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Generalized Interval-Valued Vague Soft Set

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Abstract

The concept of generalized interval-valued vague soft set is introduced along with its operations of intersection and union. An application of generalized interval-valued vague soft sets in decision making with respect to degree of preference is illustrated.

Keywords: Generalized, Interval-Valued, Vague Soft Set.

1 Introduction

Many fields in decision-making deal with certain data such as goal programming [1–11] and data envelopment analysis [12-14]. However, some fields deal with uncertain data that may not be successfully modeled by classical mathematics. Molodtsov [15] introduced the concept of soft set. Since then, the operations of soft sets and applications of soft set theory have been studied increasingly. New theoretical studies of fuzzy soft set theory and applications of these studies in decision making [16–21] were developed with application on fuzzy sets to genetic algorithms [22–23]. In this paper we define the concept of generalized interval-valued vague soft set as extension to our earlier studies on vague soft sets [24–28]. Basic operations such as union and intersection are presented. Finally, we give an application of generalized interval-valued vague soft set.

2 Generalized Interval-Valued Vague Soft Set

Let U be an initial universe and E a set of parameters.

Definition 2.1 A pair (F, A) is called a generalized interval-valued vague soft set over *U*, where F is a mapping given by $F : A \to IVV^U$ and IVV^U the collection of all interval-valued vague subset of *U*. Let $F_\alpha : A \to IVV^U \times [0,1]$ be a function defined as $F_\alpha(a) = (F(a) = \{x, t_{F(a)}, 1 - f_{F(a)}\}, \alpha(a))$.
Then F_α is called the generalized interval-valued vague soft set over (U, E).

Definition 2.2 Let (F_α, A) and (G_β, B) be two generalized interval-valued vague soft sets (F_α, A) and (G_β, B) over universe (U, E). Now (F_α, A) is called a generalized interval-valued vague soft subset of (G_β, B) if

(i) α is a vague subset of β,
(ii) $A \subseteq B$,
(iii) $\forall a \in A$, $F_\alpha(a)$ is an interval-valued vague subset of $G_\beta(a)$.

Definition 2.3 The union of two generalized interval-valued vague soft sets $\left\langle F_\alpha, A \right\rangle$ and $\left\langle G_\beta, B \right\rangle$ over a universe *U*, is a generalized interval-valued vague soft set $\left\langle H_\Omega, C \right\rangle$, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$t_{H\Omega(e)}(x) = \begin{cases} t_{F_\alpha(\varepsilon)}(x) & \text{if } \varepsilon \in A - B, x \in U \\ t_{G_\beta(\varepsilon)}(x) & \text{if } \varepsilon \in B - A, x \in U \\ [\sup(\underline{t}_{F_\alpha(\varepsilon)}(x), \underline{t}_{G_\beta(\varepsilon)}(x)), \sup(\bar{t}_{F_\alpha(\varepsilon)}(x), \bar{t}_{G_\beta(\varepsilon)}(x))] & \text{if } \varepsilon \in A \cap B, x \in U \end{cases}$$

$$1 - f_{H\Omega(e)}(x) = \begin{cases} 1 - f_{F_\alpha(\varepsilon)}(x) & \text{if } \varepsilon \in A - B, x \in U \\ 1 - f_{G_\beta(\varepsilon)}(x) & \text{if } \varepsilon \in B - A, x \in U \\ [\inf(1 - f_{F_\alpha(\varepsilon)}(x), 1 - f_{G_\beta(\varepsilon)}(x)), \inf(1 - f_{F_\alpha(\varepsilon)}(x), 1 - f_{G_\beta(\varepsilon)})(x))], & \text{if } \varepsilon \in A \cap B, x \in U \end{cases}$$

where $\Omega = \max\{\alpha, \beta\}$.

Definition 2.4 The intersection of two generalized interval-valued vague soft sets $\left\langle F_\alpha, A \right\rangle$ and $\left\langle G_\beta, B \right\rangle$ over a universe U, is a generalized interval-valued vague soft set $\left\langle H_\Omega, C \right\rangle$, where $C = A \cap B$ and $\forall \varepsilon \in C$,

$$t_{H\Omega(e)}(x)=\begin{cases} t_{F_\alpha(\varepsilon)}(x) & \text{if } \varepsilon\in A-B, x\in U \\ t_{G_\beta(\varepsilon)}(x) & \text{if } \varepsilon\in B-A, x\in U \\ [\inf(t_{F_\alpha(\varepsilon)}(x), t_{G_\beta(\varepsilon)}(x)), \inf(t_{F_\alpha(\varepsilon)}(x), t_{G_\beta(\varepsilon)}(x))] & \text{if } \varepsilon\in A\cap B, x\in U \end{cases}$$

$$1-f_{H\Omega(e)}(x)=\begin{cases} 1-\underline{f}_{F_\alpha(\varepsilon)}(x) & \text{if } \varepsilon\in A-B, x\in U \\ 1-f_{G_\beta(\varepsilon)}(x) & \text{if } \varepsilon\in B-A, x\in U \\ [\sup(1-\underline{f}_{F_\alpha(\varepsilon)}(x), 1-\underline{f}_{G_\beta(\varepsilon)}(x)), \sup(1-\overline{f}_{F_\alpha(\varepsilon)}(x), 1-\overline{f}_{G_\beta(\varepsilon)}(x))], & \text{if } \varepsilon\in A\cap B, x\in U \end{cases}$$

where $\Omega=\min\{\alpha,\beta\}$.

3 Application of Generalized Interval-Valued Vague Soft Set

We illustrate an application of generalized interval-valued vague soft set in decision making.

Example 3.1. Assume that a hotel chain wants to fill a position for its room service. There are three hotels in the universe $U=\{u_1,u_2,u_3\}$ and the parameters set $E=\{e_1,e_2,e_3\}$ where $e_i(i=1,2,3)$ stand for "experience", "efficient" and "ambience" respectively. Let the generalized interval-valued vague soft set $(\tilde{F}_\alpha, A)$ be defined as $(\tilde{F}_\alpha, E)=$

$\{\, e_1=\left(\langle u_1,[0.6,0.8],[0.7,0.8]\rangle, \langle u_2,[0.8,0.9],[1,1]\rangle, \langle u_3,[0.68,0.82],[0.9,0.8]\rangle, 0.4\right),$

$e_2=\left(\langle u_1,[0.8,0.7],[0.91,0.7]\rangle, \langle u_2,[0.6,0.7],[1,0.9]\rangle, \langle u_3,[1,0],[1,0.18]\rangle, 0.2\right),$

$e_3=\left(\langle u_1,[0.3,0.4],[0.5,0.7]\rangle, \langle u_2,[0.2,0.1],[0.8,0.9]\rangle, \langle u_3,[1,0],[1,0]\rangle\right), 0.6\,\}.$

Table 1 and Table 2 present the lower and upper memberships in generalized interval-valued vague soft set respectively, while Table 3 and Table 4 are representations of the truth and false membership functions.

Table 1: Truth-membership function

U	u_1	u_2	u_3	λ
e_1	[0.6,0.8]	[0.8,0.9]	[0.7,0.9]	0.4
e_2	[0.8,0.7]	[0.6,0.7]	[1,0]	0.2
e_3	[0.3,0.4]	[0.2,0.1]	[1,0]	0.6

Table 3: Representation of truth-membership

U	u_1	u_2	u_3	λ
e_1	-0.2	<u>-0.1</u>	-0.2	0.4
e_2	0.1	-0.1	<u>1</u>	0.2
e_3	-0.1	0.1	<u>1</u>	0.6

Table 2: False-membership function

U	u_1	u_2	u_3	λ
e_1	[0.7,0.8]	[1,1]	[0.9,0.8]	0.4
e_2	[0.91,0.7]	[1,0.9]	[1,0.18]	0.2
e_3	[0.5,0.7]	[0.8,0.9]	[1,0]	0.6

Table 4: Representation of false-membership

U	u_1	u_2	u_3	λ
e_1	-0.1	0	$\underline{0.1}$	0.4
e_2	0.21	0.1	$\underline{0.82}$	0.2
e_3	-0.2	-0.1	$\underline{1}$	0.6

The scores using data in Table 3 of truth-membership function representation are: score (u_1) =0, score (u_2) =(-0.1*0.4)= -0.04, score (u_3) =(1*0.2)+(1*0.6)= 0.8.

The scores using data in Table 4 false-membership function representation are: score (u_1) = 0, score (u_2) = 0, score (u_3) = (0.1*0.4)+(0.82*0.2)+(1*0.6)=0.804.

The final score of u_i by adding the above two scores are:
score (u_1) = 0, score (u_2) = -0.4+0 = -0.4, score (u_3) = 0.8+0.804=1.604.

Thus the hotel chain will choose to hire personnel for room service at hotel u_2.

4 Conclusion

The notion of a generalized interval-valued vague soft set as an extension to the vague soft set is introduced. The basic properties are presented and its application in decision making is illustrated.

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