

Research Article

Generalized Projective Synchronization of Chaotic Heavy Gyroscope Systems via Sliding Rule-Based Fuzzy Control

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This paper proposes the generalized projective synchronization for chaotic heavy symmetric gyroscope systems versus external disturbances via sliding rule-based fuzzy control. Because of the nonlinear terms of the gyroscope, the system exhibits complex and chaotic motions. Based on Lyapunov stability theory and fuzzy rules, the nonlinear controller and some generic sufficient conditions for global asymptotic synchronization are attained. The fuzzy rules are directly constructed subject to a common Lyapunov function such that the error dynamics of two identical chaotic motions of symmetric gyros satisfy stability in the Lyapunov sense. The proposed method allows us to arbitrarily adjust the desired scaling by controlling the slave system. It is not necessary to calculate the Lyapunov exponents and the Eigen values of the Jacobian matrix. It is a systematic procedure for synchronization of chaotic systems. It can be applied to a variety of chaotic systems no matter whether it contains external excitation or not. It needs only one controller to realize synchronization no matter how much dimensions the chaotic system contains, and the controller is easy to be implemented. The designed controller is robust versus model uncertainty and external disturbances. Numerical simulation results demonstrate the validity and feasibility of the proposed method.

1. Introduction

Dynamic chaos is a very interesting nonlinear effect which has been intensively studied during the last three decades. Chaotic phenomena can be found in many scientific and engineering fields such as biological systems, electronic circuits, power converters, and chemical systems [1].

Since the synchronization of chaotic dynamical systems has been observed by Pecora and Carroll [2] in 1990, chaos synchronization has become a topic of great interest [3–5]. Synchronization phenomena have been reported in the recent literature. Until now, different types of synchronization have been found in interacting chaotic systems, such as complete synchronization [2, 6, 7], generalized synchronization [8], phase synchronization [9], and antiphase synchronization [10]. In 1999, projective synchronization has been first reported by Mainieri and Rehacek [11] in partially linear systems that the master and slave vectors synchronize up to

a constant scaling factor α (a proportional relation). Later, some researchers have extended synchronization to a general class of chaotic systems without the limitation of partial linearity, such as non-partially-linear systems [12, 13]. After that, a new synchronization, called generalized projective synchronization (GPS), has been observed in the nonlinear chaotic systems [14–16].

On the other hand, the dynamics of a gyro is a very interesting nonlinear problem in classical mechanics. The gyro has attributes of great utility to navigational, aeronautical, and space engineering [17]. Gyros for sensing angular motion are used in airplane automatic pilots, rocket-vehicle launch guidance, space-vehicle attitude systems, ship's gyrocompasses, and submarine inertial autonavigators. The concept of chaotic motion in a gyro was first presented in 1981 by Leipnik and Newton [18], showing the existence of two strange attractors. In the past years, gyros have been found with rich phenomena which give benefit for the

understanding of gyro systems. Different types of gyros with linear/nonlinear damping are investigated for predicting the dynamic responses such as periodic and chaotic motions [17, 19, 20]. Some methods have been presented to synchronize two identical/nonidentical nonlinear gyro system such as active control [21] and neural sliding mode control [7, 8].

The goal of this paper is to synchronize two chaotic heavy symmetric gyroscope systems versus external disturbances. To achieve this goal, sliding rule-based fuzzy control is applied. In addition, the results of this paper may be extended to synchronize many classes of nonlinear chaotic systems.

This paper is organized as follows. In Section 2, dynamics of a heavy symmetric gyroscope system are described. Generalized synchronization problem is explained in Section 3. In Section 4, sliding rule-based fuzzy control is designed to chaos synchronization of chaotic gyroscopes. Simulations are presented in Section 5, to show the effectiveness of the proposed control method to chaos synchronization of chaotic gyroscope systems versus disturbances. At the end, the paper is concluded in Section 6.

2. Chaotic Gyroscope System

The symmetric gyroscope mounted on a vibrating base is shown in Figure 1. The dynamics of a symmetrical gyro with linear-plus-cubic damping of angle θ can be expressed as [17]

$$\ddot{\theta} + \alpha_1^2 \frac{(1 - \cos \theta)^2}{\sin^3 \theta} - \beta_1 \sin \theta + c_1 \dot{\theta} + c_2 \dot{\theta}^3 = f \sin \omega t \sin \theta, \quad (1)$$

where $f \sin \omega t$ is a parametric excitation, $c_1 \dot{\theta}$ and $c_2 \dot{\theta}^3$ are linear and nonlinear damping terms, respectively, and $\alpha_1^2((1 - \cos \theta)^2/\sin^3 \theta) - \beta_1 \sin \theta$ is a nonlinear resilience force. According to [17], in a symmetric gyro mounted on a vibrating base, the precession and the spin angles have cyclic motions, and hence their momentum integrals are constant and equal to each other. So the governing equations of motion depend only on the mutational angle θ . Using Routh's procedure and assuming a linear-plus-cubic form for dissipative force, (1) is obtained [17]. Given the states $x_1 = \theta$, $x_2 = \dot{\theta}$, and $g(\theta) = \alpha_1^2((1 - \cos \theta)^2/\sin^3 \theta) - \beta_1 \sin \theta$, (19) can be rewritten as follows:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= g(x_1) - c_1 x_2 - c_2 x_2^3 + (\beta + f \sin \omega t) \sin(x_1). \end{aligned} \quad (2)$$

This gyro system exhibits complex dynamics and has been studied by [20] for values of f in the range $32 < f < 36$ and constant values of $\alpha_1^2 = 100$, $\beta_1 = 1$, $c_1 = 0.5$, $c_2 = 0.05$, and ω_2 . Figure 2 illustrates the irregular motion exhibited by this system for $f = 35.5$ and initial conditions of $(x_1, x_2) = (1, -1)$.

In the next section, the chaos synchronization problem has been explained.

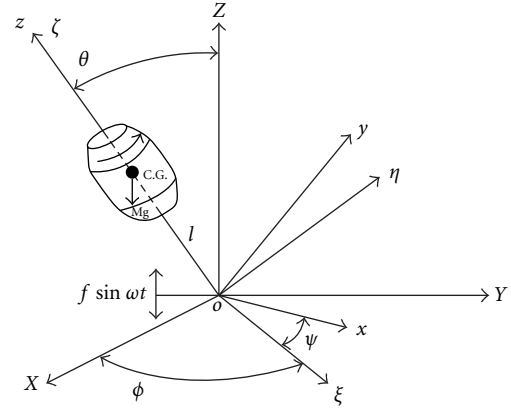


FIGURE 1: A schematic diagram of a symmetric gyroscope.

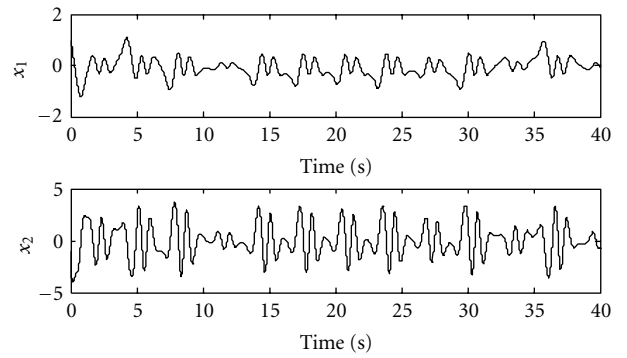


FIGURE 2: Time series of x_1 and x_2 .

3. Generalized Projective Synchronization Problem

Consider two coupled, chaotic gyroscope systems, where the master and slave systems are denoted by x and y , respectively. The master system is presented in (2). The slave system is presented as follows:

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= g(y_1) - c_1 y_2 - c_2 y_2^3 + (\beta + f \sin \omega t) \sin(y_1) = u(t). \end{aligned} \quad (3)$$

Defining the generalized synchronization errors between the master and slave systems as follows:

$$E(t) = \begin{cases} e_1(t) = y_1(t) - \alpha x_1(t), \\ e_2(t) = y_2(t) - \alpha x_2(t), \end{cases} \quad (4)$$

where $\alpha \in R$ is a scaling factor that defines a proportional relation between the synchronized systems. Then, the error dynamics can be obtained as

$$\begin{aligned} \dot{e}_1(t) &= e_2(t), \\ \dot{e}_2(t) &= (1 - \alpha)p(x_1, x_2, y_1, y_2) + u(t). \end{aligned} \quad (5)$$

In order to simplify the following procedure, a nonlinear function is defined as follows:

$$p(x_1, x_2, y_1, y_2) = \frac{g(y_1) - c_1 y_1 - c_2 y_2^3 + (\beta + f \sin \omega t) \sin(y_1)}{1 - \alpha} - \alpha \frac{g(x_1) - c_1 x_1 - c_2 x_2^3 + (\beta + f \sin \omega t) \sin(x_1)}{1 - \alpha}. \quad (6)$$

The objective of the synchronization problem is to design the appropriate control signal $u(t)$ such that for any initial conditions of the master and slave systems, the synchronization errors converge to zero such that the resulting synchronization error vector satisfies.

$$\lim_{t \rightarrow \infty} \|E(t)\| \rightarrow 0, \quad (7)$$

where $\|\cdot\|$ is the Euclidean norm of a vector. In the next section, the control input will be obtained via sliding rule-based fuzzy control to achieve the synchronization goal presented in previous section.

4. Generalized Projective Synchronization of Chaotic Gyroscopes versus Disturbances via Sliding Rule-Based Fuzzy Control

The scheme of GPS of chaotic gyroscope systems versus disturbances via the fuzzy system based on sliding mode control is shown in Figure 3. First, sliding surface is designed for chaos synchronization of chaotic gyroscope systems. An appropriate observer is designed for the linear part of the slave system. Then, sliding rule-based fuzzy control is designed as a control to synchronize the master and the slave systems, with considering the external disturbances.

4.1. Sliding Surface. Using the sliding mode control method for GPS of chaotic gyroscope systems, involves two basic steps:

- (1) selecting an appropriate sliding surface such that the sliding motion on the sliding manifold is stable and ensures $\lim_{t \rightarrow \infty} \|E(t)\| \rightarrow 0$;
- (2) establishing a robust control law which guarantees the existence of the sliding manifold $S(t) = 0$.

The sliding surfaces are defined as follows [22]:

$$S(t) = \left(\frac{d}{dt} + \delta \right)^{n-1} e(t), \quad (8)$$

where $S(t) \in R$ and δ are real positive constant parameters. Differentiating (10) with respect to time is as follows:

$$\dot{S}(t) = \left(\frac{d}{dt} + \delta \right)^n e(t). \quad (9)$$

The rate of convergence of the sliding surface is governed by the value assigned to parameter δ . Having established

appropriate sliding surfaces, the next step is to design the control input to drive the error system trajectories onto the sliding surfaces.

In this study, define a sliding surface as

$$S(t) = e_2(t) + \delta e_1(t). \quad (10)$$

Equation (10) is designed as the input of fuzzy system. Differentiating (10) with respect to time is as follows:

$$\dot{S}(t) = \dot{e}_2(t) + \delta \dot{e}_1(t). \quad (11)$$

Substituting (5) into (12), we obtain

$$\dot{S}(t) = (1 - \alpha)p(x_1, x_2, y_1, y_2) + u(t) + \delta e_2(t). \quad (12)$$

4.2. Sliding Rule-Based Fuzzy Control. A set of the fuzzy linguistic rules based on expert knowledge are applied to design the control law of fuzzy logic control. To overcome the trail-and-error tuning of the membership functions and rule base, the fuzzy rules are directly defined such that the error dynamics satisfies stability in the Lyapunov sense. The basic fuzzy logic system is composed of five function blocks [23]: (1) a rule base contains a number of fuzzy if-then rules, (2) a database defines the membership functions of the fuzzy sets used in the fuzzy rules, (3) a decision-making unit performs the inference operations on the rules, (4) a fuzzification interface transforms the crisp inputs into degrees of match with linguistic value, and (5) a defuzzification interface transforms the fuzzy results of the inference into a crisp output.

The fuzzy rule base consists of a collection of fuzzy if-then rules expressed as the form: if a is A , then b is B , where a and b denote linguistic variables, and A and B represent linguistic values that are characterized by membership functions. All of the fuzzy rules can be used to construct the fuzzy-associated memory.

In this study, the FLC is designed as follows: the signal S in (10) is as the antecedent part of the proposed FLC to design the control input u that will be used in the consequent part of the proposed FLC,

$$u = \text{FLC}(S), \quad (13)$$

where the FLC accomplishes the objective to stabilize the error dynamics (5). The i th if-then rule of the fuzzy rule base of the FLC is of the following form.

Rule i. If S is X , then

$$u_{Li} \equiv f_i(S), \quad (14)$$

where X is the input fuzzy sets, u_{Li} is the output which is the analytical function $f_i(\cdot)$ of the input variables (S).

For given input values of the process variables, their degrees of membership μ_{xi} , $i = 1, 2, \dots, n$, called rule-antecedent weights, are calculated. The centroid defuzzifier evaluates the output of all rules as follows:

$$u = \frac{\sum_{i=1, \mu_i \neq 0}^n \mu_i \cdot u_{Li}}{\sum_{i=1, \mu_i \neq 0}^n \mu_i}, \quad \mu_i = \mu_X(S). \quad (15)$$

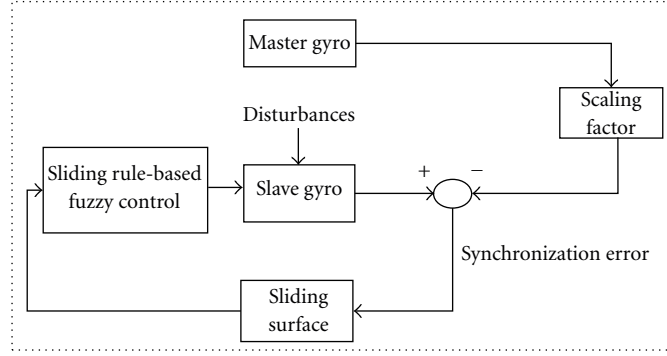


FIGURE 3: The scheme of generalized projective synchronization of chaotic gyroscopes versus disturbances via the sliding rule-based fuzzy control.

TABLE 1: Rule table of FLC.

Rule	Antecedent	Consequent
	S	u_{Li}
1	P	u_{L1}
2	Z	u_{L2}
3	N	u_{L3}

Table 1 lists the fuzzy rule base in which the input variable in the antecedent part of the rules is S , and the output variable in the consequent is u_{Li} .

Using P , Z , and N as input fuzzy sets represents “positive,” “zero,” and “negative,” respectively. The Gaussian membership function is considered. The combination of the two input variables (S) forms $n = 3$ heuristic rules in Table 1, and each rule belongs to one of the three fuzzy sets P , Z , and N . The rules in Table 1 are read as follows: taking Rule 1 in Table 1 as an example, “Rule 1: if input 1 S is P , then output is u_{L1} .”

To solve the control problem presented in (5), define a Lyapunov function as follows:

$$V(t) = \frac{1}{2} S^2(t). \quad (16)$$

Differentiating (16) with respect to times is as follows:

$$\dot{V} = S\dot{S}. \quad (17)$$

Substituting (12) into (17), then

$$\dot{V} = S \underbrace{[(1 - \alpha)p(x_1, x_2, y_1, y_2) + u(t) + \delta e_2(t)]}_A. \quad (18)$$

The corresponding requirement of Lyapunov stability is [24]

$$\dot{V} < 0. \quad (19)$$

If $A < 0$, then the Lyapunov stability will be satisfied. The following cases will satisfy all the stability conditions.

Rule 1. If $S > 0$, then $A < 0$, so consider the consequent part of Rule 1,

$$(1 - \alpha)p(x_1, x_2, y_1, y_2) + u(t) + \delta e_2(t) < 0. \quad (20)$$

Equation (20) can be simplified as follows:

$$u(t) < -\delta e_2(t) - (1 - \alpha)p(x_1, x_2, y_1, y_2). \quad (21)$$

Let us choose the control input as follows such that (21) is satisfied:

$$u_{L1} = -\delta e_2(t) - (1 - \alpha)p(x_1, x_2, y_1, y_2) - \lambda, \quad (22)$$

where λ is a positive constant value.

Rule 2. If $S \in \text{zero}$, then

$$(1 - \alpha)p(x_1, x_2, y_1, y_2) + u(t) + \delta e_2(t) = -\eta \text{sgn}(S), \quad (23)$$

where η is a positive constant value. Equation (23) can be simplified as follows:

$$u(t) = -\eta \text{sgn}(S) - (1 - \alpha)p(x_1, x_2, y_1, y_2) - \delta e_2(t). \quad (24)$$

Let us choose the control input as follows such that (24) is satisfied:

$$u_{L2}(t) = -\eta \text{sgn}(S) - (1 - \alpha)p(x_1, x_2, y_1, y_2) - \delta e_2(t). \quad (25)$$

Rule 3. If $S < 0$, then $A > 0$, so consider the consequent part of Rule 3,

$$(1 - \alpha)p(x_1, x_2, y_1, y_2) + u(t) + \delta e_2(t) > 0. \quad (26)$$

Equation (26) can be simplified as follows:

$$u(t) > -(1 - \alpha)p(x_1, x_2, y_1, y_2) - \delta e_2(t). \quad (27)$$

Let us choose the control input as follows such that (27) is satisfied:

$$u_{L3}(t) = -(1 - \alpha)p(x_1, x_2, y_1, y_2) - \delta e_2(t) + \lambda, \quad (28)$$

where λ is a positive constant value.

Therefore, all of the rules in the FLC can lead to Lyapunov stable subsystems under the same Lyapunov function (16). Furthermore, the closed-loop rule-based system equation (5) is asymptotically stable for each derivate of the Lyapunov function that satisfies $\dot{V} < 0$ in Table 1, that is, the error states guarantee convergence to zero.

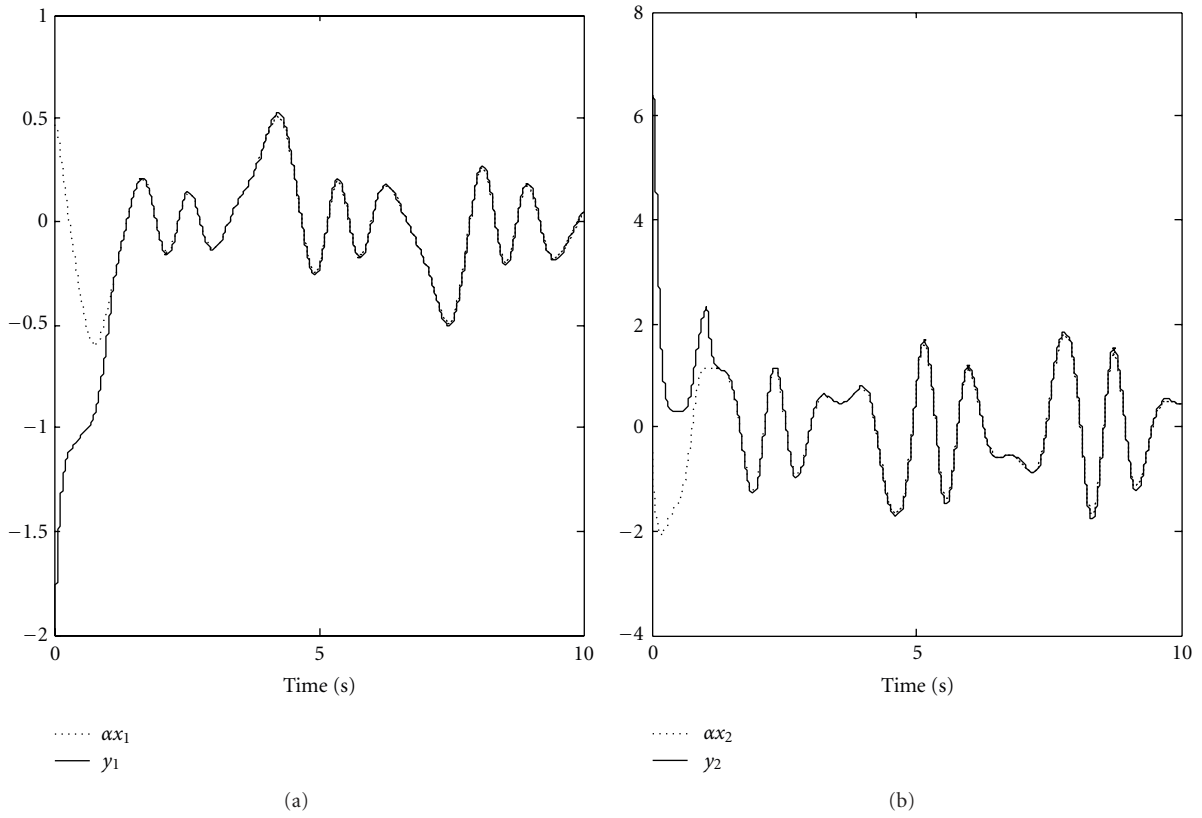


FIGURE 4: Time responses of the master and slave systems ($\alpha = 0.5$).

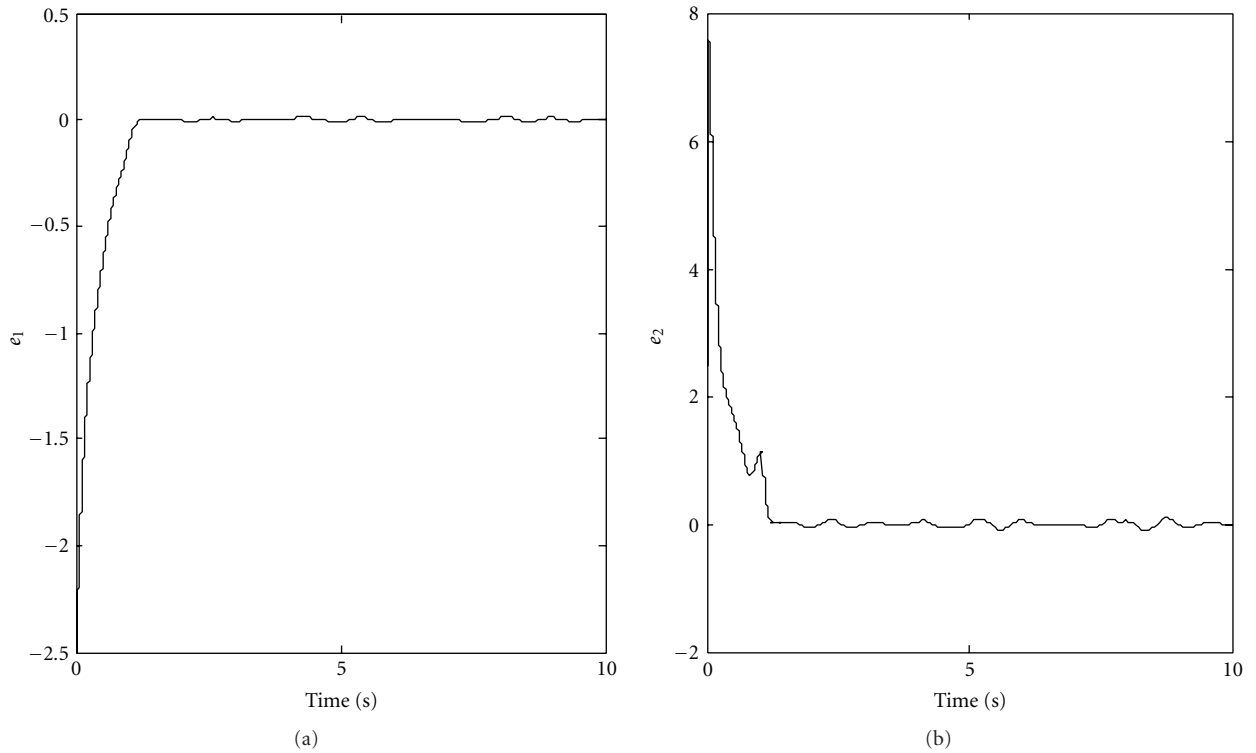


FIGURE 5: Synchronization error ($\alpha = 0.5$).

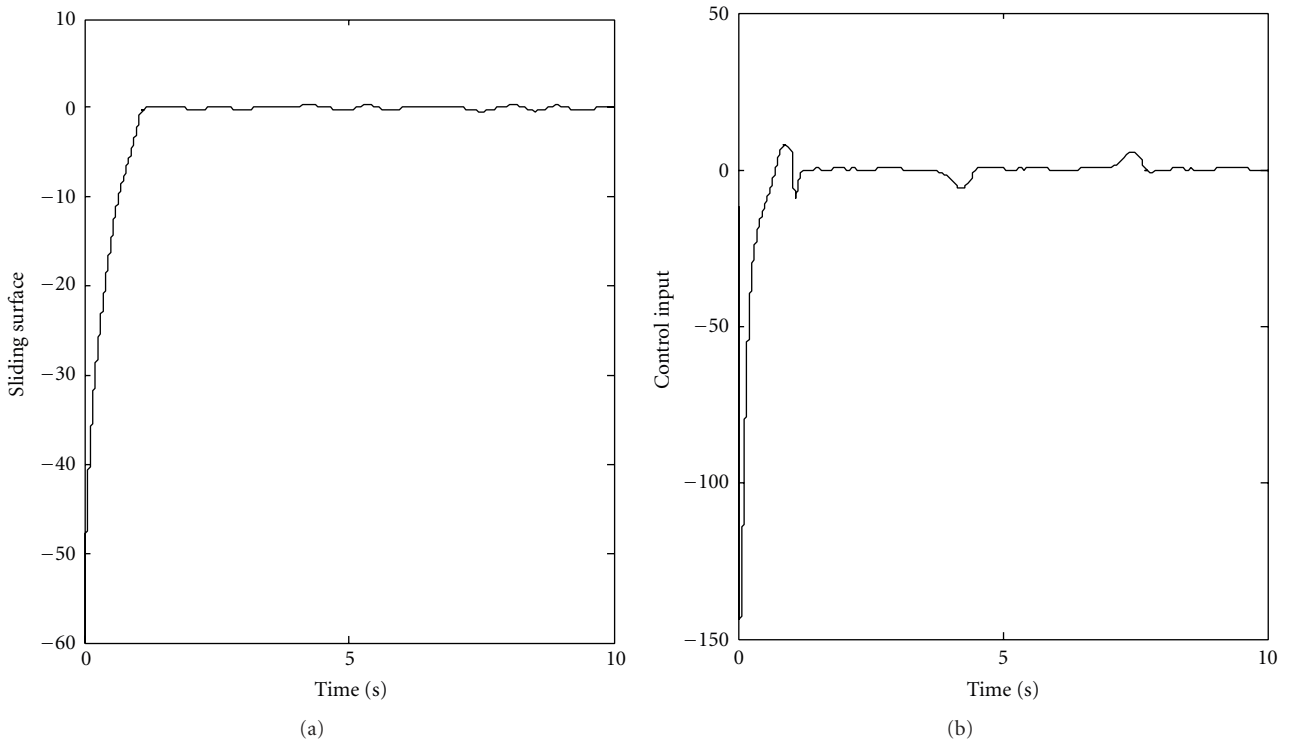


FIGURE 6: The sliding surface and input control ($\alpha = 0.5$).

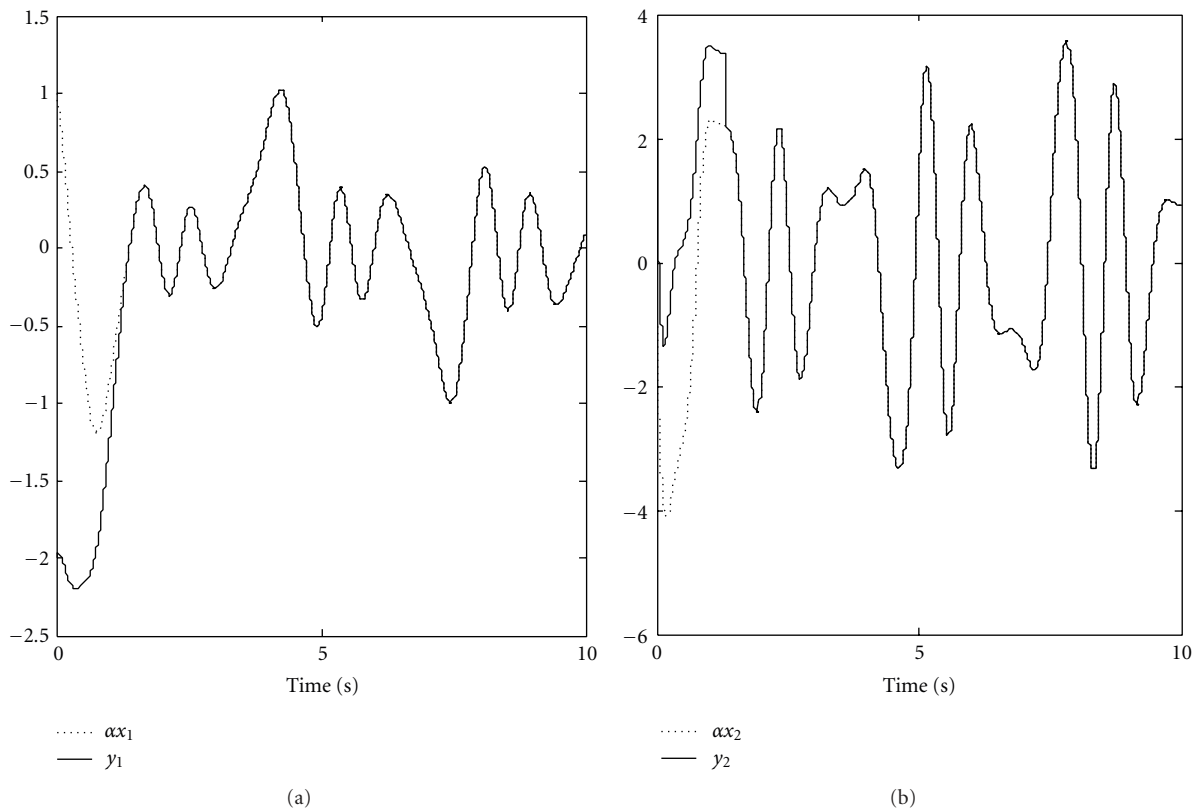


FIGURE 7: Time responses of the master and slave systems (complete synchronization $\alpha = 1$).

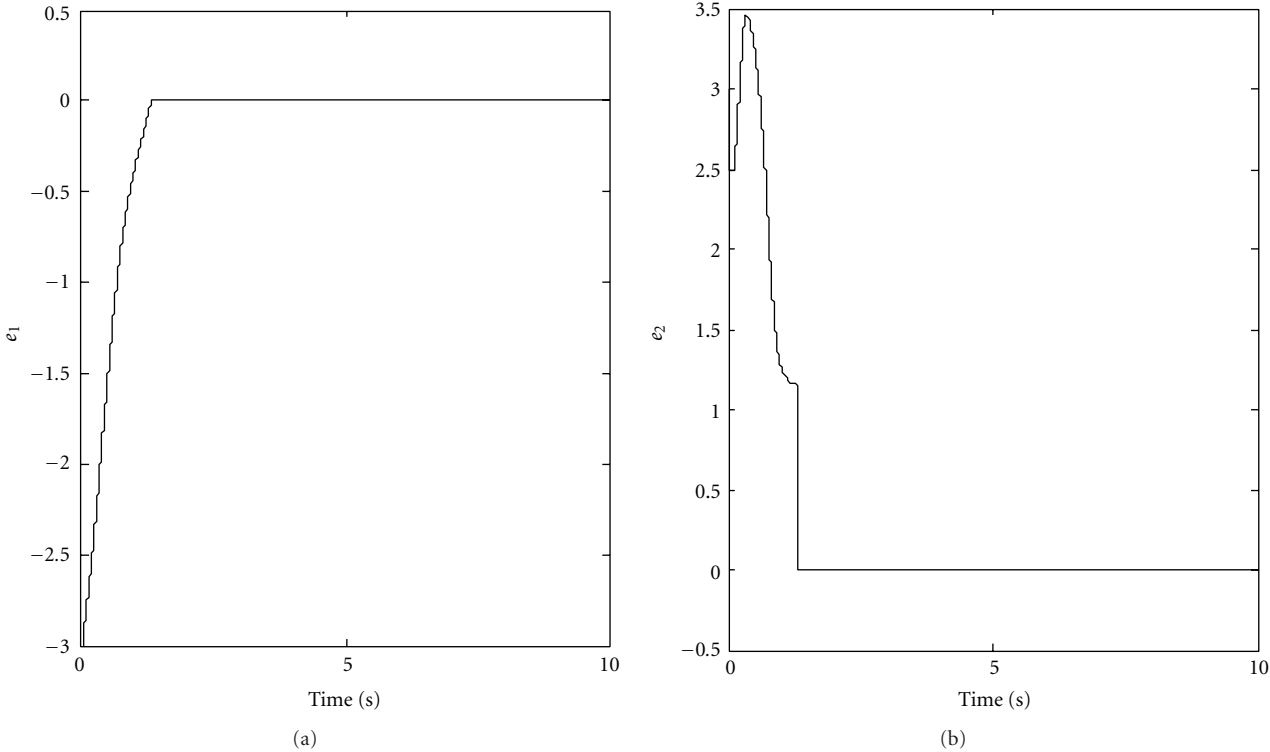


FIGURE 8: Synchronization error (complete synchronization $\alpha = 1$).

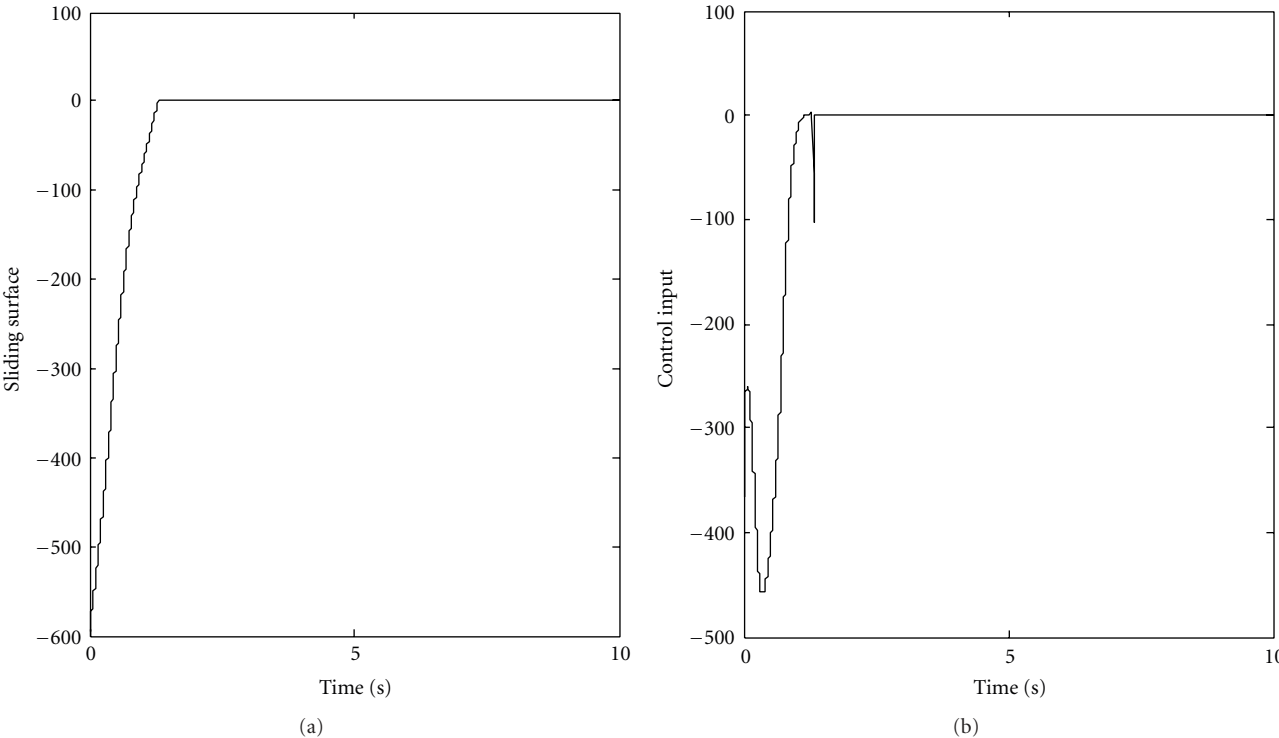


FIGURE 9: The sliding surface and input control (complete synchronization $\alpha = 1$).

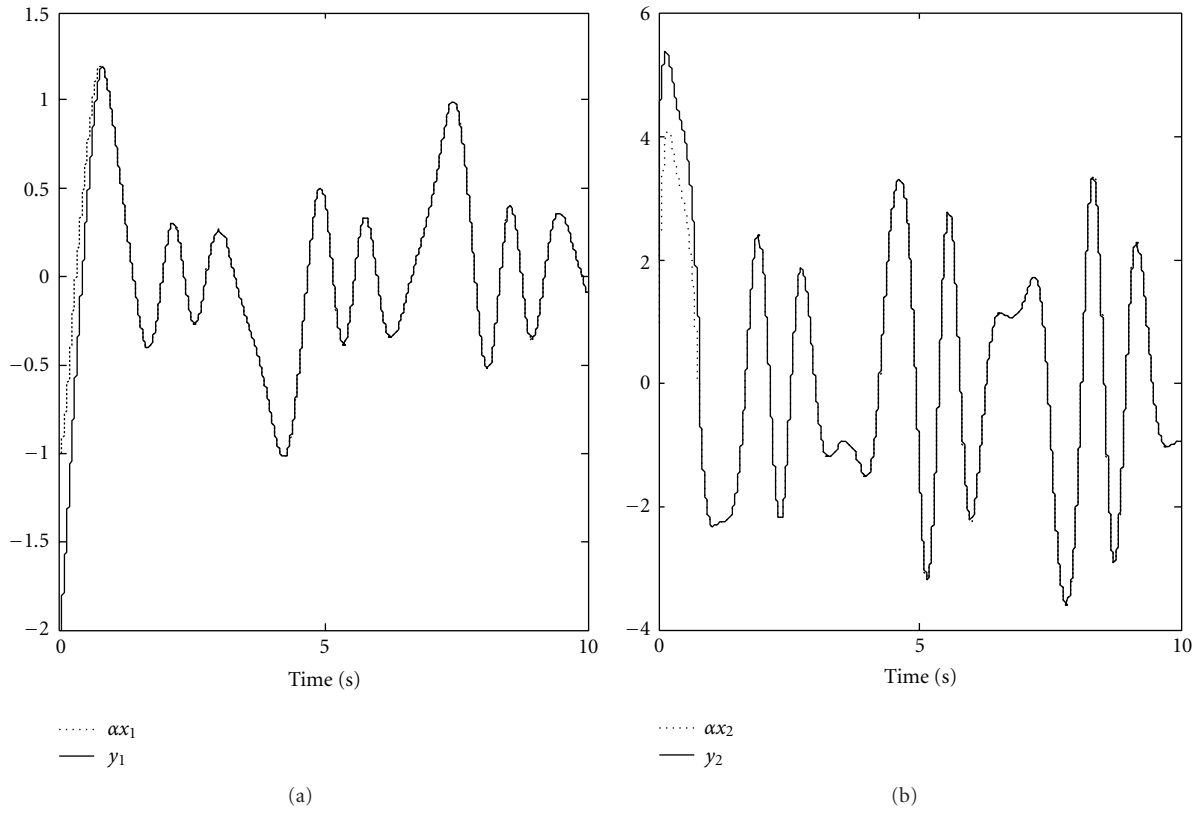


FIGURE 10: Time responses of the master and slave systems (antisyndronization $\alpha = -1$).

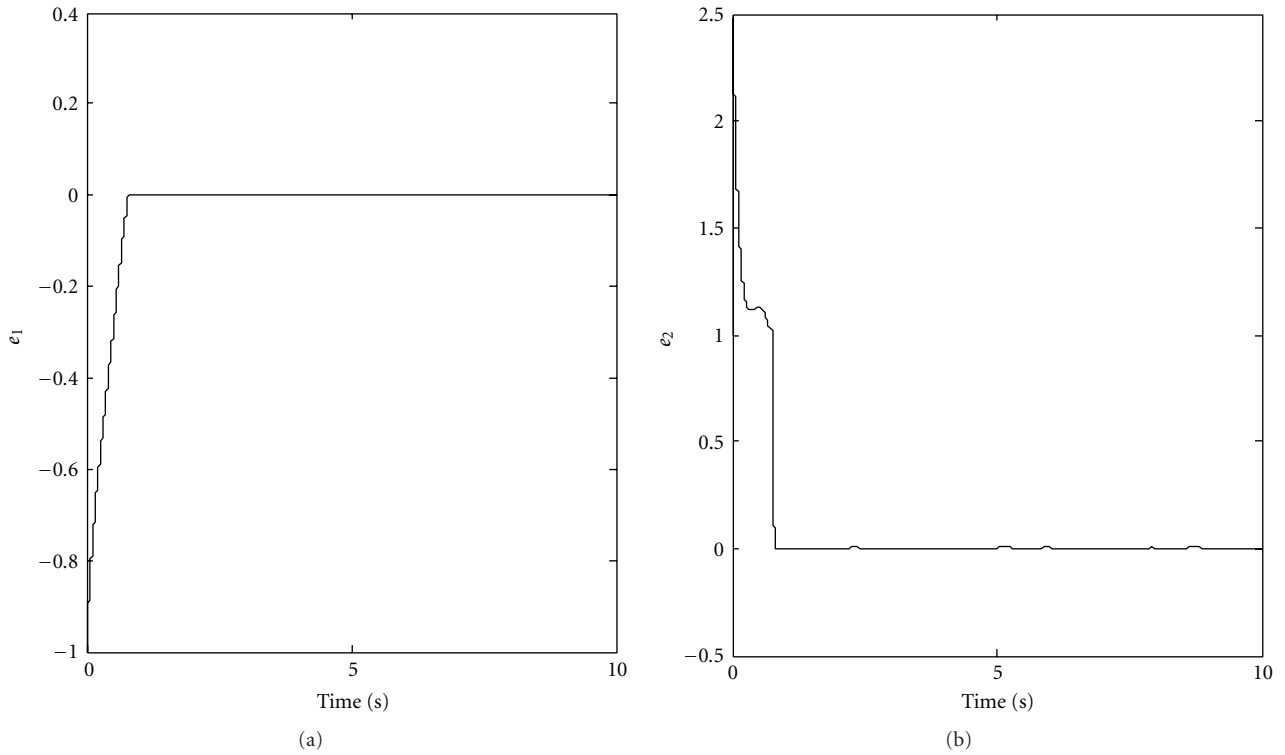


FIGURE 11: Synchronization error (antisyndronization $\alpha = -1$).

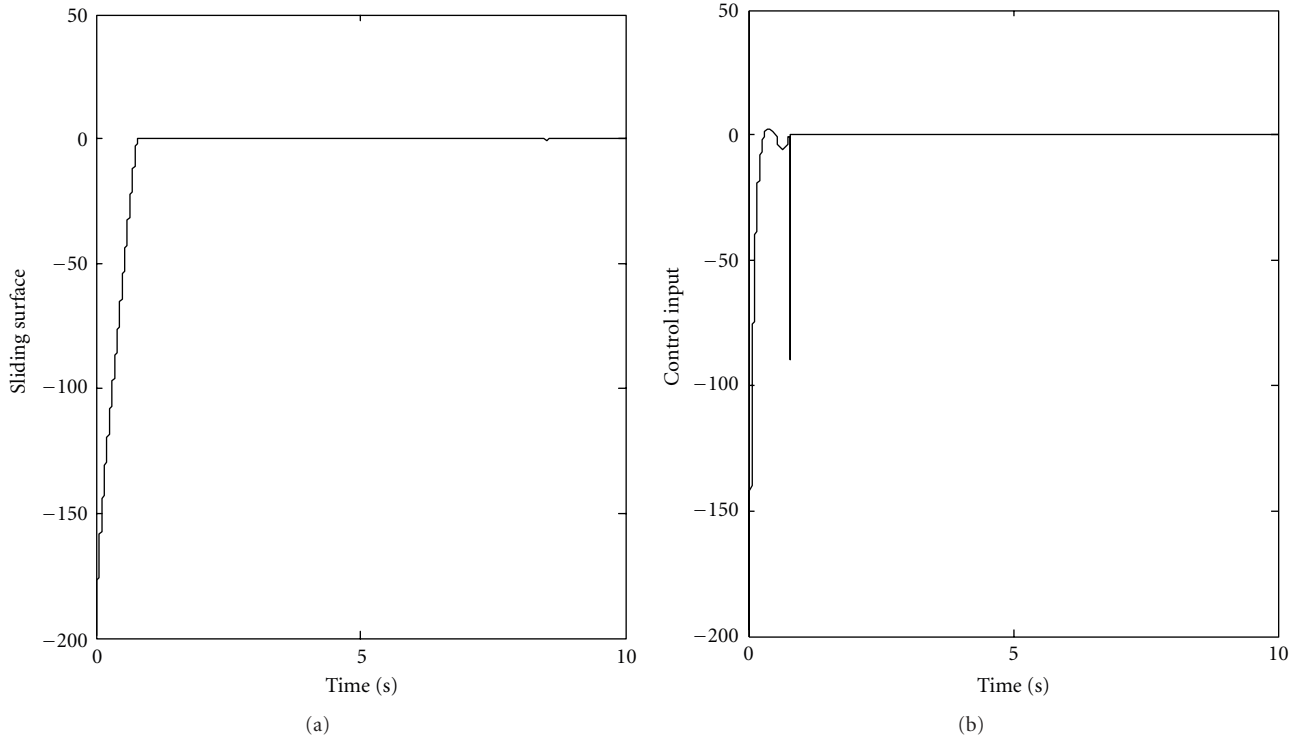


FIGURE 12: The sliding surface and input control (antisynchronization $\alpha = -1$).

5. Simulation Results

In this section, numerical simulations are given to demonstrate GPS of the chaotic gyros versus disturbances via the sliding rule-based fuzzy control. The parameters of nonlinear chaotic gyroscope systems are specified in Section 2.

The external disturbance d_1 is attached between $3 < t < 4$ and $7 < t < 8$. The initial conditions of the master and slave systems are defined as follows:

$$\begin{bmatrix} x_1(0) & x_2(0) \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}^T, \quad \begin{bmatrix} y_1(0) & y_2(0) \end{bmatrix}^T = \begin{bmatrix} -2 & 2 \end{bmatrix}^T. \quad (29)$$

Notice that, to reduce the system chattering, the sign functions are substituted with the saturation functions.

The time responses of the master and the slave system for GPS with $\alpha = 0.5$, complete synchronization $\alpha = 1$, and antisynchronization $\alpha = -1$ are shown in Figures 4, 7, and 10, respectively.

Synchronization errors for GPS with $\alpha = 0.5$, complete synchronization $\alpha = 1$, and antisynchronization $\alpha = -1$ are shown in Figures 5, 8, and 11, respectively. The errors illustrated in Figures 5, 8, and 11 converge asymptotically to zero.

In addition, the control input and sliding surface for GPS with $\alpha = 0.5$, complete synchronization $\alpha = 1$, and antisynchronization $\alpha = -1$ are shown in Figures 6, 9, and 12, respectively.

The simulation results of GPS via the sliding rule-based fuzzy control have good performances and confirm that the master and the slave systems achieve the synchronized states,

when external disturbance occurs. Also, these results demonstrate that the synchronization error states are regulated to zero asymptotically. It is observed that the proposed method is capable to GPS, when disturbances occur.

6. Conclusion

In this paper, generalized projective synchronization of chaotic gyroscope systems with external disturbances via sliding rule-based fuzzy control has been investigated. Based on Lyapunov stability theory and fuzzy rules, the nonlinear controller and some generic sufficient conditions for global asymptotic synchronization are attained. To achieve GPS, it is clear that the proposed method is capable for creating a full-range GPS of all state variables in a proportional way. It also allows us to arbitrarily adjust the desired scaling by controlling the slave system. The advantages of this method can be summarized as follows:

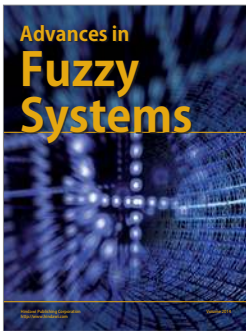
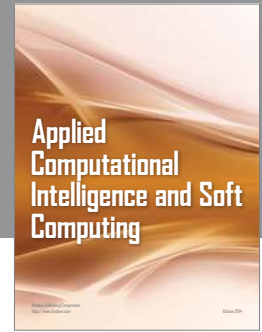
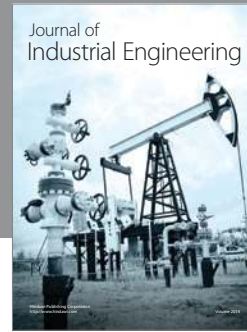
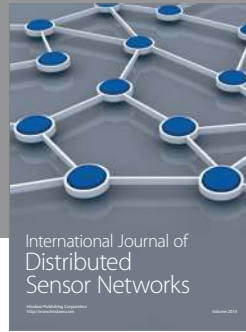
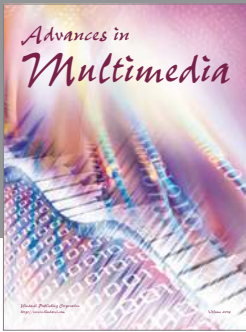
- (i) it is a systematic procedure for GPS of chaotic gyroscope system;
- (ii) the controller is easy to be implemented;
- (iii) it is not necessary to calculate the Lyapunov exponents and the eigenvalues of the Jacobian matrix, which makes it simple and convenient;
- (iv) the controller is robust versus external disturbances.

Simulations results have verified the effectiveness of this method for GPS of chaotic gyroscope systems.

Since the gyro has been utilized to describe the mode in navigational, aeronautical, or space engineering, the generalized projective synchronization procedure in this study may have practical applications in the future.

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