A New Morphological 3D Shape Decomposition: Grayscale Interframe Interpolation Method

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Abstract

One of the main image representations in Mathematical Morphology is the 3D Shape Decomposition Representation, useful for Image Compression, and Pattern Recognition. The 3D Morphological Shape Decomposition representation can be generalized a number of times, to extend the scope of its algebraic characteristics as much as possible. With these generalizations, the Morphological Shape Decomposition 's role to serve as an efficient image decomposition tool was extended to discrete images and grayscale images.
This work follows the above line, and further develops it. A new evolutionary branch is added to the 3D Morphological Shape Decomposition's development, by the introduction of a 3D Multi Structuring Element Morphological Shape Decomposition, which permits 3D Morphological Shape Decomposition of 3D binary images (grayscale iamges) into "multiparameter" families of elements. At the beginning, 3D Morphological Shape Decomposition representations are based only on "1 parameter" families of elements for image decomposition.
This paper addresses the gray scale interframe interpolation by means of mathematical morphology. The new interframe interpolation method, called 3D Shape Decomposition interpolation is based on morphological 3D Shape Decomposition. This article will present the theoretical background of the morphological interframe interpolation, deduce the new representation and show some application examples. Computer simulations could illustrate results.

1. Introduction

Image Representation is a key component in many tasks in Computer Vision and Image Processing. It consists generally of presenting an image in a form, different from the original one, in which desired characteristics of the image are emphasized and more easily accessed.
In this work, we consider morphological methods for both binary and grayscale image representation. They are based on Mathematical Morphology, which is a relatively new, and rapidly growing, nonlinear theory for Image Processing.
Mathematical Morphology is part of Set Theory, and it has a strong geometric orientation. Its theory was developed by Matheron and Serra, in the middle 60's, with the purpose of describing the shape decomposition of materials by image analysis of their sections.
Being originally developed for binary images, it was later (during the 70's) generalized for grayscale images as well.
For binary images (2D and 3D), Mathematical Morphology provides a well founded theory for analysis and processing. Therefore, Binary Morphological Representations can be developed and analyzed. Grayscale Morphological Representations are a generalization of the binary representations in 3D space, and they emphasize geometrical characteristics of the image, which are not easily accessed in a linear representation.
The main morphological representation for 3D binary images is the Shape decomposition. The 3D Shape decomposition can be calculated entirely by the basic operations of Mathematical Morphology, which makes the 3D Shape decomposition a morphological representation, suitable for analysis by morphological tools.
Based on 3D Shape decomposition a morphological representation, called 3D Morphological Shape Decomposition is presented. It consists of first calculating the ball (or other predefined convex 3D shapes) with the greatest size contained inside the shape, then taking the residue (set difference) between the original shape to the above greatest balls, and finally reiterating the above procedure on the residue until the whole shape is decomposed. The resulting decomposition elements are, therefore, disjoint.
Following the generalization of the whole morphological theory, from binary to grayscale images, the Morphological Shape Decomposition Representation could be generalized as well to grayscale images [2,3].
The motivation of this work is to investigate the use of Morphological Shape Decomposition's for binary and grayscale Image Representation, and their applications, with special interest in interframe interpolation.
Interframe interpolation problem is the process of creating intermediary frames between two gives one. Two known frames – input frame and output frame, as well as intermediary frames are of the same type.
Interpolated frame (X_i) depends on the content of input frame and output frame and the position of the interpolated

one between them. It is a number between 0 and M denoted in this paper as m, $0 \le n \le M$.
For $m = 0$ interpolated frame is equal with input frame, X_0, for $m = M$ interpolated frame is equal with output frame, X_M.
In the following pages we will consider a morphological methods for grayscale frame interpolation.
These methods are based on 3D binary mathematical morphology.
Mathematical Morphology is a general theory based of simple operators: erosion, dilation, opening and closing. The morphological operations have been successfully used in many applications including object recognition, frame enhancement, texture analysis, and industrial inspection.
The new interpolation frame method presented in this article is based on 3D morphological Shape Decomposition. It consists of changing, step by step, the Shape Decomposition subsets of input frame with the Shape Decomposition subsets of out frame [1,4,5].

2. 3D Morphological Shape Decomposition

The 3D Shape Decomposition can be calculated entirely using the basic morphological operators.
Dilation and erosion are the fundamental operators of the Mathematical Morphology. The key process in the dilation and erosion operators is the local comparison of a shape, called structuring element, with the object to be transformed.
The structuring element is a predefined shape, which is used for morphological processing of the frames. The most common shapes used as structuring elements are horizontal and vertical lines, squares, digital discs, crosses, etc.
The fundamental morphological operators are based on the operation of translation. Let *B* be a set contained in Z^3, and let *x* be a point in Z^3. The translation of the set *B* by the point x, denoted B_x, is defined as follows:

$$B_x = \left\{ b + x \,\middle|\, b \in B \right\} \tag{1}$$

The dilation of the 3D binary shape *X* by the structuring element *B*, denoted $X \oplus B$, is defined by:

$$X \oplus B = \bigcup_{x \in X} B_x \tag{2}$$

The erosion of 3D binary shape *X* by the structuring element *B*, denoted $X \ominus B$, is defined in the following way:

$$X \ominus B = \bigcap_{b \in B} X_{-b} \tag{3}$$

Based on those fundamental operators, two morphological operators are developed. These are the opening and closing operators.
The opening operator, denoted "$\circ$", can be expressed as a composition of erosion followed by dilation, both by the same input structuring element:

$$X \circ B = \left(X \ominus B \right) \oplus B \tag{4}$$

The closing operator, denoted "$\bullet$", can be expressed as composition of dilation followed by erosion by the same input structuring element:

$$X \bullet B = \left(X \oplus B \right) \ominus B \tag{5}$$

The Shape Decomposition *ST*(*X*,*B*) of 3D binary shape *X* in Z^3 can be calculated by means of morphological operations.
The Morphological Shape Decomposition is a compact error free representation of images.

2.1. Original Morphological Shape Decomposition

The 3D morphological shape decomposition is a compact error free representation of images, a property useful for image compression. The Shape decomposition is a redundant representation, i.e., some of its points may be discarded without affecting its error free characteristic.
The morphological shape decomposition representation of a binary image *X*, with a given binary 3D structuring element *B*, is a collection of sets $SD_n(X, B)$ called Shape decomposition subsets of order *n*.
Morphological shape decomposition is defined us following:
I) Initial condition
a) *N* is the maximum value defined by with

$$X \ominus nB - \left(X \ominus nB \right) \circ B = \varnothing,\ n > N \tag{6}$$

and

$$X \ominus NB - \left(X \ominus NB \right) \circ B \neq \varnothing \tag{7}$$

b) We define

$$X^{(N)} = X \tag{8}$$

$$SD_N\left(X, B \right) = X \ominus NB - \left(X \ominus NB \right) \circ B \tag{9}$$

II) Recursive relations
We can define iteratively, for $n = N, N-1, ..., 2, 1$

$$X^{(n-1)} = X^{(n)} - SD_n(X,B) \oplus nB \tag{10}$$

$$SD_n(X,B) = X \ominus nB - (X \ominus nB) \circ B \tag{11}$$

From the collection of morphological shape decomposition subsets $SD_n(X,B)$ the original shape X can be perfectly reconstruct by

$$X = \bigcup_{n \geq 0} SD_n(X,B) \oplus nB \tag{12}$$

A generalization of the 3D shape decomposition framework should be sought, based either on a topological or an algebraic approach. In the topological approach, such a generalization should aspire to solve problems like robustness, connectivity and precision as a shape descriptor; whereas, in the algebraic one, framework flexibility, self-duality, and representation efficiency are the main issues.
The 3D morphological shape decomposition satisfies the above requirements. The algebraic framework of the 3D shape decomposition will be extended several times. The purpose was always to obtain decompositions according to richer families of elements, not failing to satisfy the above algebraic requirements.
The above evolution will be described.
The morphological shape decomposition is an error-free representation since the original 3D binary image (gray scale image) X can be reconstructed from $SD_n(X,B)$.

2.2. Modified Shape decomposition

The family of 3D elements nB used in the discrete morphological shape decomposition is generated by recursively dilating the structuring element B by itself. B serves here as a generator, being constant at every step of the family generation. The family generator can have a variable size. The Modified Shape decomposition decomposes X into maximal elements from the family $\{0B, 1B, 2B, 4B, 8B, 16B, ...\}$. This family can be generated, as before, by a series of dilations, but not with a constant generator. The sizes of the generator, at the various steps, are the differences $\{1,1,2,4,8,16,...\}$ between the sizes of the elements of the family. We can observe, by comparing the conditions for the modified shape decomposition with those for the morphological shape decomposition that the only significant difference is in the family generation. The modified shape decomposition also fully represents the 3D binary original image X. Generally, coding simulations, the modified morphological shape decomposition show better results than the previous morphological shape decomposition.
For modified shape decomposition, X is bounded, the family is indexed by $n \in N$, and exponentially generated:

$$A(n) = 2^{n-1}B, \ n \geq 1 \tag{13}$$

The "single-point" element belongs to the family: $A(0) = \{(0,0)\}$. B is topologically open and contains the origin.
Generation formulas are given by:

I) Initial condition
a) N is the maximum value defined by with

$$X \ominus 2^n B - (X \ominus 2^n B) \circ B = \varnothing, \ n > N \tag{14}$$

and

$$X \ominus 2^N B - (X \ominus 2^N B) \circ B \neq \varnothing \tag{15}$$

b) We define

$$X^{(2^N)} = X \tag{16}$$

$$SD_{2^N}(X,B) = X \ominus 2^N B - (X \ominus 2^N B) \circ B \tag{17}$$

II) Recursive relations
We can define iteratively, for $n = N, N-1, ..., 2, 1$

$$\begin{aligned} & X^{(2^{n-1})} = X^{(2^n)} - SD_{2^n}(X,B) \oplus 2^n B \\ & SD_{2^n}(X,B) = X \ominus 2^n B - (X \ominus 2^n B) \circ B \\ & ... \\ & X^{(0)} = X^{(1)} - SD_1(X,B) \oplus B \\ & SD_1(X,B) = X \ominus B - (X \ominus B) \circ B \end{aligned} \tag{18}$$

2.3. Generalized-Step Shape decomposition

Not only the size of the generator can vary at each step of the family generation, but also the generator's shape.
Let $B(n)_{n \in N}$ be a series of structuring elements, topologically open all of them containing the origin and satisfying

$$B(n) \bullet B(n) = B(n) \tag{19}$$

Let this series generate a family of elements $A(n)_{n\in N}$ in the following way:

$$A(n+1) = A(n) \oplus B(n),\ n = 0,1,2,... \quad (20)$$

and

$$A(0) = A(0,0) \quad (21)$$

The collection subsets $SD_{A(n)}(X, B(n))$ is called in this case Generalized-Step Morphological Shape decomposition.

The conditions for the X bounded shape decomposition are:

I) Initial condition

a) N is the maximum value defined by with

$$X \ominus A(n) - (X \ominus A(n)) \circ B(n) = \varnothing,\ n > N \quad (22)$$

and

$$X \ominus A(N) - (X \ominus A(N)) \circ B(N) \neq \varnothing \quad (23)$$

b) We define

$$X^{(N)} = X \quad (24)$$

$$SD_N(X, B(N)) = X \ominus A(N) - (X \ominus A(N)) \circ B(N) \quad (25)$$

II) Recursive relations

We can define iteratively, for $n = N, N-1,...,2,1$

$$X^{(n-1)} = X^{(n)} - SD_n(X, B(n)) \oplus A(n) \quad (26)$$

$$SD_n(X, B(n)) = X \ominus A(n) - (X \ominus A(n)) \circ B(n) \quad (27)$$

From the collection of morphological shape decomposition subsets $SD_n(X, B(n))$ the original shape X can be perfectly reconstruct by

$$X = \bigcup_{n \geq 0} SD_n(X, B(n)) \oplus A(n) \quad (28)$$

3. Interpolation Method

The new interpolation frame method presented in this article is based on 3D binary morphological Shape Decomposition.

It consists of changing, step by step, the Shape Decomposition subsets of input frame with the Shape Decomposition subsets of out frame.

3.1. Interpolation using original shape decomposition

The method:

a) Shape Decomposition subsets $ST_n(X_0, B_0)$, for $n = N_0, N_0 - 1,...,1,0$, of input frame are computed.

The original shape X_0 can be perfectly reconstructed in the following way:

$$X_0 = \bigcup_{n=0}^{N_0} ST_n(X_0, B) \oplus nB \quad (29)$$

b) Shape Decomposition subsets $ST_n(X_M, B_M)$, for $n = N_M, N_M - 1,...,1,0$, of output frame are computed.

The output frame X_M can be perfectly reconstructed in the following way:

$$X_M = \bigcup_{n=0}^{N_M} ST_n(X_M, B) \oplus nB \quad (30)$$

c) Interpolated frame (X_i) depends on the content of input frame and output frame and the position of the interpolated one between them and must be computed using morphological formulas.

Interpolated frame, X_i, is defined, for $N = \max(N_0, N_M)$ and $M = N + 1$

$$X_i = \begin{cases} X_0, & i = 0 \\ \left(\bigcup_{n=0}^{i-1} ST_n(X_M, B) \oplus nB \right) \bigcup \left(\bigcup_{n=i}^{N} ST_n(X_0, B) \oplus nB \right), & i = 1,...,M-1 \\ X_{M,} & i = M \end{cases} \quad (31)$$

3.2. Interpolation using Generalized-Step Shape decomposition

The method:

a) Shape Decomposition subsets $ST_n\left(X_0, B_0\left(n\right)\right)$, for $n = N_0, N_0 - 1, ..., 1, 0$, of input frame are computed.

The original shape X_0 can be perfectly reconstructed in the following way:

$$X_0 = \bigcup_{n=0}^{N_0} ST_n\left(X_0, B\left(n\right)\right) \oplus A\left(n\right) \qquad (32)$$

b) Shape Decomposition subsets $ST_n\left(X_M, B_M\right)$, for $n = N_M, N_M - 1, ..., 1, 0$, of output frame are computed.

The output frame X_M can be perfectly reconstructed in the following way:

$$X_M = \bigcup_{n=0}^{N_M} ST_n\left(X_M, B\left(n\right)\right) \oplus A\left(n\right) \qquad (33)$$

c) Interpolated frame (X_i) depends on the content of input frame and output frame and the position of the interpolated one between them and must be computed using morphological formulas.

Interpolated frame, X_i, is defined, for $N = \max\left(N_0, N_M\right)$ and $M = N + 1$

$$X_i = \begin{cases} X_0, & i = 0 \\ \left(\bigcup_{n=0}^{i-1} ST_n\left(X_M, B\left(n\right)\right) \oplus A\left(n\right)\right) \bigcup \left(\bigcup_{n=i}^{N} ST_n\left(X_0, B\left(n\right)\right) \oplus A\left(n\right)\right), & i = 1, ..., M-1 \\ X_{M,} & i = M \end{cases} \qquad (34)$$

4. Conclusions

Interframe interpolation problem is the process of creating intermediary frames between two gives one.

Two known frames – input frame and output frame, as well as intermediary frames are of the same type.

Interpolated frame depends on the content of input frame and output frame and the position of the interpolated one between them. It is a number between 0 and M denoted in this paper as n, $0 \le n \le N$. For $n = 0$ interpolated frame is equal with input frame, X_0, for $n = N$ interpolated frame is equal with output frame, X_N.

This paper addresses the gray scale interframe interpolation by means of mathematical morphology. The new frame interpolation method, called Shape Decomposition interpolation is based on morphological 2D and 3D Shape Decomposition. This article presents the theoretical background of the morphological frame interpolation and deduce the new representation.

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