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GENERALIZED NEIGHBOURHOOD SYSTEMS OF FUZZY POINTS

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Abstract. We define the generalized fuzzy neighbourhood systems on the set of fuzzy points in a nonempty set X and investigate their properties by using a new interior operator. With the help of these concepts we introduce generalized fuzzy continuity, which include many of the variations of fuzzy continuity already in the literature, as special cases.

1. Introduction

A neighbourhood system assigns each object a (possibly empty, finite or infinite) family of nonempty subsets. Such subsets, called neighbourhoods, represent the semantics of *near*. Formally, neighbourhoods play the most fundemantel role in mathematical analysis. Informally, it is a common and intuitive notion. It is in databases [10,20], in rough sets [27], in logic [5], in texts of genetic algorithms [14], and many others. This paper introduces generalized neighbourhood systems on the set of fuzzy points of a nonempty set.

The fundemantal idea of fuzzy sets was first introduced by Zadeh [35]. Chang [9] is known as the initiator of the notion of fuzzy topology. In 1976, the fuzzy topology was redifined in somewhat different way by Lowen [15]. Then many attempts have been made to extend various branches of mathematics to the fuzzy settings. We focus our work to extend the notions of the generalized neighbourd system to the fuzzy settings. To generalize the notions of topolgy, the initial attempts can be seen in [18] and [16], respectively, i.e., supratopologies and minimal structures. Recently, Császár [11] introduced the notions of generalized topologies (briefly GT) and generalized neighbourhood systems (briefly GNS). In [1], fuzzy supratopology and, recently, in [26], generalized fuzzy topology were defined as generalizations of fuzzy topology introduced by Chang. In addition, as a generalization of fuzzy topology introduced by Lowen, fuzzy minimal structure was defined in [3]. The neighbourhood and q-neighbourhood of a fuzzy point in a fuzzy topological space

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in the sense of Chang was introduced by Pu and Liu [19]. An earlier study on neighbourhood of fuzzy points can be found in [13].

In this paper, we define the generalized fuzzy neighbourhood systems on the set of fuzzy points in a nonempty set X and investigate their properties by using a new interior operator which corresponds to the notion of the interior operator in general form and gives us the way to show that every generalized fuzzy topology can be generated by a generalized fuzzy neighbourhood system. In addition, we introduce generalized fuzzy continuity with the help of generalized fuzzy neighbourhood systems. These notions lead us to give a general form to various concepts discussed in the literature.

2. Preliminaries

Let X be an arbitrary nonempty set. A fuzzy set A in X is a function on X into the interval $I = [0, 1]$ of the real line. The class of all fuzzy sets in X will be denoted by I^X and symbols $A, B, ...$ is used for fuzzy sets in X. The complement of a fuzzy set A in X is $1_X - A$. The fuzzy sets in X taking on respectively the constant values 0 and 1 are denoted by 0_X and 1_X, respectively. A fuzzy set A is nonempty if $A \neq 0_X$. For two fuzzy sets $A, B \in I^X$, we write $A \leq B$ if $A(x) \leq B(x)$ for each $x \in X$. For a family $\{A_j\}_{j\in J} \subset I^X$, the union $C = \cup_j A_j$ and the intersection $D = \cap_j A_j$, are defined by $C(x) = \sup_J \{A_j(x)\}$ and $D(x) = \inf_J \{A_j(x)\}$ for each $x \in X$. For a fuzzy set A in X, the set $\{x \in X : A(x) > 0\}$ is called the support of A. A fuzzy singleton or a fuzzy point with support x and value λ $(0 < \lambda \leq 1)$ is denoted by x_λ. The fuzzy point x_λ is said to be contained in a fuzzy set A, denoted by $x_\lambda \in A$, iff $\lambda \leq A(x)$, whereas the notion $x_\lambda q A$ means that x_λ is quasi-concident with A, i.e., $x_\lambda q A$ implies $\lambda + A(x) > 1$.

Let f be a function from X to Y, $A \in I^X$ and $B \in I^Y$. Then $f^{-1}(B)$ and $f(A)$ are defined as; $f^{-1}(B)(x) = B(f(x))$ for $x \in X$ and

$$f(A)(y) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} A(x) & , f^{-1}(y) \neq \emptyset \\ 0 & , \text{otherwise} \end{cases}$$

for $y \in Y$, respectively.

Throughout this paper, by a fuzzy topological space (shortly fts) we mean a fts (X, o), as initiated by Chang [9], i.e., $o \subset I^X$ satisfy (a) 0_X, $1_X \in o$, (b) If $A_j \in o$ for each $j \in J \neq \emptyset$, then $\cup_{j\in J} A_j \in o$ and (c) If $A, B \in o$, then $A \cap B \in o$. The elements of o are called fuzzy open sets and their complements are called fuzzy closed sets. We shall denote the fuzzy interior and fuzzy closure of a fuzzy set $A \in I^X$ with $i_o A$ and $c_o A$, respectively, i.e. $i_o A = \cup\{U : U \leq A,\ A \in o\}$ and $c_o A = \cap\{F : F \geq A,\ 1_X - A \in o\}$. A fuzzy set V is called a neighbourhood of fuzzy point x_λ iff there exists $U \in o$ such that $x_\lambda \in U \leq V$ and V is called a q-neighbourhood of x_λ iff there exists $U \in o$ such that $x_\lambda q U \leq V$. The fuzzy set theoretical and fuzzy topological concepts used in this paper are standard and can be found in Zadeh [35], Chang [9], Pu and Liu [19].

The family of all fuzzy semiopen [4] (resp. fuzzy preopen [32], fuzzy α- [32], fuzzy β-open [7], fuzzy semi-preopen [34], fuzzy regular open [4]) sets of (X, o) shall be denoted by FSo (resp. FPo, $F\alpha o$, $F\beta o$, $FSPo$, FRo).

Fuzzy minimal structures are defined and investigated in [3]. A subfamily $m \subset I^X$ is said to be a fuzzy minimal structure on X iff $\lambda 1_X \in m$ for each $\lambda \in I$ and the elements of m are called fuzzy m-open sets. A fuzzy supratopology [1] is a subfamily g of I^X, satisfying 0_X, $1_X \in g$ and arbitrary union of members of g belongs to g. In addition, if g satisfies these conditions except $1_X \in g$, then g is said to be a generalized fuzzy topology (briefly GFT) in [26].

In the sequal, the set of all fuzzy points in X is denoted by $\mathcal{P}$.

3. Generalized Fuzzy Neighbourhood Systems

Let us define

$$\psi : \mathcal{P} \to 2^{I^X} \text{ satisfy } \lambda \leq V(x) \text{ for } V \in \psi(x_\lambda)$$

Then we shall say that $V \in \psi(x_\lambda)$ is a generalized fuzzy neighbourhood (briefly GFN) of the fuzzy point x_λ and ψ is a generalized fuzzy neighbourhood system (briefly GFNS) on the set of fuzzy points in X. We denote by $\Psi(\mathcal{P})$ the collection of all GFNS's on $\mathcal{P}$.

For an arbitrary fuzzy set $A \in I^X$, write $x_\lambda \in \mathcal{P}_{\psi,A}$ iff there exists $V \in \psi(x_\lambda)$ satisfying $V \leq A$.

Definition 3.1. Let $\psi \in \Psi(\mathcal{P})$ and $A \in I^X$. Then define the fuzzy set $\imath_\psi A$ as:

$$(\imath_\psi A)(x) = \begin{cases} \sup_{x_\lambda \in \mathcal{P}_{\psi,A}} \lambda & , \exists \lambda \in I \text{ satisfying } x_\lambda \in \mathcal{P}_{\psi,A} \\ 0 & , \text{otherwise} \end{cases}$$

for all x in X. $\imath_\psi A$ is called the interior of A on ψ.

Lemma 3.2. *Let $\psi \in \Psi(\mathcal{P})$. Then*
(*a*) $\imath_\psi 0_X = 0_X$,
(*b*) $\imath_\psi A \leq A$, *for* $A \in I^X$,
(*c*) $A \leq B$ *implies* $\imath_\psi A \leq \imath_\psi B$, *for all* $A, B \in I^X$.

Proof. (*a*) Since $\mathcal{P}_{\psi,0_X} = \emptyset$, we have $(\imath_\psi 0_X)(x) = 0$ for all x in X. Thus $\imath_\psi 0_X = 0_X$.

(*b*) Clearly $\imath_\psi 0_X \leq 0_X$. Let $A \neq 0_X$ and an arbitrary x in X. If $(\imath_\psi A)(x) = 0$, then $\imath_\psi A \leq A$. If $(\imath_\psi A)(x) = \sup_{x_\lambda \in \mathcal{P}_{\psi,A}} \lambda := t > 0$, then there exists $\lambda \in I$ satisfying $x_\lambda \in \mathcal{P}_{\psi,A}$. In this case, let $x_{t_j} \in \mathcal{P}_{\psi,A}$ for $j \in J \neq \emptyset$, then there exists $V_j \in \psi\left(x_{t_j}\right)$ satisfying $t_j \leq V_j(x) \leq A(x)$ for each $j \in J$. Thus $t = \sup_{j \in J} t_j \leq A(x)$. Therefore $\imath_\psi A \leq A$.

(*c*) $A \leq B$ implies $\mathcal{P}_{\psi,A} \subseteq \mathcal{P}_{\psi,B}$. Therefore $\imath_\psi A \leq \imath_\psi B$. □

Proposition 1. *Let an arbitrary* $(A_j)_{j\in J} \subset I^X$*. Then* $\bigcup_{j\in J} \imath_\psi A_j \leq \imath_\psi \left(\bigcup_{j\in J} A_j\right)$.

Proof. Clearly $A_j \leq \cup_{j\in J} A_j$ for each $j \in J$. Then $\imath_\psi A_j \leq \imath_\psi \left(\bigcup_{j\in J} A_j\right)$ for each $j \in J$ by Lemma 3.2(c). Hence $\bigcup_{j\in J} \imath_\psi A_j \leq \imath_\psi \left(\bigcup_{j\in J} A_j\right)$. □

Lemma 3.3. *Let* $\psi \in \Psi(\mathcal{P})$ *and* $g = \left\{G \in I^X : G = \imath_\psi G\right\}$*. Then* g *is a GFT on* X.

Proof. Clearly, $0_X \in g$ by Lemma 3.2(a). Let $G = \cup_{j\in J} G_j$ and $G_j \in g$ for $j \in J \neq \emptyset$. Then $\imath_\psi G \leq G$ is clear by Lemma 3.2(b). On the other hand we have $G \leq \bigcup_{j\in J} \imath_\psi G_j$ since $G_j \in g$ for each $j \in J$. In addition, $\bigcup_{j\in J} \imath_\psi G_j \leq \imath_\psi G$ is clear by Proposition1 Therefore $G \leq \imath_\psi G$. Hence $G \in g$. □

So it is clear that every GFNS generates a GFT. In this case we shall write g_ψ for this g.

Lemma 3.4. *If* g *is a GFT on* X*, then there is a* $\psi \in \Psi(\mathcal{P})$ *satisfying* $g = g_\psi$.

Proof. Let us define $V \in \psi(x_\lambda)$ iff $x_\lambda \in V \in g$. Then clearly $\psi \in \Psi(\mathcal{P})$. Now we have to prove that $g = g_\psi$:

(a) Let $G \in g$ and $G(x) = t > 0$ for an arbitrary x in X. Thus we have $x_t \in G \in g$ and so $x_t \in \mathcal{P}_{\psi,G}$ for $t \in I$. Therefore $(\imath_\psi G)(x) \geq t = G(x)$. Hence $G \in g_\psi$ by Lemma 3.2(b).

(b) Let $G \in g_\psi$. If $G = 0_X$, then $G \in g$. If $G \neq 0_X$, then there exists x in X such that $G(x) = t > 0$. Then $(\imath_\psi G)(x) = \sup_{x_\lambda \in \mathcal{P}_{\psi,G}} \lambda := t > 0$. In this case, let $x_{t_j} \in \mathcal{P}_{\psi,G}$ for $j \in J \neq \emptyset$, then there exists $V_j \in \psi\left(x_{t_j}\right)$ satisfying $V_j \leq G$ for each $j \in J$. Thus $V_x = \cup_{j\in J} V_j \leq G$ for $(V_j)_{j\in J} \subset g$ and $V_x(x) = t$. Now if we write $V = \bigcup_{G(x)>0} V_x$, then $V \in g$ and $V \leq G$. Now let an arbitrary z in X. If $G(z) = 0$, then clearly $G \leq V$. If $G(z) := l > 0$, then there exists $V_z \in g$ satisfying $V_z(z) = l \leq V(z)$. Thus $G \leq V$. Hence $G \in V$. □

Note that we shall write $\psi = \psi_g$ for the GFNS ψ defined as:

$$\psi(x_\lambda) = \{V : x_\lambda \in V \in g\}$$

for each $x_\lambda \in \mathcal{P}$.

The following result is clear by Lemma 3.4.

Corollary 1. *If* g *is a GFT on* X *and* $\psi = \psi_g$*, then* $g_\psi = g$.

Then we shall say that each fuzzy generalized topology on X can be generated by some generalized fuzzy neighbourhood system on X.

If g is a GFT on X and $A \in I^X$, then in the sense of [1] and [26] the interior of A (we shall write $i_g A$) on g is defined as the union of all $G \leq A$, $G \in g$. Then similarly we shall define i_ψ for $\psi \in \Psi(\mathcal{P})$ as $i_\psi := i_{g_\psi}$.

Proposition 2. *Let $\psi \in \Psi(\mathcal{P})$ and $A \in I^X$. Then $i_\psi A \leq \imath_\psi A$.*

Proof. Let $J \neq \emptyset$ and $(G_j)_{j \in J} \subset I^X$ denotes the elements of g_ψ satisfying $G_j \leq A$. Then $i_\psi A = \cup_{j \in J} G_j \leq \imath_\psi \left(\bigcup_{j \in J} G_j \right) = \imath_\psi (i_\psi A) \leq \imath_\psi A$ by Proposition 1 and Lemma 3.2(c). □

The following example shows that $i_\psi A \neq \imath_\psi A$, in general.

Example 3.5. Let $X = \{a, b\}$, A and B be fuzzy subsets of X defined as follows:

$$A(a) = 0.1, \; A(b) = 0.5$$
$$B(a) = 0.3, \; B(b) = 0.3$$

Now define the GFNS ψ as:

$$\psi(a_\lambda) = \{A\}, \text{ for } 0 \leq \lambda \leq 0.1 \quad \text{and} \quad \psi(b_\lambda) = \{B\}, \text{ for } 0 \leq \lambda \leq 0.3$$
$$\psi(a_\lambda) = \{1_X\}, \text{ for } 0.1 \leq \lambda \leq 1 \qquad \psi(b_\lambda) = \{1_X\}, \text{ for } 0.3 < \lambda \leq 1.$$

Then clearly $(\imath_\psi A)(a) = 0.1$, $(\imath_\psi A)(b) = 0$, while $g_\psi = \{0_X, 1_X\}$ and so $i_\psi A = 0_X$.

Note that, for a GFT $g \subset I^X$, we shall write $\psi \in \Psi_g(\mathcal{P})$ iff $V \in g$ for $V \in \psi(x_\lambda)$, $x_\lambda \in \mathcal{P}$.

Proposition 3. *If $\psi \in \Psi_g(\mathcal{P})$ for the GFT $g = g_\psi$, then $\imath_\psi = i_\psi$.*

Proof. Let $(\imath_\psi A)(x) := t > 0$ for an arbitrary x in X. Thus there exists $\lambda \in I$ such that $x_\lambda \in \mathcal{P}_{\psi,A}$. If we write $x_{t_j} \in \mathcal{P}_{\psi,A}$ for $j \in J \neq \emptyset$, then there exists $V_j \in \psi(x_{t_j})$ satisfying $V_j \leq A$ for each $j \in J$. Thus $V_x = \cup_{j \in J} V_j \leq A$ for $(V_j)_{j \in J} \subset g_\psi$. So $V_x = \imath_\psi V_x \leq \imath_\psi A$ and this implies that $V_x(x) = t$. Hence $\imath_\psi A \leq i_\psi A$. □

4. Generalized fuzzy continuity

In this section we define generalized fuzzy continuity with the help of the concepts introduced above.

Definition 4.1. Let X and X' be two sets, $\psi \in \Psi(\mathcal{P})$, $\psi' \in \Psi(\mathcal{P}')$ and a function $f : X \to X'$. Then f is said to be fuzzy (ψ, ψ')-continuous iff, for each $x_\lambda \in X$ and $V \in \psi'(f(x_\lambda))$, there is $U \in \psi(x_\lambda)$ satisfying $f(U) \leq V$.

Now let X be a set, g be GFT on X and suppose that $\kappa : I^X \to I^X$ satisfies

$$A \leq B \leq X \text{ implies } \kappa A \leq \kappa B \text{ and } A \leq \kappa A \text{ for } A \subset X.$$

Then define

$$\psi(\kappa, g)(x_\lambda) = \{V : V = \kappa G \text{ for some } G \in g \text{ such that } x_\lambda \in G\}$$

for each $x_\lambda \in X$. Clearly $\psi(\kappa, g) \in \Psi(\mathcal{P})$.

In the literature, various examples of fuzzy (ψ, ψ')-continuity can be found. Let o and o' be fuzzy topologies on X and X', respectively. The case $\psi = \psi_o$, $\psi' = \psi(c_{o'}, o')$ is called fuzzy weak continuity in [22], while $\psi = \psi_o$, $\psi' = \psi(c_{FSo'}, o')$ gives fuzzy weakly semi-continuous maps in the sense of [12], $\psi = \psi(c_{FSo}, FSo)$, $\psi' = \psi_{FSo'}$ gives fuzzy strongly irresolute maps of [17], $\psi = \psi_o$, $\psi' = \psi(c_{FSo'}, FSo')$ gives fuzzy semi-irresolute maps of [17], $\psi = \psi_o$, $\psi' = \psi(i_{o'}c_{o'}, o')$ gives fuzzy almost continuous maps of [2], $\psi = \psi_{F\beta o}$, $\psi' = \psi(i_{o'}c_{o'}, o')$ gives fuzzy almost β-continuous maps of [25], $\psi = \psi(i_o c_o, o)$, $\psi' = \psi(i_{o'}c_{o'}, o')$ gives fuzzy δ-continuous maps of [33], $\psi = \psi_{FSo}$, $\psi' = \psi_{FRo'}$ gives fuzzy almost semi-continuous maps of [32].

Let X and X' be two sets, g and g' be two generalized fuzzy topologies on X and X', respectively. In the sense of fuzzy supra-continuity [1], we shall say that $f : X \to X'$ is fuzzy (g, g')-continuous iff $G' \in g'$ implies $f^{-1}(G') \in g$.

Proposition 4. *A fuzzy (ψ, ψ')-continuous map is fuzzy $(g_\psi, g_{\psi'})$-continuous.*

Proof. Let $G' \in g_{\psi'}$ and $(f^{-1}(G'))(x) := t > 0$ for an arbitrary x in X. Then there exists $\lambda \in I$ such that $f(x)_\lambda \in \mathcal{P}'_{\psi', G'}$ since $(f^{-1}(G'))(x) = G'(f(x)) = (\imath_{\psi'} G')(f(x))$. Therefore there is a GFN $V \in \psi'(f(x)_\lambda)$ such that $V \leq G'$. Since f is fuzzy (ψ, ψ')-continuous, we have a GFN $U \in \psi(x_\lambda)$ satisfying $f(U) \leq V$ and so $U \leq f^{-1}(G')$. Thus $x_\lambda \in \mathcal{P}_{\psi, f^{-1}(G')}$. Therefore $t \leq [\imath_\psi(f^{-1}(G'))](x)$. Hence $f^{-1}(G') = \imath_\psi(f^{-1}(G'))$. □

Proposition 5. *If f is fuzzy $(g_\psi, g_{\psi'})$-continuous, $\psi = \psi_g$ and $\psi' = \psi_{g'}$ for some GFT's $g \subset I^X$ and $g' \subset I^{X'}$, respectively, then f is fuzzy (ψ, ψ')-continuous.*

Proof. Let an arbitrary $x_\lambda \in \mathcal{P}$ and $V' \in \psi'(f(x_\lambda))$. Then $\lambda \leq V'(f(x))$ and $V' \in g' = g_{\psi'}$ by Corollary 1 Therefore $\lambda \leq f^{-1}(V')(x)$ and $f^{-1}(V') \in g_\psi$. Thus $0 < \lambda \leq [\imath_\psi(f^{-1}(V'))](x)$. Then there exists $t \geq \lambda$ such that $x_t \in \mathcal{P}_{\psi, f^{-1}(V')}$, and so there is a GFN $U \in \psi(x_t)$ satisfying $U \leq f^{-1}(V')$. Also $U \in \psi(x_\lambda)$ since $\psi = \psi_g$. Therefore we have a GFN $U \in \psi(x_\lambda)$ satisfying $f(U) \leq V'$. Hence f is fuzzy (ψ, ψ')-continuous. □

Now let o and o' be fuzzy topologies on X and X', respectively. Then clearly the case $\psi = \psi_o$, $\psi' = \psi_{o'}$ is fuzzy continuity [9] in the classical sense, $\psi = \psi_{F\alpha o}$, $\psi' = \psi_{o'}$ is called fuzzy α-continuity in [32], $\psi = \psi_{FSo}$, $\psi' = \psi_{o'}$ is called fuzzy semi-contunity in [4],$\psi = \psi_{FPo}$, $\psi' = \psi_{o'}$ is called fuzzy pre-continuity in [24], $\psi = \psi_{F\beta o}$, $\psi' = \psi_{o'}$ is called fuzzy β-contunity in [7], $\psi = \psi_{F\alpha o}$, $\psi' = \psi_{FSo'}$ is called fuzzy strongly α-contunity in [29], $\psi = \psi_{FSPo}$, $\psi' = \psi_{o'}$ is called fuzzy semi-precontinuity in [34], $\psi = \psi_{F\beta o}$, $\psi' = \psi_{F\beta o'}$ is called M-β-fuzzy continuity in [7], $\psi = \psi_{FPo}$, $\psi' = \psi_{FPo'}$ is called M-fuzzy precontinuity in [8], while $\psi = \psi_{FSo}$, $\psi' = \psi_{FSo'}$ gives fuzzy irresolute maps of [21], $\psi = \psi_{F\alpha o}$, $\psi' = \psi_{F\alpha o'}$ gives fuzzy α-irresolute maps of [28], $\psi = \psi_{FPo}$, $\psi' = \psi_{FPo'}$ gives fuzzy pre-irresolute maps of [6], $\psi = \psi_{FSo}$, $\psi' = \psi_{F\alpha o'}$ gives fuzzy semi-α-irresolute maps of [31], $\psi = \psi_{F\beta o}$, $\psi' = \psi_{FPo'}$ gives fuzzy β-pre-irresolute maps of [30].

5. Conclusions

This paper presents an extended study of fuzzy topology and fuzzy continuity with respect to generalized fuzzy neighbourhood systems. Similar results can be found by considering generalized q-neighbourhood system of fuzzy points in X, i.e.

$$q\psi : \mathcal{P} \to 2^{I^X} \text{ satisfy } x_\lambda qV \text{ for } V \in q\psi\left(x_\lambda\right).$$

Then a member of $q\psi\left(x_\lambda\right)$ shall be called as a generalized fuzzy q-neighbourhood (briefly GFQN) of the fuzzy point x_λ.

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