Generalized Interval Valued Neutrosophic Graphs of First Type

Said Broumi[1], Mohamed Talea[3]
Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco
broumisaid78@gmail.com, taleamohamed@yahoo.fr

Assia Bakali[2]
Ecole Royale Navale, Boulevard Sour Jdid, B.P 16303 Casablanca, Morocco,
assiabakali@yahoo.fr

Ali Hassan[4]
Department of Mathematics, University of Punjab (New campus), Lahore, Pakistan
alihassan.iiui.math@gmail.com

Florentin Smarandache[5]
Department of Mathematics, University of New Mexico,705 Gurley Avenue, Gallup, NM 87301, USA
fsmarandache@gmail.com

Abstract— In this paper, motivated by the notion of generalized single valued neutrosophic graphs of first type, we defined a new neutrosophic graphs named generalized interval valued neutrosophic graphs of first type (GIVNG1) and presented a matrix representation for it and studied few properties of this new concept. The concept of GIVNG1 is an extension of generalized fuzzy graphs (GFG1) and generalized single valued neutrosophic of first type (GSVNG1).

Keywords— Interval valued neutrosophic graph; Generalized Interval valued neutrosophic graphs of first type; Matrix representation.

I. Introduction

Smarandache [7] grounded the concept of neutrosophic set theory (NS) from philosophical point of view by incorporating the degree of indeterminacy or neutrality as independent component to deal with problems involving imprecise, indeterminate and inconsistent information. The concept of neutrosophic set theory is a generalization of the theory of fuzzy set [17], intuitionistic fuzzy sets [14, 15], interval-valued fuzzy sets [13] and interval-valued intuitionistic fuzzy sets [16]. In neutrosophic set every element has three membership degrees including a true membership degree T, an indeterminacy membership degree I and a falsity membership degree F independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. Therefore, if their range is restrained within the real standard unit interval [0, 1], Nevertheless, NSs are hard to be apply in practical problems since the values of the functions of truth, indeterminacy and falsity lie in]−0, 1+[.The single valued neutrosophic set was introduced for the first time by Smarandache in his book [7] . Later on, Wang et al.[10] studied some properties related to single valued neutrosophic sets. In fact sometimes the degree of truth-membership, indeterminacy-membership and falsity- membership about a certain statement cannot be defined exactly in the real situations, but expressed by several possible interval values. So the interval valued neutrosophic set (IVNS) was required. For this purpose, Wang et al.[11] introduced the concept of interval valued neutrosophic set (IVNS for short), which is more precise and more flexible than the single valued neutrosophic set. The interval valued neutrosophic sets (IVNS) is a generalization of the concept of single valued neutrosophic set, in which three membership (T, I, F) functions are independent, and their values belong to the unite interval [0 , 1]. Some more literature about neutrosophic sets, interval valued neutrosophic sets and their applications in various fields can be found in [32, 34, 46].

Graphs are the most powerful and handful tool used in representing information involving relationship between objects and concepts. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes values from [0, 1].The concept fuzzy graphs, intuitionistic fuzzy graphs and their extensions such interval valued fuzzy graphs [2, 3, 12, 20], interval valued intuitionitic fuzzy graphs [41], and so on, have been studied deeply in over hundred papers. All these types of graphs have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects.

In 2016, Samanta et al [37] proposed a new concept called the generalized fuzzy graphs (GFG) and studied some major properties such as completeness and regularity with proved results. The authors classified the GFG into two type. The first type is called generalized fuzzy graphs of first type (GFG1). The second is called generalized fuzzy graphs of second type 2 (GFG2). Each type of GFG are represented by matrices similar to fuzzy graphs. The authors have claimed that fuzzy graphs defined by several researches are limited to represent for some systems such as social network.

When description of the object or their relations or both is indeterminate and inconsistent, it cannot be handled by fuzzy, intuitionistic fuzzy, interval valued fuzzy and interval valued

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intuitionstic fuzzy graphs . So, for this purpose, Smaranadache [9] proposed the concept of neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. Many book on neutrosophic graphs based on literal indeterminacy (I) was completed by Smarandache and Vandasamy [45]. Later on, Smarandache [5, 6] gave another definition for neutrosphic graph theory using the neutrosophic truth-values (T, I, F) without and constructed three structures of neutrosophic graphs: neutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. Later on Smarandache [8] proposed new version of neutrosophic graphs such as neutrosophic offgraph, neutrosophic bipolar/tripola/ multipolar graph. In a short period of time, few authors have focused deeply on the study of neutrosophic vertex-edge graphs and explored diverse types of different neutrosophic graphs.

In 2016, using the concepts of single valued neutrosophic sets, Broumi et al.[27] introduced the concept of single valued neutrosophic graphs, and introduced certain types of single valued neutrosophic graphs (SVNG) such as strong single valued neutrosophic graph, constant single valued neutrosophic graph, complete single valued neutrosophic graph and investigate some of their properties with proofs and examples. Later on, Broumi et al.[28] also introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. In addition, Broumi et al.[29] proved a necessary and sufficient condition for a single valued neutrosophic graph to be an isolated single valued neutrosophic graph. The same authors [35] defined the concept of bipolar single neutrosophic graphs as the generalization of bipolar fuzzy graphs, N-graphs, intuitionistic fuzzy graph, single valued neutrosophic graphs and bipolar intuitionistic fuzzy graphs. In addition, the same authors [36] proposed different types of bipolar single valued neutrosophic graphs such as bipolar single valued neutrosophic graphs, complete bipolar single valued neutrosophic graphs, regular bipolar single valued neutrosophic graphs and investigate some of their related properties. In [30, 31, 47], the authors initiated the idea of interval valued neutrosophic graphs and the concept of strong interval valued neutrosophic graph, where different operations such as union, join, intersection and complement have been investigated.

Nasir et al. [22, 23] proposed a new type of graph called neutrosophic soft graphs and have established a link between graphs and neutrosophic soft sets. The authors also, defined some basic operations of neutrosophic soft graphs such as union, intersection and complement.

Akram et al.[18] proposed a new type of single valued neutrosophic graphs different that the concepts proposed in [22,27] and presented some fundamental operations on single-valued neutrosophic graphs. Also, the authors presented some interesting properties of single-valued neutrosophic graphs by level graphs.

In [19] Malik and Hassan introduced the concept of single valued neutrosophic trees and studied some of their properties. Also, Hassan et Malik [1] proposed some classes of bipolar single valued neutrosophic graphs and investigated some of their properties.

Dhavaseelan et al. [26] introduced the concept of strong neutrosophic graph and studied some interesting properties of strong neutrosophic graphs. P. K. Singh [24] has discussed adequate analysis of uncertainty and vagueness in medical data set using the properties of three-way fuzzy concept lattice and neutrosophic graph introduced by Broumi et al. [27].

Fathhi et al.[43] computed the dissimilarity between two neutrosophic graphs based on the concept of Haussdorff distance.

Ashraf et al.[40], proposed some novels concepts of edge regular, partially edge regular and full edge regular single valued neutrosophic graphs and investigated some of their properties. Also the authors, introduced the notion of single valued neutrosophic digraphs (SVNDGs) and presented an application of SVNDG in multi-attribute decision making.

Mehra and Singh [39] introduced the concept of single valued neutrosophic signed graphs and examined the properties of this concept with examples. Ulucay et al.[44] introduced the concept of neutrosophic soft expert graph and have established a link between graphs and neutrosophic soft expert sets [21] and studies some basic operations of neutrosophic soft experts graphs such as union, intersection and complement.

Recently, Naz et al. [42] defined basic operations on SVNGs such as direct product, Cartesian product, semi-strong product, strong product, lexicographic product, union, ring sum and join and provided an application of single valued neutrosophic digraph (SVNDG) in travel time.

Similar to the interval valued fuzzy graphs and interval valued intuitionistic fuzzy graphs, which have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects. Also, the interval valued neutrosophic graphs presented in the literature [30, 31] have a common property, that edge membership value is less than the minimum of it's end vertex values. Whereas the edge indeterminacy-membership value is less than the maximum of it's end vertex values or is greater than the maximum of its's end vertex values. And the edge non-membership value is less than the minimum of it's end vertex values or is greater than the maximum of its's end vertex values.

Broumi et al [38] have discussed the removal of the edge degree restriction of single valued neutrosophic graphs and presented a new class of single valued neutrosophic graph called generalized single valued neutrosophic graph type1, which is a is an extension of generalized fuzzy graph type1 [37]. with the following

Based on generalized single valued neutrosophic graph of type1(GSVNG1) introduced in [38]. The main objective of this paper is to extend the concept of generalized single valued neutrosophic graph of first type to interval valued neutrosophic graphs first type (GIVNG1) to model systems having an indeterminate information and introduced a matrix representation of GIVNG1. This paper has been arranged as the following:

In Section 2, some fundamental concepts about neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic graph and generalized single valued neutrosophic graphs type 1 are presented which will employed in later sections. In Section 3, the concept of generalized interval valued neutrosophic graphs type 1 is given with an illustrative example. In section 4 a representation matrix of generalized interval valued neutrosophic graphs type 1 is introduced.. Conclusion is also given at the end of section 5.

II. Preliminaries

This section contains some basic definitions from [7, 10, 30, 38] about neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic graphs and generalized single valued neutrosophic graphs type 1, which will helpful for rest of the sections.

Definition 2.1 [7]. Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >, x ∈ X}, where the functions T, I, F: X→]$^-$0,1$^+$[define respectively the truth-membership function, indeterminacy-membership function, and falsity-membership function of the element x ∈ ∈ X to the set A with the condition:

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of]$^-$0,1$^+$[.

Since it is difficult to apply NSs to practical problems, Smarandache [7] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [10]. Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1]. A SVNS A can be written as

$$A=\{< x: T_A(x),\ I_A(x),\ F_A(x) >,\ x \in X\} \quad (2)$$

Definition 2.10[30]. By an interval-valued neutrosophic graph of a graph G* = (V, E) we mean a pair G = (A, B), where A =< [T_{AL}, T_{AU}], [I_{AL}, I_{AU}], [F_{AL}, F_{AU}]> is an interval-valued neutrosophic set on V, and B =<[T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}]> is an interval-valued neutrosophic relation on E satisfying the following condition:

1.V= {$v_1, v_2, \ldots, v_n$} such that T_{AL}:V→[0, 1], T_{AU}:V→[0, 1], I_{AL}:V→[0, 1], I_{AU}:V→[0, 1], and F_{AL}:V→[0, 1], F_{AU}:V→[0, 1] denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element y ∈ V, respectively, and

$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ for all v_i ∈ V (i=1, 2, …, n). (3)

2. The functions T_{BL}:V x V →[0, 1], T_{BU}:V x V →[0, 1], I_{BL}:V x V →[0, 1], I_{BU}:V x V →[0, 1] andF_{BL}:V x V →[0,1], F_{BU}:V x V →[0, 1] are such that:

$T_{BL}(v_i, v_j) \leq$ min [$T_{AL}(v_i$, $T_{AL}(v_j)$], $T_{BU}(v_i, v_j) \leq$ min [$T_{AU}(v_i), T_{AU}(v_j)$] ,

$I_{BL}(v_i, v_j) \geq$ max [$I_{BL}(v_i)$, $I_{BL}(v_j)$], $I_{BU}(v_i, v_j) \geq$ max[$I_{BU}(v_i), I_{BU}(v_j)$] and

$F_{BL}(v_i, v_j) \geq$ max [$F_{BL}(v_i)$, $F_{BL}(v_j)$], $F_{BU}(v_i, v_j) \geq$ max[$F_{BU}(v_i), F_{BU}(v_j)$] (4)

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge (v_i,v_j) ∈ E respectively, where:

$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$, for all ($v_i, v_j$) ∈ E (i, j = 1, 2,…, n). (5)

They called A the interval valued neutrosophic vertex set of V, and B the interval valued neutrosophic edge set of E, respectively; note that B is a symmetric interval valued neutrosophic relation on A.

Example 2.3 [30] Figure 1 is an example for IVNG, G=(A, B) defined on a graph G* = (V, E) such that V= {v_1, v_2, v_3}, E= {$v_1v_2, v_2v_3, v_3v_4, v_4v_1$}, A is an interval valued neutrosophic set of V

A={ < v_1,([0.3, 0.5], [0.2, 0.3],[0.3, 0.4])>, <v_2, ([0.2, 0.3], [0.2, 0.3],[0.1, 0.4]) >, <v_3, ([0.1, 0.3], [0.2, 0.4],[0.3, 0.5]) >}, and B an interval valued neutrosophic set of E ⊆ V x V

B={ < v_1v_2,([0.1, 0.2], [0.3, 0.4],[0.4, 0.5])>, <v_2v_3, ([0.1, 0.3], [0.4, 0.5],[0.4, 0.5]) >, <v_3v_1, ([0.1, 0.2], [0.3, 0.5],[0.4, 0.6]) >}

Remark 2.4: -The underlying set V is vertex set of usual graph that we use it in neutrosophic graph as vertex.

- G*=(V,E) denoted a usual graph where a neutrosophic graphs obtained from it that truth –membership, indeterminacy –membership and non-membership values are 0 to 1.

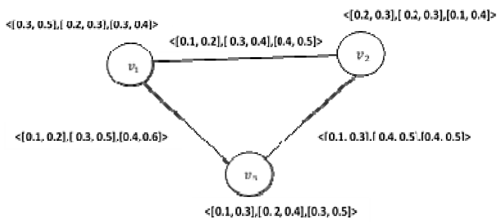

Fig.1. interval valued neutrosophic graph

Definition 2.5 [38]. Let V be a non-void set. Two function are considered as follows:

ρ=(ρ_T, ρ_I, ρ_F):V → [0, 1]^{3}and

ω=($\omega_T, \omega_I, \omega_F$):VxV → [0, 1]^3 . We suppose

A= $\{(\rho_T(x), \rho_T(y)) \mid \omega_T(x, y) \geq 0\}$,
B= $\{(\rho_I(x), \rho_I(y)) \mid \omega_I(x, y) \geq 0\}$,
C= $\{(\rho_F(x), \rho_F(y)) \mid \omega_F(x, y) \geq 0\}$,
We have considered ω_T, ω_I and $\omega_F \geq 0$ for all set A,B, C , since its is possible to have edge degree = 0 (for T, or I, or F).

The triad (V, ρ, ω) is defined to be generalized single valued neutrosophic graph of first type (GSVNG1) if there are functions

α:A→ [0, 1] , β:B→ [0, 1] and δ:C→ [0, 1] such that
$\omega_T(x, y) = \alpha((\rho_T(x), \rho_T(y)))$
$\omega_I(x, y) = \beta((\rho_I(x), \rho_I(y)))$
$\omega_F(x, y) = \delta((\rho_F(x), \rho_F(y)))$
Where x, y∈ V.

Here $\rho(x)$=($\rho_T(x)$, $\rho_I(x)$, $\rho_F(x)$), x∈ V are the membership, indeterminacy and non-membership of the vertex x and $\omega(x, y)$=($\omega_T(x, y)$, $\omega_I(x, y)$, $\omega_F(x, y)$), x, y∈ V are the membership, indeterminacy and non-membership values of the edge (x, y).

III. Generalized Interval Valued Neutrosophic Graph of First Type

In this section, based on the generalized single valued neutrosophic graphs of first type proposed by Broumi et al.[38], the definition of generalized interval valued neutrosophic graphs first type is defined as follow:

Definition 3.1. Let V be a non-void set. Two function are considered as follows:
ρ=([ρ_T^L, ρ_T^U], [ρ_I^L, ρ_I^U], [ρ_F^L, ρ_F^U]):V → $[0, 1]^3$and
ω=([ω_T^L, ω_T^U], [ω_I^L, ω_I^U], [ω_F^L, ω_F^U]):VxV → $[0, 1]^3$. We suppose
A= $\{([\rho_T^L(x), \rho_T^U(x)], [\rho_T^L(y), \rho_T^U(y)]) \mid \omega_T^L(x, y) \geq 0$ and $\omega_T^U(x, y) \geq 0\}$,
B= $\{([\rho_I^L(x), \rho_I^U(x)], [\rho_I^L(y), \rho_I^U(y)]) \mid \omega_I^L(x, y) \geq 0$ and $\omega_I^U(x, y) \geq 0\}$,
C= $\{([\rho_F^L(x), \rho_F^U(x)], [\rho_F^L(y), \rho_F^U(y)]) \mid \omega_F^L(x, y) \geq 0$ and $\omega_F^U(x, y) \geq 0\}$,
We have considered ω_T^L, ω_T^U, ω_I^L, ω_I^U, ω_F^L, $\omega_F^U \geq 0$ for all set A, B, C , since its is possible to have edge degree = 0 (for T, or I, or F).

The triad (V, ρ, ω) is defined to be generalized interval valued neutrosophic graph of first type (GIVNG1) if there are functions

α:A→ [0, 1] , β:B→ [0, 1] and δ:C→ [0, 1] such that
$\omega_T^L(x, y) = \alpha((\rho_T^L(x), \rho_T^L(y)))$, $\omega_T^U(x, y) = \alpha((\rho_T^U(x), \rho_T^U(y)))$,
$\omega_I^L(x, y) = \beta((\rho_I^L(x), \rho_I^L(y)))$, $\omega_I^U(x, y) = \beta((\rho_I^U(x), \rho_I^U(y)))$,
$\omega_F^L(x, y) = \delta((\rho_F^L(x), \rho_F^L(y)))$, $\omega_F^L(x, y) = \delta((\rho_F^U(x), \rho_F^U(y)))$
Where x, y∈ V.

Here $\rho(x)$=($\rho_T(x)$, $\rho_I(x)$, $\rho_F(x)$), x∈ V are the interval membership, interval indeterminacy and interval non-membership of the vertex x and $\omega(x, y)$=($\omega_T(x, y)$, $\omega_I(x, y)$, $\omega_F(x, y)$), x, y∈ V are the interval membership, interval indeterminacy membership and interval non-membership values of the edge (x, y).

Example 3.2 : Let the vertex set be V={x, y, z, t} and edge set be E={(x, y),(x, z),(x, t),(y, t)}

	x	y	z	t
[ρ_T^L, ρ_T^U]	[0.5, 0.6]	[0.9 , 1]	[0.3, 0.4]	[0.8, 0.9]
[ρ_I^L, ρ_I^U]	[0.3, 0.4]	[0.2, 0.3]	[0.1, 0.2]	[0.5, 0.6]
[ρ_F^L, ρ_F^U]	[0.1, 0.2]	[0.6, 0.7]	[0.8, 0.9]	[0.4, 0.5]

Table 1: interval membership, interval indeterminacy and interval non-membership of the vertex set.

Let us consider functions $\alpha(m, n)$= m ∨ m= $\beta(m, n)$= $\delta(m, n)$
Here, A={([0.5, 0.6], [0.9, 1]), ([0.5, 0.6], [0.3, 0.4]), ([0.5, 0.6], [0.8, 0.9]), ([0.9, 1.0], [0.8, 0.9])}
B = {([0.3, 0.4], [0.2, 0.3]), ([0.3, 0.4], [0.1, 0.2]), ([0.3, 0.4], [0.5, 0.6]), ([0.2, 0.3], [0.5, 0.6])}
C = {([0.1, 0.2], [0.6, 0.7]), ([0.1, 0.2], [0.8, 0.9]), ([0.1, 0.2], [0.4, 0.5]), ([0.6, 0.7], [0.4, 0.5])}.Then

ω	(x, y)	(x, z)	(x, t)	(y, t)
[ω_T^L, ω_T^U]	[0.9, 1]	[0.5, 0.6]	[0.8,0.9]	[0.9,1]
[ω_I^L, ω_I^U]	[0.3,0.4]	[0.3,0.4]	[0.5,0.6]	[0.5, 0.6]
[ω_F^L, ω_F^U]	[0.6, 0.7]	[0.8,0.9]	[0.4,0.5]	[0.6,0.7]

Table 2: membership, indeterminacy and non-membership of the edge set.

The corresponding generalized single valued neutrosophic graph is shown in Fig.2

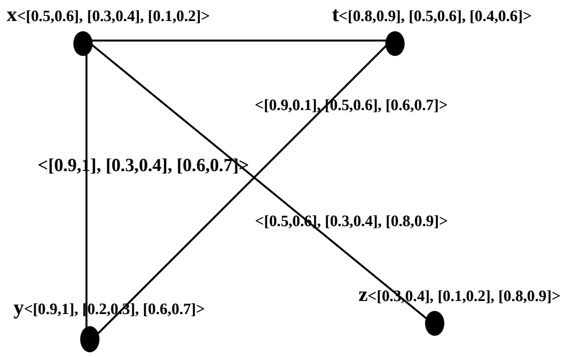

Fig 2.GIVNG of first type.

The easier way to represent any graph is to use the matrix representation. The adjacency matrices, incident matrices are the widely matrices used. In the following section GIVNG1 is represented by adjacency matrix.

IV. Matrix Representation of Generalized Interval Valued Neutrosophic Graph of First Type

Because Interval membership, interval indeterminacy membership and interval non-membership of the vertices are considered independents. In this section, we extended the representation matrix of generalized single valued neutrosophic graphs first type proposed in [38] to the case of generalized interval valued neutrosophic graphs of first type.

The generalized interval valued neutrosophic graph (GIVNG1) has one property that edge membership values (T, I, F) depends on the membership values (T, I, F) of adjacent vertices . Suppose ξ=(V, ρ, ω) is a GIVNG1 where vertex set V={v_1, v_2,…, v_n}. The functions

α :A→ (0, 1] is taken such that

$\omega_T^L(x,y) = \alpha((\rho_T^L(x), \rho_T^L(y)))$, $\omega_T^U(x,y) = \alpha((\rho_T^U(x), \rho_T^U(y)))$,

Where x, y∈ V and A= {([$\rho_T^L(x)$, $\rho_T^U(x)$], [$\rho_T^L(y)$, $\rho_T^U(y)$]) |ω_T^L(x, y) ≥ 0 and ω_T^U(x, y) ≥ 0 }

,

β :B→ (0, 1] is taken such that

$\omega_I^L(x,y) = \beta((\rho_I^L(x), \rho_I^L(y)))$, $\omega_I^U(x,y) = \beta((\rho_I^U(x), \rho_I^U(y)))$,

Where x, y∈ V and B= {([$\rho_I^L(x)$, $\rho_I^U(x)$], [$\rho_I^L(y)$, $\rho_I^U(y)$]) |ω_I^L(x, y) ≥ 0 and ω_I^U(x, y) ≥ 0 }

and

δ :C→ (0, 1] is taken such that

$\omega_F^L(x,y) = \delta((\rho_F^L(x), \rho_F^L(y)))$, $\omega_F^U(x,y) = \delta((\rho_F^U(x), \rho_F^U(y)))$,

Where x, y∈ V and C= {([$\rho_F^L(x)$, $\rho_F^U(x)$], [$\rho_F^L(y)$, $\rho_F^U(y)$]) | ω_F^L(x, y) ≥ 0 and ω_F^U(x, y) ≥ 0 }. The GIVNG1 can be represented by (n+1) x (n+1) matrix $M_{G_1}^{T,I,F}$=[$a^{T,I,F}$(i, j)] as follows:

The interval membership (T), interval indeterminacy-membership (I) and the interval non-membership (F) values of the vertices are provided in the first row and first column. The (i+1, j+1)- th-entry are the membership (T), indeterminacy-membership (I) and the non-membership (F) values of the edge (x_i, x_j), i, j=1,…,n if i≠j.

The (i, i)-th entry is $\rho(x_i)$=($\rho_T(x_i)$, $\rho_I(x_i)$, $\rho_F(x_i)$), where i=1,2,…,n. The interval membership (T), interval indeterminacy-membership (I) and the interval non-membership (F) values of the edge can be computed easily using the functions α, β and δ which are in (1,1)-position of the matrix. The matrix representation of GIVNG1, denoted by $M_{G_1}^{T,I,F}$, can be written as three matrix representation $M_{G_1}^T$, $M_{G_1}^I$ and $M_{G_1}^F$. For convenience representation $v_i(\rho_T(v_i))$ =[$\rho_T^L(v_i)$, $\rho_T^U(v_i)$], for i=1, …., n

The $M_{G_1}^T$ can be represented as follows

α	$v_1(\rho_T(v_1))$	$v_2(\rho_T(v_2))$	$v_n(\rho_T(v_n))$
$v_1(\rho_T(v_1))$	$[\rho_T^L(v_1), \rho_T^U(v_1)]$	$\alpha(\rho_T(v_1), \rho_T(v_2))$	$\alpha(\rho_T(v_1), \rho_T(v_n))$
$v_2(\rho_T(v_2))$	$\alpha(\rho_T(v_2), \rho_T(v_1))$	$[\rho_T^L(v_2), \rho_T^U(v_2)]$	$\alpha(\rho_T(v_2), \rho_T(v_2))$
…	….	…	…
$v_n(\rho_T(v_n))$	$\alpha(\rho_T(v_n), \rho_T(v_1))$	$\alpha(\rho_T(v_n), \rho_T(v_2))$	$[\rho_T^L(v_n), \rho_T^U(v_n)]$

Table3. Matrix representation of T-GIVNG1

The $M_{G_1}^I$ can be represented as follows

β	$v_1(\rho_I(v_1))$	$v_2(\rho_I(v_2))$	$v_n(\rho_I(v_n))$
$v_1(\rho_I(v_1))$	$[\rho_I^L(v_1), \rho_I^U(v_1)]$	$\beta(\rho_I(v_1), \rho_I(v_2))$	$\beta(\rho_I(v_1), \rho_I(v_n))$
$v_2(\rho_I(v_2))$	$\beta(\rho_I(v_2), \rho_I(v_1))$	$[\rho_I^L(v_2), \rho_I^U(v_2)]$	$\beta(\rho_I(v_2), \rho_I(v_2))$
…	….	…	…
$v_n(\rho_I(v_n))$	$\beta(\rho_I(v_n), \rho_I(v_1))$	$\beta(\rho_T(v_n), \rho_I(v_2))$	$[\rho_I^L(v_n), \rho_I^U(v_n)]$

Table4. Matrix representation of I-GIVNG1

The $M_{G_1}^I$ can be represented as follows

δ	$v_1(\rho_F(v_1))$	$v_2(\rho_F(v_2))$	$v_n(\rho_F(v_n))$
$v_1(\rho_F(v_1))$	$[\rho_F^L(v_1), \rho_F^U(v_1)]$	$\delta(\rho_F(v_1), \rho_F(v_2))$	$\delta(\rho_F(v_1), \rho_F(v_n))$
$v_2(\rho_F(v_2))$	$\delta(\rho_F(v_2), \rho_F(v_1))$	$[\rho_F^L(v_2), \rho_F^U(v_2)]$	$\delta(\rho_F(v_2), \rho_F(v_2))$
…	….	…	…
$v_n(\rho_F(v_n))$	$\delta(\rho_F(v_n), \rho_F(v_1))$	$\delta(\rho_F(v_n), \rho_F(v_2))$	$[\rho_F^L(v_n), \rho_F^U(v_n)]$

Table5. Matrix representation of F-GIVNG1

Remark1 : if $\rho_I^L(x)$= $\rho_I^U(x)$=0 and $\rho_F^L(x)$= $\rho_F^U(x) = 0$ the generalized interval valued neutrosophic graphs type 1 is reduced to generalized fuzzy graphs type 1 (GFG1).

Remark 2: if $\rho_T^L(x)$ = $\rho_T^U(x)$, $\rho_I^L(x)$ = $\rho_I^U(x)$ and $\rho_F^L(x)$= $\rho_F^U(x)$, the generalized interval valued neutrosophic graphs type 1 is reduced to generalized single valued graphs type 1 (GSVNG1).

Here the generalized Interval valued neutrosophic graph of first type (GIVNG1) can be represented by the matrix representation depicted in table 9. The matrix representation can be written as three interval matrices one containing the entries as T, I, F (see table 6, 7 and 8).

α = max(x, y)	x([0.5,0.6])	y([0.9,1])	z(0.3,0.4])	t([0.8,0.9])
x([0.5,0.6])	[0.5,0.6]	[0.9, 1.0]	[0.5, 0.6]	[0.8,0.9]
y([0.9,1])	[0.9, 1.0]	[0.9,1]	[0, 0]	[0.9,1.0]
z([0.3,0.4])	[0.5, 0.6]	[0, 0]	[0.3,0.4]	[0, 0]
t([0.8,0.9])	[0.8, 0.9]	[0.9, 1.0]	[0, 0]	[0.8,0.9]

Table 6:Lower and upper Truth- matrix representation of GIVNG1

β = max(x, y)	x([0.3,0.4])	y([0.2,0.3])	z([0.1,0.2])	t([0.5,0.6])
x([0.3,0.4])	[0.3,0.4]	[0.3,0.4]	[0.3,0.4]	[0.5,0.6]
y([0.2,0.3])	[0.3,0.4]	[0.2,0.3]	[0, 0]	[0.5,0.6]
z([0.1,0.2])	[0.3,0.4]	[0, 0]	[0.1,0.2]	[0, 0]
t([0.5,0.6])	[0.5,0.6]	[0.5,0.6]	[0, 0]	[0.5,0.6]

Table 7: lower and upper Indeterminacy- matrix representation of GIVNG1

δ= max(x, y)	x([0.1,0.2])	y([0.6,0.7])	z([0.8,0.9])	t([0.4,0.6])
x([0.1,0.2])	[0.1,0.2]	[0.6,0.7]	[0.8,0.9]	[0.4,0.6]
y([0.6,0.7])	[0.6,0.7]	[0.6,0.7]	[0, 0]	[0.6,0.7]
z([0.8,0.9])	[0.8,0.9]	[0, 0]	[0.8,0.9]	[0, 0]
t([0.4,0.6])	[0.4,0.6]	[0.6,0.7]	[0, 0]	[0.4,0.6]

Table 8: Lower and upper Falsity- matrix representation of GIVNG1

The matrix representation of GIVNG1 can be represented as follows:

(α,β,δ)	x(0.5,0.3,0.1)	y(0.9,0.2,0.6)	z(0.3,0.1,0.8)	t(0.8,0.5,0.4)
x(**[0.5,0.6], [0.3,0.4], [0.1,0.2]**)	**<[0.5,0.6], [0.3,0.4], [0.1,0.2]>**	**<[0.9,1.0], [0.3,0.4], [0.6,0.7]>**	**<[0.5,0.6], [0.3,0.4], [0.8,0.9]>**	**<[0.8,0.9], [0.5,0.6], [0.4,0.6]>**
y(**[0.9, 1.0], [0.2, 0.3], [0.6, 0.7]**)	**<[0.9,1.0], [0.3,0.4], [0.6,0.7]>**	**<[0.9, 1.0], [0.2, 0.3], [0.6, 0.7]>**	**<[0, 0], [0, 0], [0, 0]>**	**<[0.9, 1.0], [0.5, 0.6], [0.6, 0.7]>**
z(**[0.3,0.4], [0.1,0.2], [0.8,0.9]**)	**<[0.5,0.6], [0.3,0.4], [0.8,0.9]>**	**<[0, 0], [0, 0], [0, 0]>**	**<[0.3,0.4], [0.1,0.2], [0.8,0.9]>**	**<[0, 0], [0, 0], [0, 0]>**
t(**[0.8,0.9], [0.5,0.6], [0.4,0.6]**)	**<[0.8,0.9], [0.5,0.6], [0.4,0.6]>**	**<[0.9,1.0], [0.5,0.6], [0.6,0.7]>**	**<[0, 0], [0, 0], [0, 0]>**	**<[0.8,0.9], [0.5,0.6], [0.4,0.6]>**

Table 9: Matrix representation of GIVNG1.

Theorem 1. Let $M_{G_1}^T$ be matrix representation of T-GIVNG1, then the degree of vertex $D_T(x_k)$ =$[\sum_{j=1,j\neq k}^n a_T^L(k+1,j+1), \sum_{j=1,j\neq k}^n a_T^U(k+1,j+1)]$, $x_k \in$ V or
$D_T(x_p)$ = $[\sum_{i=1,i\neq p}^n a_T^L(i+1,p+1)$, $\sum_{i=1,i\neq p}^n a_T^U(i+1,p+1)$ $x_p \in$ V.

Proof :is similar as in theorem 1 of [37].

Theorem 2. Let $M_{G_1}^I$ be matrix representation of I-GIVNG1, then the degree of vertex $D_I(x_k)$ =$[\sum_{j=1,j\neq k}^n a_I^L(k+1,j+1), \sum_{j=1,j\neq k}^n a_I^U(k+1,j+1)]$, $x_k \in$ V
or $D_I(x_p)$ =$[\sum_{i=1,i\neq p}^n a_I^L(i+1,p+1)$,$\sum_{i=1,i\neq p}^n a_I^U(i+1,p+1)$ $x_p \in$ V.
Proof :is similar as in theorem 1 of [37].

Theorem 3. Let $M_{G_1}^F$ be matrix representation of F-GIVNG1, then the degree of vertex
$D_F(x_k)$ =[$\sum_{j=1,j\neq k}^n a_F^L(k+1,j+1)$, $\sum_{j=1,j\neq k}^n a_F^U(k+1,j+1)$], $x_k \in$ V or
$D_F(x_p)$ = $[\sum_{i=1,i\neq p}^n a_F^L(i+1,p+1)$, $\sum_{i=1,i\neq p}^n a_F^U(i+1,p+1)$ $x_p \in$ V.

Proof :is similar as in theorem 1 of [37].

Theorem4. Let $M_{G_1}^{T,I,F}$ be matrix representation of GIVNG1, then the degree of vertex D(x_k) =($D_T(x_k), D_I(x_k), D_F(x_k)$) where
$D_T(x_k)$ =[$\sum_{j=1,j\neq k}^n a_T^L(k+1,j+1)$, $\sum_{j=1,j\neq k}^n a_T^U(k+1,j+1)$] $x_k \in$ V.
$D_I(x_k)$ ==[$\sum_{j=1,j\neq k}^n a_I^L(k+1,j+1)$, $\sum_{j=1,j\neq k}^n a_I^U(k+1,j+1)$] $x_k \in$ V.
$D_F(x_k)$ ==[$\sum_{j=1,j\neq k}^n a_F^L(k+1,j+1)$, $\sum_{j=1,j\neq k}^n a_F^U(k+1,j+1)$] $x_k \in$ V.
Proof: the proof is obvious.

V. CONCLUSION

In this article, we have extended the concept of generalized single valued neutrosophic graph type 1 (GSVNG1) to generalized interval valued neutrosophic graph type 1 (GIVNG1) and presented a matrix representation of it. In the future works, we plan to study the concept of completeness, the concept of regularity and to define the concept of generalized interval valued neutrosophic graphs type 2.

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