UNCLASSIFIED
security classification of this page (whon defa Eniered)

| REPORT DOCUMENTATION PAGE | READ INSTRUC TIONS BEFORE COMPLETING PORM |
| :---: | :---: |
|  | 3. recipient's catalog jumbez |
| 4. TITLE (and Subitite) <br> Generalized Rank Annihilation Factor Analysis | 5. TYPE OF REPOAT \& PE IIOJ COVERED Technical Report - Interin <br> 6. PERFORMING ORG. REPIIRT NUMBER |
| 7. AUTHOR(s) <br> Eugenio Sanchez and Bruce R. Kowalski | $\begin{aligned} & \text { B. CONTRACT OR GRANT NJM BER(T) } \\ & \text { NOOO14-75-C-0536 } \end{aligned}$ |
| 9. PERFORMING ORGANIzATION NAME ANO ADOAES Laboratory for Chemometrics Department of Chemistry BG-10 | 10. PROGRAM ELEMENT PHOJECT, TASK AREAA WORKUNTTUUYERS NR 051-565 |
| 11. Controlling office name and adoress Materials Sciences Division | 12. REPORT OATE October 15, 1985 |
| Office of Naval Research <br> Arlington, Virginia 22217 | 13. NUMBER OF PAGES 13 |
| 14. MONITORING AGENCY NAME A ADORESSII dilformi from Conitrolline Oillico) | 15. SECUAITY CLASS. (ot thto roport) UNCLASSIFIED |
|  | 15. OEECASSIFICATION/DOTNGRADING |

16. DISTRIBUTION STATEMENT (ot ithe Ropart)

This document has been approved for public release and sale;
its distribution is unlimited.

$\rightarrow$ Multivariate Analysis;
Bilinear Form, Bilinear Data
Principal Component Regression (PCR),$\longrightarrow$ Two-diminsional Data;
Rank Annihilation Factor Analysis Singular Value Decomposition Singular Value Decomposition, Pseudoinverse - ${ }^{\leftarrow}$ Background Correction 20. ABSTRACT (Conilinue on roverae alde lf noceseary and idonilty by block mumber) $\rightarrow$ The method of rank annihilation is shown to be a particular case of a more general method for quantitation in bilinear data arrays such as LC/UV, GC/MS or emission-excitation fluorescence. Generalized rank annihilation is introduced as a calibration method that allows for simultaneous quantitative determination of all of the analytes of interest in a mixture of unknowns. Only one calibration mixture is required. The silinear spectra of both unknown and calibration sample must be obtained. Bilinear target factor analysis is introduced as a projection of a target bilinear matrix onto another principal component bilinear

# OFFICE OF NAVAL RESEARCH <br> Contract N00014-75-C-0536 <br> Task No. NR 051-565 <br> TECHNICAL REPORT NO. 34 <br> Generalized Rank Annihilation Factor Analysis <br> by <br> Eugenio Sanchez and Bruce R. Kowalski 

Prepared for Publication
in

## Analytical Chemistry

University of Washington
Department of Chemistry BG-10
Seattle, Washington 98195

October 15, 1985

Reproduction in whole or in part is permitted for any purpose of the United States Government

This document has been approved for public release and sale; its distribution is unlimited

## Generalized Rank Annihilation Factor Analysis

sir:The anslytical chemist is irequently confronted with the problem of analyzing complex mixtures for which only concentrations of a few components are of interest. In these cases, it is desirable to be able to obtain quantitative information for the anslytes of interest without concern for the rest of the components in the sample. Second order bilinear sensors, i.e. sensors that yield a two dimensional data matrix of the form $M_{i j}=\sum_{k} \beta_{k} x_{i k} y_{j k}$, are specially suited for this purpose, and the preferred technique for quantitation is known as rank annihilation factor analysis, RAFA (1,2). So far this method has been applied to excitation-emission fluorescence ( $1-3$ ), LC/UV (4) and TLC-reflectance imsging spectrophotometry (5) with good results. It is important to realize that not all two dimerisional techniques yield bilinear dats arrays: e.g. 2D-NMR or MS/MS data in their raw forms are not bilineer.

A limitation of rank annihilation as originally formulated is that an iterative solution requiring many matrix diagonalizations is necessary (1). Lorber (6) has reported a non-iterative solution presenting the problem as a generalized eigenvalue-eigenvector equation for which a direct solution is found by using the singular value decomposition. With his method, to obtain the concentrations of the $p$ analytes of interest in the sample, its bilinear spectrum and the $p$ calibration spectra for each pure analyte must be recorded to obtain the concentrations. Analysis for each analyte requires a separste calculation. This letter presents the generalized rank annihilstion methad, of which Lorber's non-iterative method is only a particular case, that allows simultaneous quantitation of analytes in a sample using just

one bilinear calibration spectrum obtained from a mixture of standards, one standard for each analyte.

Generalized rank anninilation can determine the bilinear spectrum and the relative concentration for each analyte in the unknown mixture. The colculated spectra are next matched to those of the standards. It is then straightforward to determine the actual concentration of each analyte from its relative concentration and the concentration of the corresponding standard. The full bilinear spectrum of each analyte is not actually required for identification. One need only use a single order (e.g. only the UV spectrum in the LC/UY case) for the match. This is an unusual type of analysis as in most cases, analyte concentrations are estimated one at a time thereby precluding identification.

## THEORY AND DISCUSSION

Any bilinear data matrix $M$ can be expressed as a linear combination of the $n$ pure-component, bilinear spectrs $\mu_{k}$ :

$$
M=\sum_{k}^{n} \beta_{k} \mu_{k} \quad \text { where } \mu_{k}=x_{k} y_{k}^{\top} ;\left(\mu_{i j}\right)_{k}=x_{i k} y_{j k}
$$

The $x_{k}$ are column vectors with information in one order, e.g. excitation spectra, and the $y_{k}{ }^{\top}$ are row vectors with information in the second order, e.g. emission spectra. If we define the $\mu_{k}$ as unitary-concentration, purecomponent, bilinear spectra, $B_{k}$ is the concentration of the $k^{\text {th }}$ compound in M. We can rewrite eq 1 in matrix notation as

$$
\begin{equation*}
M=X B Y^{\top} \tag{2}
\end{equation*}
$$

Where $X$ is a matrix whose columns are the $n X_{k_{1}}$ yectors, $Y^{T}$ is a matrix Whose rows are the $n y_{k}{ }^{\top}$ vectors and $B$ is a diagonal matrix with diagorial elements thot are the concentrations, $B_{k}$.

In general we will haye two data matrices, the unknown concentrations data matrix $M$ and the calibration data matrix $\boldsymbol{N}$. The bilinear calibration data matrix $\mathbf{N}$ can similarly be represented in matrix notation as

$$
\begin{equation*}
N=X \xi Y \top \tag{3}
\end{equation*}
$$

Where $X$ and $Y^{\top}$ are the same matrices defined for eq 2 and $\xi$ is a diagonal matrix whose diagonal elements are the concentrations $\xi_{k}$ for the calitration matrix.

The matrices $M$ and $N$ have in common the $X$ and $Y^{\top}$ blocks, e.g. the excitation and emission spectra are the same, differing only in their concentration matrices, $B$ and $\xi$ respectively. Therefore, solving for $X$ in eq 2 and eq 3 we obtain

$$
\begin{align*}
& X B=M(Y \top)^{+}  \tag{4}\\
& X \xi=N(Y T)^{+} \tag{5}
\end{align*}
$$

where $(Y T)+$ represents the pseudoinverse ( 7 ) of the matrix $Y^{\top}$. Now we right-multiply eq 4 by $\xi$ and eq 5 by $B$ and combine to get:
$N(\boldsymbol{Y} \boldsymbol{T})+\boldsymbol{B}=\mathbf{M}(\boldsymbol{Y} \boldsymbol{T})+\xi$
defining $Z \equiv\left(Y^{\top}\right)^{+}$,

$$
\begin{equation*}
N Z B=M Z \xi \tag{7}
\end{equation*}
$$

We only know $M, N$ and $\xi$, thus we must to solve for $Z$ and $B . E q(7)$ is similar to the generalized eigenyalue-eigenvector problem, but can not be solyed by standard methods since $\mathbf{N}$ and $\boldsymbol{M}$ are not necessarily square matrices. A solution of this equation will be discussed in the next sections, for the following different possible cases:
[1] The calibration data matrix $N$ has just anecomponent, that is present in the sample doto matrix $M$,

$$
\begin{align*}
& \operatorname{diagonal}(B)=\left\{\beta_{1}, \beta_{2}, \ldots, \beta_{n}\right\} \quad n \geq 1  \tag{8}\\
& \text { diagonal }(\xi)=\left\{\xi_{1}, 0, \ldots, 0\right\} \tag{9}
\end{align*}
$$

This is the standard RAFA problem as discussed by Lorber (6).
[2] The calibration data matrix $N$ has severo/components, that are a sulsset of the components present in the sample dato matrix $M$,

| diagonal $(B)=\left\{B_{1}, B_{2}, \ldots, B_{r}, B_{r+1}, \beta_{r+2}, \ldots, B_{r+s}\right\}$ | $r \geq 0$ |
| :--- | :--- |
| diagonal $(\xi)=\left\{0,0, \ldots, 0, \xi_{r+1}, \xi_{r+2}, \ldots, \xi_{r+s}\right\}$ | $\varepsilon \geq 1$ |

Here, $r$ is the number of components in the somple $M$ that are not present in the calibration N , and s is the number of common components.
[3] The components in the sample data matrix $M$ are a subset of the components present in the calibration data matrix $N$,

$$
\begin{array}{ll}
\operatorname{diagona}(B)=\left\{\beta_{1}, B_{2}, \ldots, B_{s}, 0, \ldots, 0\right\} & s \geq 1 \\
\text { diagonal }(\xi)=\left\{\xi_{1}, \xi_{2}, \ldots, \xi_{s}, \xi_{s+1}, \ldots, \xi_{s+t}\right\} & \leq>1 \tag{13}
\end{array}
$$

Again, $s$ is the number of common components, and $t$ is the number of components in the calibration $\boldsymbol{N}$ that are absent from the sample $\mathbf{M}$.
[4] The most general case would be when there are analytes in the unknown sample that are not present in the calibration sample and vice verso,

$$
\begin{aligned}
& \text { diagonal }(B)=\left\{\beta_{1}, \beta_{2}, \ldots, \beta_{r}, \beta_{r+1}, \beta_{r+2}, \ldots, \beta_{r+\varepsilon}, 0, \ldots, 0\right\}(14) \\
& \text { diagonal }(\xi)=\left\{0,0, \ldots, 0, \xi_{r+1}, \xi_{r+2}, \ldots, \xi_{r+\varepsilon}, \xi_{r+\xi+1}, \ldots, \xi_{r+\varepsilon+t}\right\}(15)
\end{aligned}
$$

Here, $r$ is the number of components in the sample $M$ that are not present in the calibration $N, s$ is the number of common components, and $t$ is the number of components in the calibration data matrix $N$ that are absent in the unknown sample $M$.

## [1] FIRST CASE: One Component Quantitation

In this case, the calibration data matrix $\mathbf{N}$ has just one component, $\mu_{k}$, that is also present in the sample data matrix. The solution for this case has been reported by Lorber ( 6 ) and will be included here for completeness.

The irst step in solving eq 7 is to spply principal components analysis ( 8 ) to the sample matrix $\mathbf{M}$, and then express the matrices in terms of these principal components. The principal components of $M$ are obtained by applying singular value decomposition (7)

$$
\begin{equation*}
M=U S V^{\top} \tag{16}
\end{equation*}
$$

where
$M V=S U$
$M^{\top} U=5 V$
$M^{\top} M V=\mathbf{S}^{2} \mathbf{V}$
eigen-equations in $V$ space.
$M M^{\top} U=S^{2} U$
eigen-equations in $U$ space.
The next step is to estimate the number of principal components that are significant using abstract foctor analysis (8) or cross validation $(9,10)$. In the ideal case, this number is equal to the number of components $n$ in the sample mixture. The number of significant principal components will allow reduction to the deterministic information contained in the $M$ matrix, with random error discarded in the lesser factors. To do this, a new matrix $M$ is generated from the first $n$ "significant" columns of $U, V$ and the upper left corner $n$ by $n$ part of $S$,

$$
\begin{equation*}
\underline{M}=\underline{U} \underline{\mathbf{S}} \underline{\varphi}^{\top} \tag{21}
\end{equation*}
$$

Noweq 7 can be rewritten as

$$
\begin{equation*}
N Z B=\underline{M} Z \xi=\underline{U} \underline{\mathbf{S}} \underline{\underline{v}}^{\top} Z \xi \tag{22}
\end{equation*}
$$

If we substitute $\mathbf{Z}=\underline{\mathbf{V}} \mathbf{S}^{-1} \mathbf{Z}^{*}$, where $\mathbf{Z}^{*} \equiv \underline{\underline{S}} \mathbf{Y}^{\top} \mathbf{Z}$
$N\left(\underline{\boldsymbol{V}} \underline{\underline{S}}^{-1} \mathbf{Z}^{*}\right) B=\underline{\boldsymbol{U}} \boldsymbol{\underline { S }} \underline{Y}^{\top}\left(\dot{\boldsymbol{Y}} \underline{\underline{S}}^{-1} \mathbf{Z}^{*}\right) \xi$
using the orthogonality properties of $\underline{\underline{V}} \underline{\underline{V}} \underline{V}=1=$ Identity matrix in the upper left $n$ by $n$ corner and zeros in the rest, so:

which reduces to
$\left(N \underline{\underline{S}} \underline{\mathbf{S}}^{-1}\right) \mathbf{Z}^{*} \boldsymbol{B}=\underline{\mathbf{U}} \mathbf{Z}^{*} \boldsymbol{\xi}$
Left-multiplying by $\underline{U}^{\top}$ and right-multiplying by $B^{-1}$ gives
$\left(\underline{U}^{\top} N \underline{Y} \underline{\mathbf{S}}^{-1}\right) Z^{*} B B^{-1}=\left(\underline{U}^{\top} \underline{U}\right) Z^{*} \xi B^{-1}=Z^{*} \lambda \quad \lambda \equiv \xi B^{-1}$
or, finally,
$\left(\underline{U}^{\top} N \underline{\underline{S}} \underline{S}^{-1}\right) Z^{*}=Z^{*} \boldsymbol{\lambda}$
which is the usual eigenvalue-eigenvector equation, because the matrix ( $\underline{U}^{\top} \mathbf{N}_{k} \underline{\underline{S}} \underline{S}^{-1}$ ) is square. The eigenvectors $\mathbf{Z}^{*}$ are not perpendicular because the matrix ( $\underline{U}^{\top} \boldsymbol{N}_{\mathbf{k}} \underline{\mathbf{V}} \underline{\mathbf{S}}^{-1}$ ) is not symmetric. Because the rank of $N$ is one, there will be $0-1$ zero solutions for the eigenvalues $\lambda_{k}$. Therefore, the only non-zero solution will be equal to the trace of the motrix ( $\underline{U}^{\top} \mathcal{N}_{k} \underline{\mathcal{Y}} \underline{\underline{S}}^{-1}$ ). By calculating the trace of this matrix, i.e. $\lambda_{k}$, the concentration $\beta_{k}$ of the $k^{\text {th }}$ component is solved directly as $\beta_{k}=\xi_{k} / \lambda_{k}$.

If the unknown sample does not have the component that is present in the calibration sample, we cannot expand $N$ in terms of $X$ and $Y$ (eq 3), therefore eq 25 b is not valid. This is an example of the fourth case introduced in the previous section, which will be considered later in this paper. In practice, a non-zero concentration value $\beta_{k}$ will be obtained, so the validity of eq 3 must be verified before applying eq 25 b. Using target factor analysis $(8,11)$, madified for bilinear data, it is possible to check if $N$ is included in $M$ (see appendix for the details of bilinear target factor anglysis). The projection matrices $\underline{\mathbb{U}} \underline{U}^{\top}$ and $\underline{\underline{V}} \underline{V}^{\top}$ should leave $\mathbb{N}$ unchariged:

## $\underline{\underline{U}} \underline{U}^{\top} \mathbf{N} \underline{V} \underline{V}^{\top}=\mathbf{N}$

As pointed out by Larber (6), if the colitretion matrix $N$ hes mare than one component, i.e. its rank is greater than one, several solutions will be obtained for the concentrations $\beta_{1}, \beta_{2}, \ldots, \beta_{n}$, but there will be no way to match which concentration corresponds to which chemical component. The proposed alternative is to obtain the spectrum of all the components separately, and estimate their concentration one by one. A solution to this problem is described in the next section, using the eigenvectors matrix $\mathbf{Z}$ in eq 7 , which was defined as the pseudainverse of the $Y$, i.e. the generalized irverse of the pure component's emission spectra.

## [2] SECOND CASE: Simultaneous Quantitation of Several Components

In this case, the calibration dato matrix $N$ has several components, that are a subset of the components present in the sample data matrix M. In the first place, it is necessary to check that the components in N are 8 subset of the cornponents present in the sample data matrix M, applying bilinear target factor analysis to the matrix $N$, i.e. eq 26 should be true.

If more than one component is represented in the calibration matrix, eq 25b has several non-zero eigenvalues. The solution is a set of eigenvalues $\boldsymbol{\lambda}$ and their corresponding eigenvectors $\mathbf{Z}^{*}$. The eigenvectors allow us to calculate the pure spectra matrices $X$ and $Y^{\top}$, e.g. excitation and ernission spectra:

$$
\begin{align*}
& Z^{*}=\underline{S}_{\underline{Y}}{ }^{\top} Z=\underline{S} \underline{Y}^{\top}\left(Y^{\top}\right)^{+}  \tag{27}\\
& Y^{\top}=\left(\underline{V} \underline{\underline{S}} \underline{S}^{-1} Z^{*}\right)^{+} \tag{28}
\end{align*}
$$

Using the definition of $M=X B Y^{\top}=\underline{\mathbf{U}} \underline{\mathbf{S}} \underline{\underline{V}}^{\top}$

$$
\begin{equation*}
X B=M\left(Y^{\top}\right)^{+}=\underline{U} \underline{S} \underline{Y}^{\top} \underline{Y} \underline{S}^{-1} Z^{*}=\underline{U} Z^{*} \tag{29}
\end{equation*}
$$

The eigenvalues $\lambda_{k}$ are the ratio of concentrations $\xi_{k} / \beta_{k}$ for each component, i.e. colibrotion/unknown. Heving the pure spectro $x_{k}$ or $y_{k}{ }^{\top}$, it is easy to match which concentration $\xi_{k}$ corresponds to which ratio $\lambda_{k}$, therefore the concentrations $\beta_{k}$ con be estimated $\beta_{k}=\xi_{k} / \lambda_{k}$.

## [3] THIRD CASE: Calibration as a Base

When the somple dato matrix $M$ is a subset of the components in the calibration $N$, we must invert the procedure. The principal components of the matrix $M$ do not form a basis for the representation of the matrix $N$, therefore eq 25b is not valid in this case. The principal components of $N$ are estimated $N=\underline{\Psi}_{N} \boldsymbol{S}_{N} \boldsymbol{Y}_{N}{ }^{\top}$, ond equations similor to eq 25b, 28, 29 are obtained:

$$
\begin{align*}
& \left(\underline{U}_{N}^{\top} M \underline{X}_{N} \underline{S}_{N}{ }^{-1}\right) Z_{N^{*}}^{*}=Z_{N^{*}} \lambda_{N}  \tag{30}\\
& Y^{\top}=\left(\underline{\underline{X}}_{N} \underline{S}_{N_{N}}^{-1} Z_{N^{*}}\right)^{+}  \tag{31}\\
& X B=\underline{U}_{N} Z_{N}^{*} \tag{32}
\end{align*}
$$

The eigenvalues ( $\left.\lambda_{\mu}\right)_{k}$ are not defined os they were before. Now the $\left(\lambda_{N}\right)_{k}$ are the ratio or concentrotions $\beta_{k} / \xi_{k}$ for each component, i.e. unknown sample/colibration.

Bilinear target factor analysis can the used to test instances of the third case. The projection of the matrix $M$ in the spaces defined by $N$ should leave $M$ unchanged:

$$
\begin{equation*}
\underline{U}_{N}{\underline{u_{N}}}^{\top} M \underline{X}_{N}{\underline{y_{N}}}^{\top}=M \tag{33}
\end{equation*}
$$

If both this test and eq 26 1ail, then we are dealing with the fourth cose, discussed in the next section. In practice, the third case con be solved using
principal components analysis or multiple linear regression, because the spectre of all the components are known.

## [4] FOURTH CASE: The General Condition

In this case, the calibration sample will have some components that are not present in the unknown somple, and there will be some componerits in this unknown sample not present in the calibration sample. Projection of one matrix onto the principal components of the other matrix will change its information; eq 25b and eq 30 will not be valid.

A solution to this problem can be obtained using the principal components of the sum of the matrices $M$ and $N$, defining $W \equiv M+N$, $W=\underline{U}_{W} \underline{S}_{w} \underline{V}_{W^{\top}}{ }^{\top}$
( $\left.\underline{U}_{W}^{\top} M \underline{U}_{W} \underline{S}_{W^{-1}}{ }^{1}\right) Z_{W}{ }^{*}=Z_{W}^{*} \lambda_{W}$
$Y^{\top}=\left(\underline{V}_{w \mid} \underline{S}_{w_{i}}{ }^{-1} Z_{w^{*}}\right)^{*}$
$X B=\underline{U}_{W} Z_{W}{ }^{*}$
The eigenvalues $\lambda_{k}$ are the ratio of concentrations $\Omega_{k} /\left(\xi_{k}+\beta_{k}\right)$. For all the components present in both mixtures, the concentration in the unknown is $\beta_{k}=\lambda_{k} \xi_{k} /\left(1-\lambda_{k}\right)$. When one component is not present in the calibration sample, $\xi_{k}=0$, and $\lambda_{k}=1$.

The solution presented for this case con be applied to all the previous cases, and no testing with target factor analysis is necessary. An artificial matrix $\boldsymbol{W}$ is generated to perform the calculations. This suggests that one could instead generate the $w$ matrix simply by making a single standard addition containing known amounts of all analytes to the unknown sample. In this way the calitration mixture is added to the unknown mixture, and the $W$ matrix is measured directly. Duantitation by RAFA with
the standard addition method (SAM) has been discussed by Lorber (14) for single anolyte addition. This procedure would extend the opplicability of his method to the quantitation of several analytes ot a time, correcting for matrix effects and thereby represents an extension of the generalized standard addition method, GSAM ( 12,13 ), to second-order tensor data.

If we have several calibration matrices $N_{1}, N_{2}, \ldots, N_{q}$, we can apply the method to all of them, one at a time, or we can hande it as a three waty factor analysis problem, using all of the information in one calculation. We are currently working in this problem, which will be the subject of another publication.

## APPENDIX

Target Factor Analysis $(8,11)$ can be applied to bilinear data in a similar way that it is used to one dimensional data. For the test vectors $\mathbf{x}_{i}$ or $y_{i}$, TFA can be expressed as:

$$
\begin{equation*}
\underline{U} \underline{U}^{\top} x_{i}=x_{i} \quad \text { or } \quad y_{1}^{\top} \underline{\underline{V}} \underline{V}^{\top}=y_{i}^{\top} \tag{38}
\end{equation*}
$$

every test vector $x_{i}$ or $y_{i}$ generates a predicted targent vector $x_{i}$ or $y_{i}$. If the test vectors are present in the matrix M, i.e. If the $i^{\text {th-component, which }}$ spectrum is $X_{i} y_{i}$, is present in $M$, then the predicted targent vectors should be equal to the test vectors: $\mathrm{x}_{\mathrm{i}}=x_{i} ; y_{i}=y_{i}$; therefore
$\underline{\mathbf{U}} \underline{U}^{\top} \boldsymbol{x}_{i}=\boldsymbol{x}_{i} \quad$ or $\quad \boldsymbol{y}_{i}{ }^{\top} \underline{\mathbf{V}} \underline{\mathbf{V}}^{\top}=\boldsymbol{y}_{i}{ }^{\top}$
using the definition of $X$ and $Y$ we can similarly write
$\underline{U} \underline{U}^{\top} \mathbf{X}=\boldsymbol{X} \quad \boldsymbol{Y}^{\top} \underline{\boldsymbol{V}} \underline{\boldsymbol{Y}}^{\top}=\boldsymbol{Y}^{\top}$
now, if $N=X \boldsymbol{Y} Y^{\top}$, then
$\underline{U} \underline{U}^{\top} N \underline{V} \underline{V}^{\top}=\left(\underline{U} \underline{U}^{\top} X\right) \xi\left(\mathbf{Y}^{\top} \underline{V} \underline{V}^{\top}\right)=X \xi Y^{\top}=N$
this is,
$\underline{\mathbf{U}} \underline{U}^{\top} \mathbf{N} \underline{V} \underline{\mathbf{V}}^{\top}=\mathbf{N}$.
this equation def ines bilinear target factor analysis. Note that
$\underline{\boldsymbol{U}} \underline{U}^{\top} \mathbf{N}=\mathbf{N} . \quad$ and $\quad \mathbf{N} \underline{\mathbf{V}} \underline{\underline{T}}^{\boldsymbol{T}}=\mathbf{N}$.
in practice, due to random nolse, equations 42-44 are aproximate.

## ACKNOWLEDGMENT

The authors gratefully acknowledge Scott Ramos for his assistance in writing this manuscript.

## LITERATURE CITED

[1] Ho, C-N.; Christian, G.D.; Davidson, E.R. Anol. Chem 1978, 50, 1108-1 113 .
[2] Ho, C-N.; Christian, G.D.; Davidson, E.R. Ansl. Chem. 1980, 52, 1071-1079.
[3] Ho, C-N.; Christian, G.D.; Davidson, E.R. Anel. Chem. 1981, 53, 9298.
[4] McCue, M.; Malinowski, E.R.; at Chramotagr: sci. 1983, 21, 229234.
[5] Gianelli, M.L.; Burns, D.H.; Callis, J.B.; Christian, G.D.; Andersen, N.H. thai. Chem. 1983, 55, 1858-1862.
[6] Lorber, A. Anal. Chim Acto 1984, 164, 293-297.
[7] Lawson, C.L.; Hanson, RJ. "Solving Least Squares Problems"; Prentice-Hall: Englewood Cliffs NJ, 1974.
[8] Malinowski, E.R.; Howery, D.G. "Factor Analysis in Chemistry"; Wiley: New York, 1980.

19] Wold, S. Technometrics 1978, 20, 397-405.
[10] Eastment, H.T.; Krzanorrski, W.J. Technometrics 1982, 24, 73-77.
[11] Lorber, A. Ahol. Chem 1984, 56, 1004-1010.
[12] Saxberg, B.E.H.; Kowalski,B.R. Anal. Chem. 1979, 51, 1031-1038

# [13] Jochum, C.; Jochum, P.; Kowalski, B.R. Anol. Chem. 1981, 53, 8592 

[14] Lorber, A. And. Chem. 1985, 56, in press

## Eugenio Sánchez

Bruce R. Kowalski

Laboratory for Chemometrics
Department of Chemistry, BG-10
University of Washington
Seattle, Washington 98195

This work was supported, in part, by the Office of Naval Research. One of the authors (E.S.) is grateful to the Venezuelan Fundación "Gran Mariscal de Ayacucho" for the award of a Scholarship.

## TECHNICAL REPORT OISTRIBUTION LIST, GËN

No. ..... No.
Copies ..... Copies

Office of Naval Research
Attn: Code 413
800 N. Quincy Street
Arlington, Virginia 22217
Or. Bernard Douda
Naval Weapons Support Center
Code 5042
Crane, Indiana 47522
Commander, Naval Air Systems Command
Attn: Code 310C (H. Rosenwasser)
Washington, D.C. 20360
Naval Civil Engineering Laboratory 1
Attn: Dr. R. W. Drisko
Port Hueneme, California 93401

Defense Technical Information Center 12
Building 5, Cameron Station
Alexandria, Virginia 22314

DTNSRDC
Attn: Dr. G. Bosmajian
Applied Chemistry Division
Annapolis, Maryland 21401
Or. William Tolles
Superintendent
1
Chemistry Division, Code 6100
Naval Research Laboratory
Washington, D.C. 20375

1
Dr. David Young ..... 1

2 Code 334

NORDA
NSTL, Mississippi 39529
Naval Weapons Center
Attn: Or. A. B. Amster Chemistry Division
China Lake, California 93555
Scientific Advisor
Commandant of the Marine Corps Code RD-1
Washington, D.C. 20380
U.S. Army Research Office

Attn: CRD-AA-IP
P.O. Box 12211

Research Triangle Park, NC 27709
Mr. John Boyle
Materials Branch
Naval Ship Engineering Center
Philadelphia, Pennsylvania 19112
Naval Ocean Systems Center

Marine Sciences Division
San Diego, California 91232

## ABSTRACTS OISTRIBUTION LIST, O51B

Dr. R. A. Osteryoung
Department of Chemistry
State University of New YorkBuffalo, New York 14214
Dr. J. Osteryoung
Department of Chemistry
State University of New YorkBuffalo, New York 14214
Dr. H. Chernoff
Department of Mathematics
Massachusetts Institute of Technology
Cambridge, Massachusetts ..... 02139
Dr. A. ZirinoNaval Undersea Center
San Diego, California ..... 92132
Dr. George H. Morrison
Department of Chemistry
Ithaca, New York ..... 14853
Or. Alan Bewick
Department of Chemistry
Southampton University
Southampton, Hampshire
ENGLAND SO9 5NHOr. M. B. DentonDepartment of Chemistry
University of Arizona
Tucson, Arizona 85721
Dr. S. P. Perone
Lawrence Livermore National
Laboratory L-370
P.0. Box 808
Livermore, California ..... 94550

Dr. G. M. Hieftje
Department of Chemistry
Indiana University
Bloomington, Indiana 47401
Or. Christie G. Enke
Department of Chemistry
Michigan State University
East Lansing, Michigan 48824
Walter G. Cox, Code 3632
Naval Underwater Systems Center
Building 148
Newport, Rhode Island 02840
Professor Isiah M. Warner
Department of Chemistry
Empry University
Atlanta, Georgia 30322
Or. Kent Eisentraut
Air Force Materials Laboratory
Wright-Patterson AFB, Ohio 45433
Dr. Adolph B. Amster
Chemistry Oivision
Naval Weapons Center
China Lake, California 93555
Dr. B. E. Douda
Chemical Sciences Branch
Code 50 C
Naval Weapons Support Center
Crane, Indiana 47322
Or. John Eyler
Department of Chemistry
University of Florida
Gainesville, Florida 32611

## ABSTRACTS DISTRIBUTION LIST, 051B

Professor J. Janata
Department of Bioengineering
University of Utah
Salt Lake City, Utah 84112
Dr. J. DeCorpo
NAVSEA
Code 05R14
Washington, D.C. 20362
Dr. Charles Anderson
Analytical Chemistry Division
Athens Environmental Laboratory
College Station Road
Athens, Georgia 30613
Dr. Ron Flemming
8108 Reactor
National Bureau of Standards
Washington, D.C. 20234
Dr. Frank Herr
Office of Naval Research
Code 422CB
800 N. Quincy Street
Arlington, Virginia 22217
Professor E. Keating
Department of Mechanical Engineering
U.S. Naval Academy

Annapolis, Maryland 21401
Dr. M. H. Miller
1133 Hampton Road
Route 4
U.S. Naval Academy

Annapolis, Maryland 21401
Dr. Clifford Spiegelman
National Bureau of Standards
Room A337 Bldg. 101
Washington, D.C. 20234

Dr. Denton Elliott
AFOSR/NC
Bolling AFB
Washington, D.C. 20362
Dr. B. E. Spielvogel
Inorganic and Analytical Branch P.O. Box 12211

Research Triangle Park, NC 27709
Ms. Ann De Witt
Maierial Science Department
160 Fieldcrest Avenue
Raritan Center
Edison, New Jersey 08818
Dr. A. Harvey
Code 6110
Naval Research Laboratory
Washington, D.C. 20375
Mr. S. M. Hurley
Naval Facilities Engineering Command
Code 032P
200 Stovall Street
Alexandria, Virginia 22331
Ms. W. Parkhurst
Naval Surface Weapons Center
Code R33
Silver Spring, Maryland 20910
Or. M. Robertson
Electrochemical Power Sources Division Code 305
Naval Weapons Support Center
Crane, Indiana 47522
Or. Andrew T. Zander PI204
Perkin-Elmer Corporation
901 Ethan Allen Highway/MS905
Ridgefield, Connecticut 06877

## ABSTRACTS DISTRIBUTION LIST, 051E

Dr. Marvin Wilkerson
Naval Weapons Support Center
Code 30511
Crane, Indiana 47522
Or. J. Wyatt
Naval Research Laboratory
Code 6110
Washington, D.C. 20375
Or. J. MacDonald
Code 6110
Naval Research Laboratory
Washington, D.C. 20375

Or. Marvin Wilkerson Naval Weapons Support Center Code 30511
Crane, Indiana 47522
Dr. J. Wyatt
Naval Research Laboratory Code 6110 Washington, D.C. 20375

Or. J. MacDonald
Code 6110
Naval Research Laboratory Washington, D.C. 20375

Or. H. Wohltjen
Naval Research Laboratory Code 6170
Washington, D.C. 20375
Or. John Hoffsommer
Naval Surface Weapons Center
Building 30 Room 208
Silver Spring, Maryland 20910
Dr. Robert W. Shaw
U.S. Army Research Office

Box 12211
Research Triangle Park, NC 27709

