Generalized Principal Component Analysis

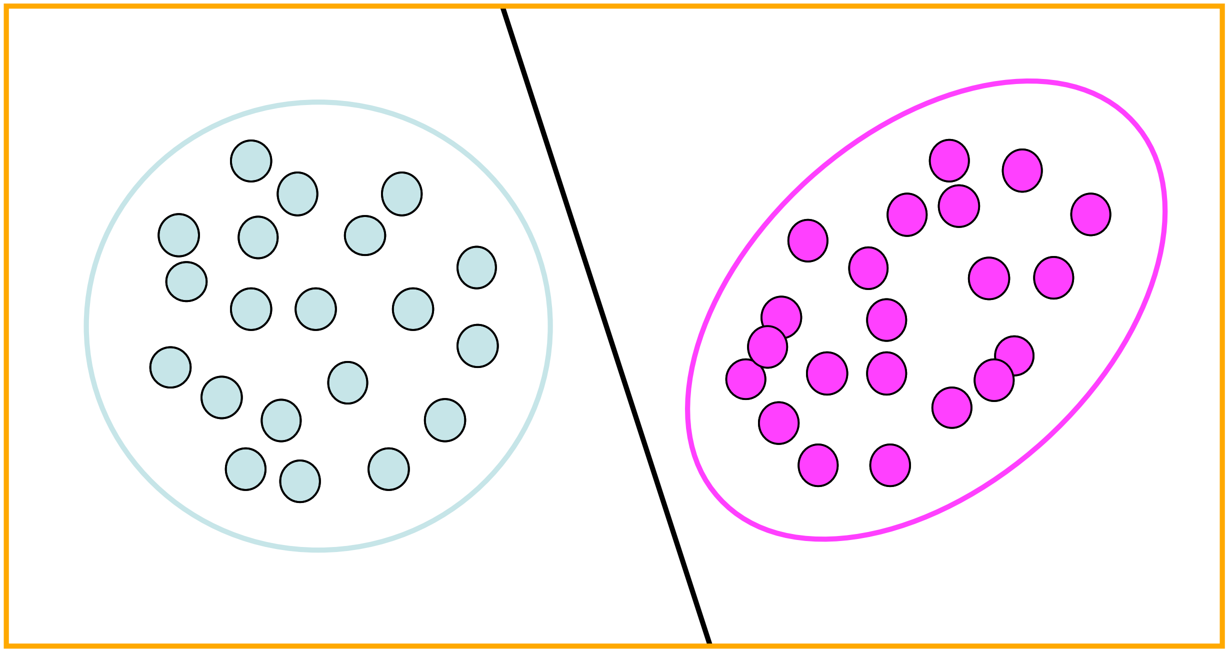
Tutorial @ CVPR 2007

Yi Ma
ECE Department
University of Illinois
Urbana Champaign

René Vidal
Center for Imaging Science
Institute for Computational Medicine
Johns Hopkins University

Data segmentation and clustering

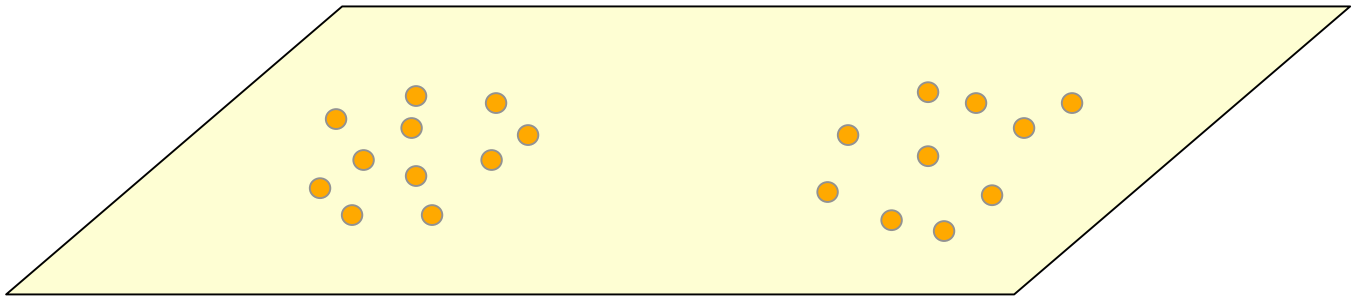
- Given a set of points, separate them into multiple groups



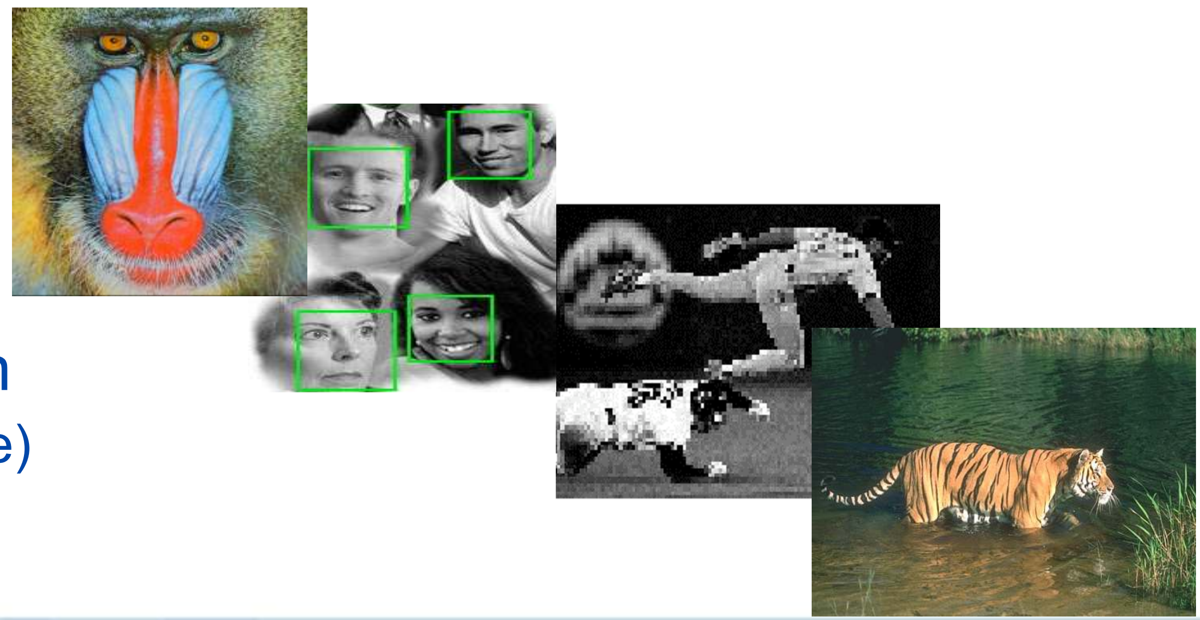
- Discriminative methods: learn boundary
- Generative methods: learn mixture model, using, e.g. Expectation Maximization

Dimensionality reduction and clustering

- In many problems data is high-dimensional: can reduce dimensionality using, e.g. Principal Component Analysis

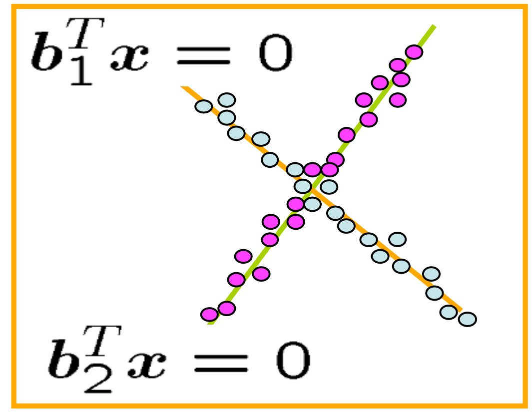


- Image compression
- Recognition
  - Faces (Eigenfaces)
- Image segmentation
  - Intensity (black-white)
  - Texture

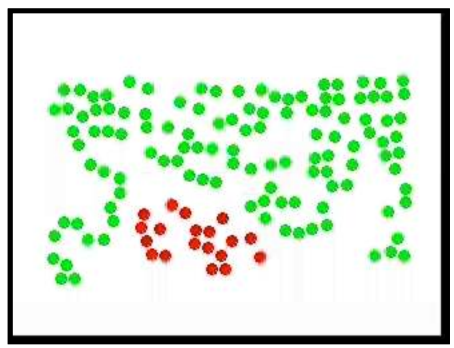


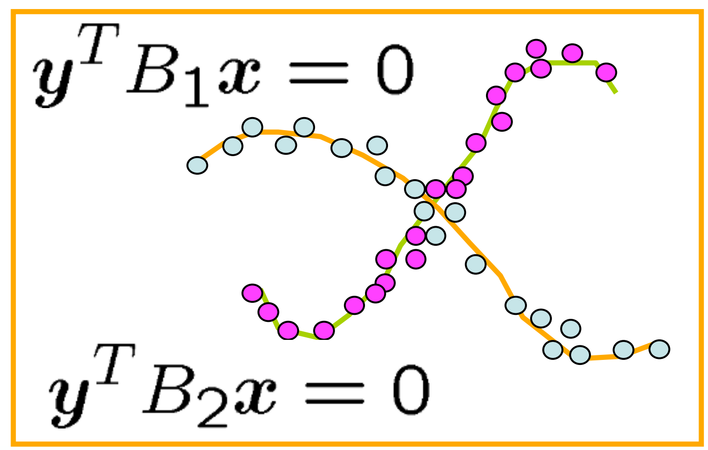
Segmentation problems in dynamic vision

- Segmentation of video and dynamic textures



- Segmentation of rigid-body motions



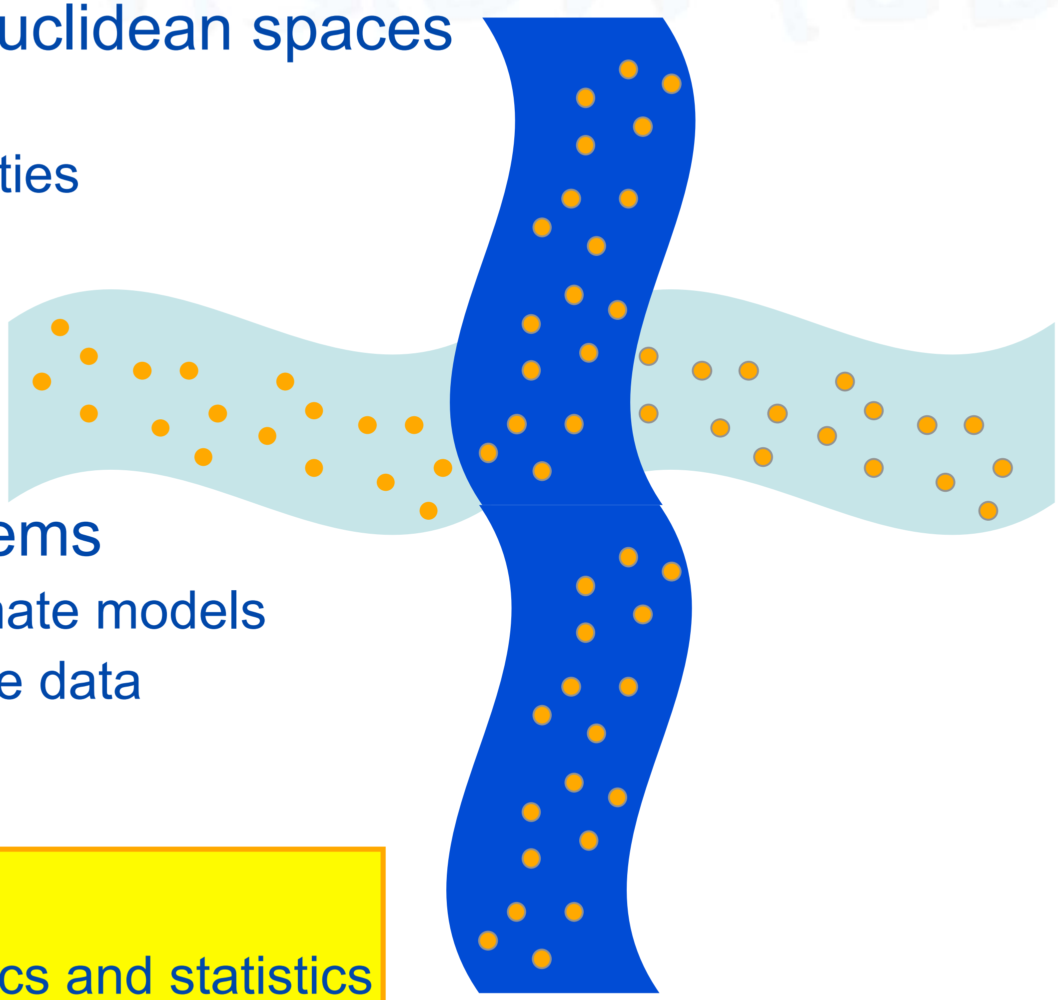


Segmentation problems in dynamic vision

- Segmentation of rigid-body motions from dynamic textures

Clustering data on non Euclidean spaces

- Clustering data on non Euclidean spaces
  - Mixtures of linear spaces
  - Mixtures of algebraic varieties
  - Mixtures of Lie groups



- “Chicken-and-egg” problems
  - Given segmentation, estimate models
  - Given models, segment the data
  - Initialization?

- Need to combine
  - Algebra/geometry, dynamics and statistics

Outline of the tutorial

- Part I: Theory (8.30-10.00)
  - Introduction to GPCA (8.30-8.40)
  - Basic GPCA theory and algorithms (8.40-9.20)
  - Advanced statistical and algebraic methods for GPCA (9.30-10.20)
- Break (10.00-10.30)
- Part II: Applications (10.30-12.10)
  - Applications to motion and video segmentation (10.30-11.20)
  - Applications to image representation & segmentation (11.20-12.10)
- Questions (12.10-12.30)

Part I: Theory

- Introduction to GPCA (8.30-8.40)

- Basic GPCA theory and algorithms (8.40-9.20)
  - Review of PCA and extensions
  - Introductory cases: line, plane and hyperplane segmentation
  - Segmentation of a known number of subspaces
  - Segmentation of an unknown number of subspaces

- Advanced statistical and algebraic methods for GPCA (9.20-10.00)
  - Model selection for subspace arrangements
  - Robust sampling techniques for subspace segmentation
  - Voting techniques for subspace segmentation

Part II: Applications in computer vision

- Applications to motion & video segmentation (10.30-11.20)
  - 2-D and 3-D motion segmentation
  - Temporal video segmentation
  - Dynamic texture segmentation

- Applications to image representation and segmentation (11.20-12.10)
  - Multi-scale hybrid linear models for sparse image representation
  - Hybrid linear models for image segmentation

References: Springer-Verlag 1998

Generalized Principal Component Analysis

Estimation & Segmentation of Geometric Models

René Vidal (Biomedical Engineering, Johns Hopkins University)

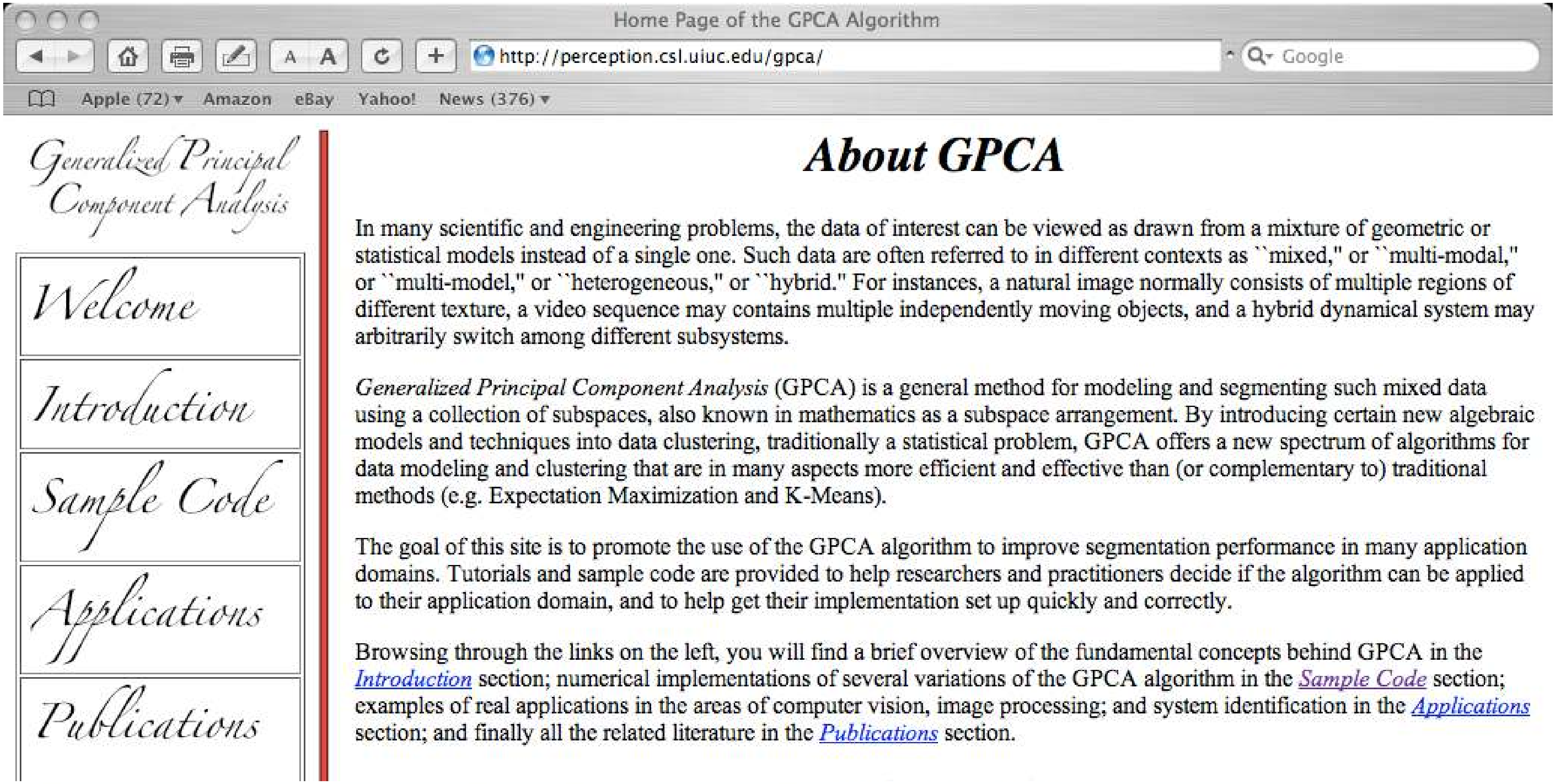
Yi Ma (ECE, University of Illinois at Urbana-Champaign)

S. Shankar Sastry (EECS, University of California at Berkeley)

Center for Imaging Science

Slides, MATLAB code, papers

http://perception.csl.uiuc.edu/gpca



About GPCA

In many scientific and engineering problems, the data of interest can be viewed as drawn from a mixture of geometric or statistical models instead of a single one. Such data are often referred to in different contexts as ``mixed,'' or ``multi-modal,'' or ``multi-model,'' or ``heterogeneous,'' or ``hybrid.'' For instances, a natural image normally consists of multiple regions of different texture, a video sequence may contains multiple independently moving objects, and a hybrid dynamical system may arbitrarily switch among different subsystems.

Generalized Principal Component Analysis (GPCA) is a general method for modeling and segmenting such mixed data using a collection of subspaces, also known in mathematics as a subspace arrangement. By introducing certain new algebraic models and techniques into data clustering, traditionally a statistical problem, GPCA offers a new spectrum of algorithms for data modeling and clustering that are in many aspects more efficient and effective than (or complementary to) traditional methods (e.g. Expectation Maximization and K-Means).

The goal of this site is to promote the use of the GPCA algorithm to improve segmentation performance in many application domains. Tutorials and sample code are provided to help researchers and practitioners decide if the algorithm can be applied to their application domain, and to help get their implementation set up quickly and correctly.

Browsing through the links on the left, you will find a brief overview of the fundamental concepts behind GPCA in the Introduction section; numerical implementations of several variations of the GPCA algorithm in the Sample Code section; examples of real applications in the areas of computer vision, image processing; and system identification in the Applications section; and finally all the related literature in the Publications section.