer for useful conversations and showing the data at 200 GeV/c and moreover, is indebted to Professor Y. Kano (Nagoya Univ.) for his kind correspondence. Dr. N. Stelte and Dr. R. Beckmann have helped him in numerical computations. He is also indebted to the Yamada Science Foundation for financial support.

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