# Generalized Relaying in the Presence of Interference

Ivana Marić, Ron Dabora, and Andrea Goldsmith
Stanford University, Stanford, CA
{ivanam,ron,andrea}@wsl.stanford.edu

***Abstract*—Capacity gains from cooperation in a network with two source-destination pairs and a relay are analyzed. Scenarios in which the relay decodes both messages are considered. An achievable rate region is derived and evaluated for Gaussian channels. A simple encoding scheme is employed that does not include rate-splitting at the encoders and/or the relay. The obtained results demonstrate the gains from interference forwarding in certain scenarios: the relay optimally splits its power between sending the desired message and the interference. Thus, instead of only message forwarding, the relay uses some of its power to facilitate interference cancelation at the destination node.**

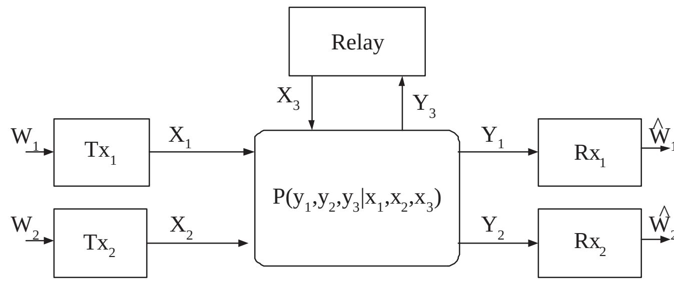


Fig. 1. Interference channel with a relay.

## I. Introduction

Cooperation via relays that forward information improves the rate of the helped communicating pair. Several cooperative strategies have been proposed to forward the desired information and thus increase the achievable rate at the destination node [1], [2], [3]. In networks with multiple sources, the presence of multiple messages opens up the possibility for the relay to perform joint encoding. The smallest network that captures relaying for multiple sources is shown in Fig. 1. We refer to this network as the *interference channel with a relay (ICR)*. The ICR has elements of interference, multiaccess, relay and broadcast channels. As in the interference channel the encoders, as well as the relay, can employ rate-splitting to facilitate partial interference cancelation [4]. Since the relay is broadcasting information to two receivers, it can employ the binning strategy proposed for the broadcast channel [5]. The relay can adopt either the decode-and-forward (DF), compress-and-forward (CF) or simply amplify-and-forward (AF) relaying to forward messages. Using DF, the relay decodes and transmits messages to their intended receivers. Alternatively, adopting CF, the relay quantizes the observed signal that contains channel inputs from both sources and forwards it. Some of these approaches have been analyzed for the ICR in [6], [7].

In general, forwarding a message to one receiver increases the interference for another - an aspect not present when relaying for a single communicating pair. We have previously identified scenarios of the ICR in which forwarding interfering messages to unintended receivers can be beneficial, as it allows the receivers to decode unwanted messages and cancel interference. We referred to this strategy as *interference forwarding* [8], [9]. In the scenarios considered in previous work, the relay was only able to forward interfering messages and not the desired ones. In this paper, we generalize the results on the achievable rate region of [8], [9]. We then examine further possibilities of interference forwarding: if the relay can choose between forwarding the intended message and the interfering one, would it ever be beneficial to send the latter one? In particular, we consider scenarios in which the relay decodes both the desired and the interfering message, but can forward them to only one receiver. We show that it is not always optimal for the relay to use all of its power to forward the desired message to the destination; the relay should allocate some portion of its power for sending the interference.

The encoding scheme considered in this paper does not include rate-splitting at the encoders and/or the relay, which would enable partial interference cancelation at the destinations. Our approach is thus expected to yield the best performance when the interference is strong, because then the interference cancelation can readily be realized via interference forwarding at the relay. We have not considered rate-splitting in order to identify more easily the scenarios in which interference forwarding can bring benefits. In future work, we will investigate the gains of interference forwarding when accompanied by rate-splitting.

## II. Channel Model

*The discrete interference channel with a relay (ICR)* consists of three finite input alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3$, three finite output alphabets $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3$, and a probability distribution $p(y_1, y_2, y_3|x_1, x_2, x_3)$. Each encoder $t$, $t = 1, 2$, wishes to send a message $W_t \in \mathcal{W}_t = \{1, \ldots, 2^{nR_t}\}$ to decoder $t, t = 1, 2$ (see Fig. 1). The channel is memoryless and time-

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invariant in the sense that

$$p(y_{1,i}, y_{2,i}, y_{3,i}|x_1^i, x_2^i, x_3^i, y_1^{i-1}, y_2^{i-1}, y_3^{i-1}, w_1, w_2)$$
$$= p_{Y_1,Y_2,Y_3|X_1,X_2,X_3}(y_{1,i}, y_{2,i}, y_{3,i}|x_{1,i}, x_{2,i}, x_{3,1}). \quad (1)$$

We will follow the convention of dropping subscripts of probability distributions if the arguments of the distributions are lower case versions of the corresponding random variables.

An $(R_1, R_2, n)$ code for the ICR consists of two message sets $\mathcal{W}_1$, $\mathcal{W}_2$, two encoding functions at the encoders,

$$X_1^n = f_1(W_1)$$
$$X_2^n = f_2(W_2), \quad (2)$$

an encoding function at the relay

$$X_{i,3} = f_{3,i}(Y_3^{i-1}), \quad (3)$$

and two decoding functions

$$\hat{W}_t = g_t(Y_t^n). \quad (4)$$

The average error probability of the code is given by

$$P_e = P\left[\hat{W}_1 \neq W_1 \cup \hat{W}_2 \neq W_2\right]. \quad (5)$$

A rate pair $(R_1, R_2)$ is achievable if, for any $\epsilon > 0$, there exists, for a sufficiently large $n$, a code $(R_1, R_2, n)$ such that $P_e \leq \epsilon$. The capacity region is the closure of the set of all achievable pairs $(R_1, R_2)$.

## III. Achievable Rate Region

*Theorem 1:* Any rate pair $(R_1, R_2)$ that satisfies

$$R_1 \leq I(X_1, X_3; Y_1|U_2, X_2) \quad (6)$$
$$R_2 \leq I(X_2, X_3; Y_2|U_1, X_1) \quad (7)$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_1) \quad (8)$$
$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_2) \quad (9)$$
$$R_1 \leq I(X_1; Y_3|X_2, U_1, U_2) \quad (10)$$
$$R_2 \leq I(X_2; Y_3|X_1, U_1, U_2) \quad (11)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_3|U_1, U_2) \quad (12)$$

for a joint distribution that factors as

$$p(u_1, x_1)p(u_2, x_2)f(x_3|u_1, u_2)p(y_1, y_2, y_3|x_1, x_2, x_3)$$

is achievable in the ICR. $f$ is a deterministic function.

*Proof:* The proof outline is given in the appendix. ■

*Remark 1:* Bounds (10)-(12) are required in order to provide reliable decoding at the relay. Since the relay decodes both indexes, possible error events at the relay are the same as in the multiaccess channel (MAC) [10]. Bounds (6)-(9) are due to decoding constraints at two destination nodes. In the encoding strategy of Thm. 1 rate-splitting is not used. Instead, each destination node jointly decodes messages $(W_1, W_2)$ as in the MAC. Compared to the MAC rate constraints, the error in decoding the unwanted message at a destination node is ignored and therefore, at each decoder, there is one less rate constraint when compared to the MAC rate bounds.

*Remark 2:* For the special case $U_1 = 0, X_3 = U_2$ we recover the rates from [9, Thm. 1].

## IV. Gaussian Channels

The Gaussian channel is described by the following input-output relationship:

$$Y_1 = h_{11}X_1 + h_{12}X_2 + h_{13}X_3 + Z_1$$
$$Y_2 = h_{21}X_1 + h_{22}X_2 + h_{23}X_3 + Z_2$$
$$Y_3 = h_{31}X_1 + h_{32}X_2 + Z_3 \quad (13)$$

where $Z_t \sim \mathcal{N}[0, 1]$, $E[X_t^2] \leq P_t, t = 1, 2$, and $\mathcal{N}[0, \sigma^2]$ denotes the normal distribution with zero mean and variance $\sigma^2$. We evaluate region (6)-(12) by choosing Gaussian inputs as:

$$U_1 \sim \mathcal{N}[0, \alpha P_1],\ X_{10} \sim \mathcal{N}[0, \bar{\alpha}P_1],\ X_1 = X_{10} + U_1$$
$$U_2 \sim \mathcal{N}[0, \beta P_2],\ X_{20} \sim \mathcal{N}[0, \bar{\beta}P_2],\ X_2 = X_{20} + U_2.$$

Thus, the encoders 1 and 2 split their power between sending new information (respectively with $\bar{\alpha}P_1$ and $\bar{\beta}P_2$ ) and between cooperating with the relay. The power at the relay is split between forwarding messages $W_1, W_2$ as:

$$X_3 = \sqrt{\frac{\gamma P_3}{\alpha P_1}}U_1 + \sqrt{\frac{\bar{\gamma}P_3}{\beta P_2}}U_2$$

where $0 \leq \alpha, \beta, \gamma \leq 1$. Parameter $\gamma$ determines how the relay splits its power for forwarding $W_1, W_2$. A higher $\gamma$ results in more power dedicated for forwarding $W_1$.

The rate region of Thm. 1 becomes

$$R_1 \leq C(h_{11}^2P_1 + h_{13}^2\gamma P_3 + 2h_{11}h_{13}\sqrt{\alpha P_1\gamma P_3})$$
$$R_2 \leq C(h_{22}^2P_2 + h_{23}^2\bar{\gamma}P_3 + 2h_{22}h_{23}\sqrt{\beta P_2\bar{\gamma}P_3})$$
$$R_1 + R_2 \leq C(h_{21}^2 P_1 + h_{12}^2P_2 + h_{13}^2P_3 + 2h_{11}h_{13}\sqrt{\alpha P_1\gamma P_3} + 2h_{12}h_{13}\sqrt{\beta P_2\bar{\gamma}P_3})$$
$$R_1 + R_2 \leq C(h_{21}^2P_1 + h_{22}^2P_2 + h_{23}^2P_3 + 2h_{21}h_{23}\sqrt{\alpha P_1\gamma P_3} + 2h_{22}h_{23}\sqrt{\beta P_2\bar{\gamma}P_3})$$
$$R_1 \leq C(h_{31}^2\bar{\alpha}P_1)$$
$$R_2 \leq C(h_{32}^2\bar{\beta}P_2)$$
$$R_1 + R_2 \leq C(h_{31}^2\bar{\alpha}P_1 + h_{32}^2\bar{\beta}P_2) \quad (14)$$

where $C(x) = 0.5\log(1 + x)$.

We are interested in investigating whether the relay - being able to forward both the desired message and the interfering message to a destination - should ever allocate power to forward interference. For that reason, we consider the special case scenario $h_{23} = 0$, so that the relay cannot help decoder 2. We next illustrate that forwarding $W_2$ can be beneficial for decoder 1. For simplicity, we assume that $h_{11} = h_{22} = 1$. We are interested in the 'strong' interference at the destination 2, i.e., we assume $h_{21} > 1$. The rates (14) reduce to

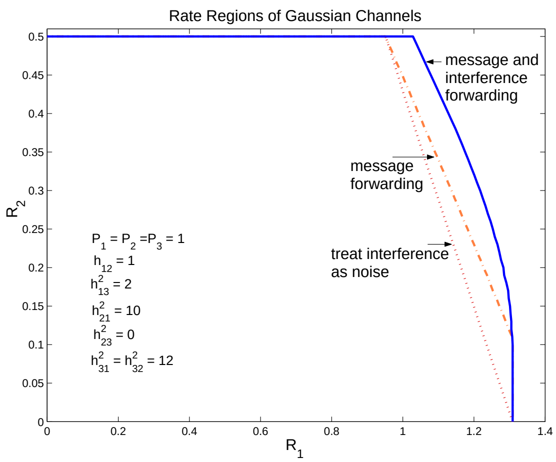


Fig. 2. Rate regions of a Gaussian ICR channel with and without interference forwarding are shown with respective solid and dot-dashed lines. The difference between two regions illustrates the gains of interference forwarding. The dotted region shows the rates achievable when decoder 1 treats interference as noise. In this example, $h_{12} = 1$.

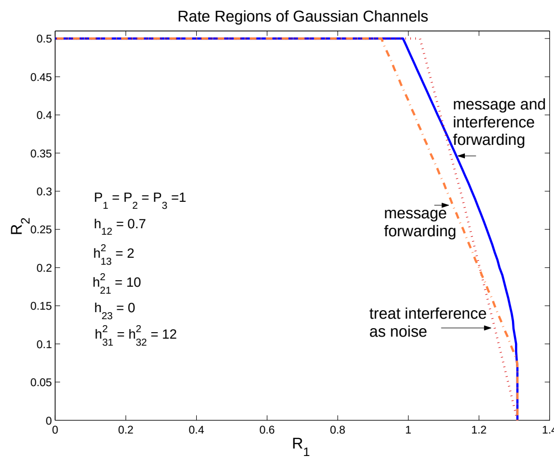


Fig. 3. Rate regions of a Gaussian ICR channel with and without interference forwarding shown with respective solid and dot-dashed lines. The difference between two regions illustrates the gains of interference forwarding. The dotted region shows the rates achievable when decoder 1 treats interference as noise. In this example, $h_{12} = 0.7$.

$$\begin{aligned}
&R_1 \le C(P_1 + h_{13}^2\gamma P_3 + 2h_{13}\sqrt{\alpha P_1\gamma P_3})\\
&R_2 \le C(P_2)\\
&R_1 + R_2 \le C(P_1 + h_{12}^2P_2 + h_{13}^2P_3 + 2h_{13}\sqrt{\alpha P_1\gamma P_3}\\
&\qquad\qquad + 2h_{12}h_{13}\sqrt{\beta P_2\bar\gamma P_3})\\
&R_1 + R_2 \le C(h_{21}^2P_1 + P_2)\\
&R_1 \le C(h_{31}^2\bar\alpha P_1)\\
&R_2 \le C(h_{32}^2\bar\beta P_2)\\
&R_1 + R_2 \le C(h_{31}^2\bar\alpha P_1 + h_{32}^2\bar\beta P_2). \qquad (15)
\end{aligned}$$

The rate region is shown in Figure 2 for $P_1 = P_2 = P_3 = 1$.

Since the encoders do not perform rate-splitting, the receivers cannot partially decode unwanted messages. Thus, the other decoding option is for decoders to treat the signal carrying the unwanted message as noise. Since we consider the case of strong interference at decoder 2, i.e. $h_{21} > 1$, this approach would result in a lower rate $R_2$. Therefore, we compare rates (15) to the case when decoder 1 treats the signal received from encoder 2 as noise. In this case, the relay forwards only the desired message $W_1$ (i.e. $\gamma = 1$). The achievable rates are given by

$$\begin{aligned}
&R_1 \le C\left(\frac{P_1 + h_{13}^2P_3 + 2h_{13}\sqrt{\alpha P_1P_3}}{1 + h_{12}^2P_2}\right)\\
&R_2 \le C(P_2)\\
&R_1 + R_2 \le C(h_{21}^2P_1 + P_2)\\
&R_1 \le C(h_{31}^2\bar\alpha P_1)\\
&R_1 + R_2 \le C(h_{31}^2\bar\alpha P_1 + h_{32}^2P_2). \qquad (16)
\end{aligned}$$

The benefit of one vs. the other strategy depends on the interference level at decoder 1 (i.e. on $h_{12}$) as illustrated in Figures 2 and 3. The difference between the two figures is in the interference level experienced at decoder 1. As the interference gets smaller (i.e. $h_{12}$ decreases), decoding it becomes less beneficial. In particular, the first sum rate bound in (15), which comes from a constraint on deciding on both $(W_1, W_2)$ at decoder 1, becomes smaller as it depends on $h_{12}$. Then, treating interference as noise at decoder 1 performs better. In this approach, we have $\beta = 0$ since the power split at encoders is only used for facilitating cooperation with the relay.

## V. Conclusion

It has been previously shown that forwarding interference at the relay can bring benefit to decoders by facilitating interference cancelation. In previously considered scenarios, the relay did not have an option of forwarding the desired message. In this paper, we considered scenarios in which the relay decodes both the desired and the interfering message. The relay hence has an option to forward the desired message and/or the interference to its destined receiver. We identified situations in which forwarding both messages at the relay to a receiver improves the rate. Hence, it turns out that it is not always optimal for the relay to use all of its power to forward the desired message to the destination; the relay should allocate some portion of its power to send interference instead. We expect that our conclusion generalizes to larger networks.

We point out that we considered an encoding scheme that does not include rate-splitting at the encoders and/or the relay. Our approach is thus expected to yield the best performance when the interference is strong, because then the interference cancelation can readily be realized via interference forwarding at the relay. This assumption was made in order to more easily identify the scenarios in which interference forwarding brings benefits. Rate-splitting also facilitates (partial) interference cancelation. In future work, we will investigate the gains of interference forwarding when accompanied by rate-splitting.

## VI. APPENDIX

*Proof: (outline).* **Code construction:** Choose a distribution $p(u_1)p(x_1|u_1)p(u_2)p(x_2|u_2)$ and $f(x_3|u_1,u_2)$.

- Generate $2^{nR_1}$ codewords $u_1^n(w_1')$, $w_1' = 1, \ldots, 2^{nR_1}$, by choosing $u_{1,i}(w_1')$ independently according to $P_{U_1}(\cdot)$.
- For each $w_1'$: Generate $2^{nR_1}$ codewords $x_1^n(w_1', w_1)$ using $\prod_{i=1}^n P_{X_1|U_1}(\cdot|u_{1,i}(w_1'))$, $w_1 = 1, \ldots, 2^{nR_1}$.
- Generate $2^{nR_2}$ codewords $u_2^n(w_2')$, $w_2' = 1, \ldots, 2^{nR_2}$, by choosing $u_{2,i}(w_2')$ independently according to $P_{U_2}(\cdot)$.
- For each $w_2'$: Generate $2^{nR_2}$ codewords $x_2^n(w_2', w_2)$ using $\prod_{i=1}^n P_{X_2|U_2}(\cdot|u_{2,i}(w_2'))$, $w_2 = 1, \ldots, 2^{nR_2}$.
- For each pair $(w_1', w_2')$ : Generate $x_3^n(w_1', w_2')$ where $x_3$ is a deterministic function of $(u_1, u_2)$.

**Encoders:** (See Fig. 4). Encoder 1 transmits $x_1^n(w_1', w_1)$.
Encoder 2: Transmit $x_2^n(w_2', w_2)$.
Relay: Transmit $x_3^n(w_1', w_2')$.

**Decoders:** Decoders 1 and 2 use backward decoding to decode $(w_1, w_2)$. The relay jointly decodes $(w_1, w_2)$. In particular, in block $B$:

Decoder 1: Given $y_1^n$, choose $(\hat{w}_1', \hat{w}_2')$ if $(u_1^n(\hat{w}_1'), x_1^n(\hat{w}_1', 1), u_2^n(\hat{w}_2'), x_2^n(\hat{w}_2', 1), x_3^n(\hat{w}_1', \hat{w}_2'), y_1^n) \in T_\epsilon(P_{U_1X_1U_2X_2X_3Y_1})$. If there is more than one such pair, choose one. If there is no such pair, choose $(1,1)$. Given $y_2^n$, the same decoding is done at destination 2.

**Analysis:** Suppose $(1,1)$ was sent. The error events at encoder 1 are $E_1 = \{\hat{w}_1' \neq 1, \hat{w}_2' \neq 1\}$ and $E_2 = \{\hat{w}_1' \neq 1, \hat{w}_2' = 1\}$.

Consider the probability of event $E_1$ :

$$
\begin{aligned}
P[\hat{W}_1' \neq 1, \hat{W}_2' \neq 1] = \sum_{w_1'=2}^{2^{nR_1}} \sum_{w_2'=2}^{2^{nR_2}} P[(U_1^n(w_1'), X_1^n(w_1', 1),\\
U_2^n(w_2'), X_2^n(w_2', 1), X_3^n(w_1', w_2'), Y_1^n) \in T_\epsilon]\\
\leq 2^{-n[I(U_1, U_2, X_1, X_2, X_3; Y_1) - (R_1 + R_2) - \delta]} \qquad (17)
\end{aligned}
$$

by [10, Thm.8.6.1]. From (17), achieving arbitrarily small error probability of $E_1$ requires

$$R_1 + R_2 < I(X_1, X_2, X_3; Y_1). \qquad (18)$$

Consider the probability of event $E_2$ :

$$
\begin{aligned}
P[\hat{W}_1' \neq 1, \hat{W}_2' = 1] = \sum_{w_1'=2}^{2^{nR_1}} P[(U_1^n(w_1'),\\
X_1^n(w_1', 1), U_2^n(1), X_2^n(1,1), X_3^n(w_1', 1), Y_1^n) \in T_\epsilon]\\
\leq 2^{-n[I(U_1, X_1, X_3; Y_1|U_2, X_2) - R_1 - \delta]} \qquad (19)
\end{aligned}
$$

by [10, Thm.8.6.1]. From (17), achieving arbitrarily small error probability of $E_1$ requires

$$R_1 < I(U_1, X_1, X_3; Y_1|U_2, X_2) \qquad (20)$$

or equivalently

$$R_1 < I(X_1, X_3; Y_1|U_2, X_2). \qquad (21)$$

Similar analysis holds for decoder 2. The relay constraints follow from the analysis for the error events in the MAC [10].

■

| | block 1 | block 2 | block 3 | block 4 |
|---|---|---|---|---|
| source 1 | $x_1(1, w_1(1))$ | $x_1(w_1(1), w_1(2))$ | $x_1(w_1(2), w_1(3))$ | $x_1(w_1(3), 1)$ |
| source 2 | $x_2(1, w_2(1))$ | $x_2(w_2(1), w_2(2))$ | $x_2(w_2(2), w_2(3))$ | $x_2(w_2(3), 1)$ |
| relay | $x_3(1,1)$ | $x_3(w_1(1), w_2(1))$ | $x_3(w_1(2), w_2(2))$ | $x_3(w_1(3), w_2(3))$ |

Fig. 4. Encoding at the sources and at the relay.

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