Mathematical Modelling and Analysis
Abstracts of the 9th International Conference MMA2004, May 29-31, 2004, Jūrmala, Latvia

GENERALIZED RIESZ METHOD AND CONVERGENCE ACCELERATION

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In many fields of numerical mathematics we have some problems of convergence acceleration. Though mainly are used non-linear methods (see [3], in several cases are preferred linear methods. In this talk we deal with the possibilities to use generalized Riesz method for acceleration.

Let X and Y be Banach spaces and $\mathcal{L}(X,Y)$ be a space of linear bounded operators from X into Y. A sequence $x = (\xi_k)$ $(\xi_k \in X)$ is called λ-bounded (λ-convergent) if $\beta_k = O(1)$ $(\exists \lim \beta_k)$, while $\beta_k = \lambda_k (\xi_k - \xi)$ with $\xi = \lim \xi_k$, $\lambda = (\lambda_k)$ and $0 < \lambda_k \nearrow$. Let m_X^λ (c_X^λ) be a set of all λ-bounded (λ-convergent) sequences. A sequence $x = (\xi_k)$ is called summable by a generalized method $\mathcal{A} = (A_{nk})$ if $y = (\eta_n)$ with $\eta_n = \sum_k A_{nk}\xi_k$ and $A_{nk} \in \mathcal{L}(X,Y)$ is convergent. The transformation A is called accelerating λ- boundedness (λ-convergence) if $\mathcal{A}m_X^\lambda \subset m_Y^\mu$ with $\lim \mu_n/\lambda_n = \infty$. Let us denote by $(\Re, P_n)$ or shortly by $\Re$ the generalized Riesz method, defined by

$$R_{nk} = \begin{cases} R_n P_k & k = 0, 1, \ldots, n, \\ \theta & k > n, \end{cases}$$

where $P_k,\ R_n \in \mathcal{L}(X,X)$, while R_n is determined by

$$R_n \sum_{k=0}^{n} P_k \zeta = \zeta \quad (\zeta \in X,\ n \in \mathbf{N}_0).$$

In [2] are proved sufficient conditions for the inclusion $\Re m_X^\lambda \subset m_X^\mu$. In [1] are studied several inverse theorems, so-called Tauberian theorems for generalized Riesz method $\Re$ in the case $\lambda_n = O(1)$ or $\mu_n = O(1)$. We study the case $\lambda_n \neq O(1)$ or $\mu_n \neq O(1)$ and prove so-called Tauberian remainder theorems for the method $\Re$.

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