



1 of 1

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GENERALIZED SQUEEZED STATES FROM GENERALIZED COHERENT STATES

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ABSTRACT

Both the coherent states and also the squeezed states of the harmonic oscillator have long been understood from the three classical points of view: the 1) displacement operator, 2) annihilation- (or ladder-) operator, and minimum-uncertainty methods. For general systems, there is the same understanding except for ladder-operator and displacement-operator squeezed states. After reviewing the known concepts, I propose a method for obtaining generalized minimum-uncertainty squeezed states, give examples, and relate it to known concepts. I comment on the remaining concept, that of general displacement-operator squeezed states.

1 Introduction

As we all know, Glauber, Klauder, and Sudarshan produced the modern era's seminal works on coherent states of the harmonic oscillator [1, 2, 3, 4]. There came to be three classical definitions of these coherent states, from 1) *the displacement-operator method*, 2) *the annihilation-operator method* which I will relabel *the ladder-operator method*, and 3) *the minimum-uncertainty method*. Today coherent states are important in many fields of theoretical and experimental physics [5, 6].

Generalizations of these states appeared in two areas. One was to "two-photon states" [7], states which were rediscovered under many names. In 1979 they were first called "squeezed states" [8]. In recent times these states have become of more and more interest [9, 10]. This is especially true in the fields of quantum optics [11] and gravitational wave detection [12]. The squeezed states of the harmonic oscillator

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can also be equivalently defined by appropriate generalizations of the three classical definitions of the coherent states.

The other generalization was to non-harmonic oscillator systems. From the group theory or operator point of view, generalized coherent states for Lie groups were widely studied from the displacement operator and annihilation operator methods [3, 13].

In 1978 I, along with Mike Simmons and Vincent Gutschick, began a program to find the coherent states for arbitrary potentials from a minimum-uncertainty point of view. You give us the potential, be it Morse, Pöschl-Teller, Coulomb, or whatever, and we'll give you the coherent states. What came as a byproduct was the realization that along the way we had also found the appropriate generalization to the squeezed state. Further, we found that for arbitrary systems the three methods no longer necessarily gave the same coherent and/or squeezed states.

This all is in the **past**, from the theme of this Symposium. It leaves appropriate generalizations of squeezed states from the ladder-operator and displacement-operator points of view to be found.

The **present** deals with work which Rod Truax and I have recently reported on [14]. (Indeed, in this Proceedings I draw upon much of the contents of Ref. [14], especially in Section 3.) We proposed a generalization of squeezed states using the ladder-operator method. This method was also connected to the minimum-uncertainty method and some aspects of special-case displacement-operator squeezed states which have been obtained. It was also found that, as expected, the generalized squeezed states from the different methods could be equivalent, but need not be. (I note that connections of these ideas to Rydberg wave packets and other quantum systems have been made elsewhere [15, 16].)

This leaves the **future**. What remains to be done is to find generalized squeezed states from the displacement-operator method.

2 Past

2.1 Coherent states for the harmonic oscillator

I begin by reviewing the coherent states for the harmonic oscillator. As is well-known, there are three standard definitions of these states, which are equivalent.

1) *Displacement-Operator Method*. The coherent states can be obtained by applying the unitary displacement operator on the ground state [3, 2]:

$$|\alpha\rangle \equiv D(\alpha)|0\rangle = \exp[\alpha a^\dagger - \alpha^* a]|0\rangle = \exp\left[-\frac{1}{2}|\alpha|^2\right] \sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle, \quad (1)$$

where $|n\rangle$ are the number states.

2) *Ladder- (or Annihilation-) Operator Method.* The coherent states can also be defined as the eigenstates of the destruction operator:

$$a|\alpha\rangle = \alpha|\alpha\rangle. \quad (2)$$

This follows from Eq. (1), since

$$0 = D(\alpha)a|0\rangle = (a - \alpha)D(\alpha)|0\rangle = (a - \alpha)|\alpha\rangle. \quad (3)$$

These states are the same as the displacement-operator coherent states.

3) *Minimum-Uncertainty Method.* This method harks back to Schrödinger's discovery of the coherent states [17]. Recall that Schrödinger wanted to find states which maintained their shapes and followed the classical motion. For the harmonic oscillator, these are the states which minimize the uncertainty relation

$$[x, p] = i\hbar, \quad (\Delta x)^2(\Delta p)^2 \geq \frac{1}{4}\hbar^2, \quad (4)$$

subject to the constraint that the ground state is a member of the set. In wave function language ($\hbar = \omega = m = 1$) they are described by

$$\psi_{cs}(x) = [\pi]^{-1/4} \exp\left[-\frac{(x - x_0)^2}{2} + ip_0x\right]. \quad (5)$$

That the states of Eqs. (1) and (5) are the same can be demonstrated by using the generating function for the Hermite polynomials along with the identifications

$$\sqrt{2}\Re(\alpha) = x_0, \quad \sqrt{2}\Im(\alpha) = p_0. \quad (6)$$

Observe that of the four original parameters, $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, and $\langle p^2 \rangle$, only two remain, $\Re(\alpha)$ and $\Im(\alpha)$. That is firstly because the inequality in the uncertainty relation has been satisfied. The remaining three parameter set of states is restricted to two parameters by demanding that the ground state (which corresponds to zero motion) must be a member of the set.

2.2 Squeezed states of the harmonic oscillator

The above three methods also yield equivalent squeezed states for the harmonic oscillator.

1) *Displacement-Operator Method.* In this method one applies the “squeeze” or $SU(1,1)$ displacement operator on the coherent state:

$$D(\alpha)S(z)|0\rangle = |(\alpha, z)\rangle, \quad S(z) = \exp[zK_+ - z^*K_-], \quad (7)$$

where K_+ , K_- , and K_0 form an $\mathfrak{su}(1,1)$ algebra amongst themselves:

$$K_+ = \frac{1}{2}a^\dagger a^\dagger, \quad K_- = \frac{1}{2}aa, \quad K_0 = \frac{1}{2}(a^\dagger a + \frac{1}{2}), \quad (8)$$

$$[K_0, K_\pm] = \pm K_\pm, \quad [K_+, K_-] = -2K_0. \quad (9)$$

The ordering of DS vs. SD in Eq. (7) is unitarily equivalent, amounting to a change of parameters:

$$D(\alpha)S(z) = S(z)D(\gamma), \quad \gamma = \alpha \cosh r - \alpha^* e^{i\theta} \sinh r, \quad (10)$$

where $z = re^{i\theta}$.

2) *Ladder-Operator Method.* For the harmonic oscillator, this method again follows from the displacement-operator method. Combining the Bogoliubov transformation [18],

$$S^{-1}aS = (\cosh r)a + e^{i\theta}(\sinh r)a^\dagger, \quad (11)$$

with Eq. (10), one has that

$$[(\cosh r)a - e^{i\theta}(\sinh r)a^\dagger] |(\alpha, z)\rangle = \gamma |(\alpha, z)\rangle. \quad (12)$$

One now sees why I relabel this method the ladder-operator method. For the squeezed states one needs both the raising (creation) and lowering (annihilation) operators.

3) *Minimum-Uncertainty Method.* From this point of view, the transition from coherent to squeezed states is intuitively simple. These states minimize the $x - p$ uncertainty relation, without the added restriction that the ground state (Gaussian) is a member of the set. That is, these are a *three* parameter set of states, which are the Gaussians of all widths:

$$\psi_{s,s}(x) = [\pi s^2]^{-1/4} \exp \left[-\frac{(x - x_0)^2}{2s^2} + ip_0 x \right], \quad (13)$$

These squeezed states are equivalent to those obtained from the other formulations. This can be verified by combining Eqs. (12) and (13) with the relationships

$$x = \frac{(a + a^\dagger)}{\sqrt{2}}, \quad p = \frac{(a - a^\dagger)}{i\sqrt{2}}. \quad (14)$$

The remaining relationships among the parameters are

$$z = re^{i\theta}, \quad r = \ln s. \quad (15)$$

(The phase, θ , is an initial time-displacement.)

2.3 Generalized (displacement- and ladder-operator) coherent states

When one considers coherent states for general systems one finds that the coherent states from all three methods are not, in general, equivalent, although they may be in particular cases.

1) *Displacement-Operator Method.* The generalization of this method to arbitrary Lie groups has a long history [5, 6, 3, 13]. (Supersymmetric extensions of it also exist [19].) One simply applies the displacement operator, which is the unitary exponentiation of the factor algebra, on to an extremal state. That is, let T be a unitary irreducible representation of the group G on a Hilbert space and let $|\psi_0\rangle$ be a fixed vector in the space. Let G_0 be the stability group; i.e.,

$$T(G_0)|\psi_0\rangle = e^{i\alpha(G_0)}|\psi_0\rangle. \quad (16)$$

Then

$$|\psi_g\rangle = T(G/G_0)|\psi_0\rangle \quad (17)$$

are the coherent states.

2) *Ladder-Operator Method.* The generalization to arbitrary Lie groups is straight forward, and has also been widely studied [5, 6]. One obtains the eigenstates of the generalized lowering-operator (assuming there is an extremal state below):

$$A_-|\delta\rangle = \delta|\delta\rangle. \quad (18)$$

2.4 Generalized (minimum-uncertainty) coherent and squeezed states

In the program I described in the introduction, we wanted to find the coherent states for arbitrary potential systems. We were motivated by the physics of Schrödinger. We wanted to find states which follow the classical motion as well as possible and which maintain their shapes as well as possible. We felt that meant there was an associated uncertainty relation, but in which variables? Our answer was found in a manner completely backwards from the way I now present it, but such is often the case with physics. In fact, along the way, this method also yields the squeezed states for general potential systems [20, 21].

One starts with the classical Hamiltonian problem and transforms it into the "natural classical variables," X_c and P_c , which vary as the sin and the cos of the classical ωt . The Hamiltonian is therefore of the form $P_c^2 + X_c^2$. One then takes these natural classical variables and transforms them into "natural quantum operators." Since these are quantum operators, they have a commutation relation and uncertainty relation:

$$[X, P] = iG, \quad (\Delta X)^2(\Delta P)^2 \geq \frac{1}{4}\langle G \rangle^2. \quad (19)$$

The states that minimize this uncertainty relation are given by the solutions to the equation

$$Y\psi_{ss} \equiv \left(X + \frac{i\langle G \rangle}{2(\Delta P)^2} P \right) \psi_{ss} = \left(\langle X \rangle + \frac{i\langle G \rangle}{2(\Delta P)^2} \langle P \rangle \right) \psi_{ss}. \quad (20)$$

Note that of the four parameters $\langle X \rangle$, $\langle P \rangle$, $\langle P^2 \rangle$, and $\langle G \rangle$, only three are independent because they satisfy the equality in the uncertainty relation. Therefore,

$$(X + iBP)\psi_{ss} = C\psi_{ss}, \quad B = \frac{\Delta X}{\Delta P}, \quad C = \langle X \rangle + iB\langle P \rangle. \quad (21)$$

Here B is real and C is complex. We called these states, $\psi_{ss}(B, C)$, the minimum-uncertainty states for general potentials [20, 21]. However, using the parlance accepted later, they are the squeezed states for general potentials [10]. Then B can be adjusted to B_0 so that the ground eigenstate of the potential is a member of the set. Then these restricted states, $\psi_{ss}(B = B_0, C) = \psi_{cs}(B_0, C)$, are the minimum-uncertainty coherent states for general potentials.

It can be intuitively understood that $\psi_{ss}(B, C)$ and $\psi_{cs}(B_0, C)$ are the squeezed and coherent states by recalling the situation for the harmonic oscillator. The coherent states are the displaced ground state. The squeezed states are Gaussians that have widths different than that of the ground state Gaussian, which are then displaced.

3 Present: Generalized Ladder-Operator Squeezed States

As we have discussed, general annihilation-operator (or ladder-operator) coherent states are the eigenstates of the lowering operator (given a lowest extremal state). We now propose a generalization to squeezed states, including those for arbitrary symmetry systems: *The general ladder-operator squeezed states are the eigenstates of a linear combination of the lowering and raising operators.*

I note that the success of this method will not be totally surprising. In many exactly solvable potential systems, the natural quantum operators of the minimum-uncertainty method were found to be Hermitian combinations of the n -dependent raising and lowering operators [20, 21]. Here, however, one must generalize to full operators: $n \rightarrow n(H)$. Furthermore, in other harmonic-oscillator-like systems, with a Bogoliubov transformation, this method applies. (See below.)

I will proceed by showing how the minimum-uncertainty method for obtaining generalized squeezed states can be used as an intuitive tool to aid in understanding the ladder-operator method for obtaining generalized squeezed states. I will do this with two specific examples. Once that is done, the ladder operator method can

be applied to general symmetry systems, independent of whether they come from a Hamiltonian system in the manner of the minimum-uncertainty method above. Such is our third example.

3.1 Example I: The harmonic oscillator.

First let us re-examine the harmonic oscillator, starting from the minimum-uncertainty method. Here X and P are obviously x and p . Then we have

$$Y = x + s^2 \frac{d}{dx}, \quad (22)$$

where we have presciently labeled B as s^2 . (For the limit to coherent states, it turns out that $B = 1$.)

Now writing x and p in terms of creation and annihilation operators, we find

$$\sqrt{2} \left[a \left(\frac{1+s^2}{2} \right) + a^\dagger \left(\frac{1-s^2}{2} \right) \right] \psi_{s,s}(s^2, x_0 + is^2 p_0) = [x_0 + is^2 p_0] \psi_{s,s}(s^2, x_0 + is^2 p_0). \quad (23)$$

Therefore, the squeezed states are eigenstates of a linear combination of the annihilation and creation operators. Specifically, these states are as given in Eq. (13).

$$\psi_{s,s}(x) = [\pi s^2]^{-1/4} \exp \left[-\frac{(x-x_0)^2}{2s^2} + ip_0 x \right], \quad (24)$$

3.2 Example II: The harmonic oscillator with centripetal barrier.

I now discuss the symmetry of the harmonic oscillator with centripetal barrier. Previously, the coherent states for this particular example were found with the minimum-uncertainty method, but not the squeezed states [21]. Therefore, it is a useful system since, at the end, we can relate our results to the coherent states obtained from the minimum-uncertainty method.

This system contains an $su(1,1)$ algebra [22]. Its elements are

$$L_{\pm} = \frac{1}{4\nu} \frac{d^2}{dz^2} \mp \frac{1}{2} z \frac{d}{dz} \mp \frac{1}{4} + \frac{\nu}{4} z^2 - \frac{\nu}{4z^2}, \quad (25)$$

$$L_0 = \frac{H}{4\nu} + \frac{\nu}{2}, \quad H = -\frac{d^2}{dz^2} + \nu^2 \left(\frac{1}{z} - z \right)^2. \quad (26)$$

In terms of the X and P minimum-uncertainty operators [21], we find

$$X = \frac{L_- + L_+}{\nu} = z^2 - \left(1 + \frac{H}{2\nu^2} \right), \quad P = \frac{2(L_- - L_+)}{i} = \frac{1}{i} \left[2z \frac{d}{dz} + 1 \right]. \quad (27)$$

Therefore, the squeezed states for this system are formed by the solution to the equation

$$0 = \left[y \frac{d^2}{dy^2} + \left(\frac{1}{2} + 2\nu B y \right) \frac{d}{dy} + \frac{1}{4} \left(y - \frac{\nu^2}{y} + 2B\nu \right) - \frac{\nu C}{2} \right] \psi_{ss}, \quad (28)$$

where we have changed variables to $y = \nu z^2$. The squeezed state solutions to this equation are

$$\psi_{ss} = N \exp[-y(\nu B + \gamma)] [y^{\lambda + \frac{1}{2}}] \Phi \left(\left[\frac{\nu C}{4\gamma} + \frac{1}{2} \left(\lambda + \frac{3}{2} \right) \right], \left[\lambda + \frac{3}{2} \right]; 2\gamma y \right), \quad (29)$$

where $\Phi(a, b; c)$ is the confluent hypergeometric function $\sum_{n=0}^{\infty} \frac{(a)_n c^n}{(b)_n n!}$, $\gamma = \sqrt{\nu^2 B^2 - \frac{1}{4}}$,

and $\lambda(\lambda + 1) = \nu^2$. In the limit where $B \rightarrow 1/(2\nu)$, these become the coherent states given in Ref. [21].

$$\psi_{cs} = \left[\frac{2\nu^{1/2} e^{-\nu \Re(C)}}{I_{\lambda+1/2}(\nu|C|)} \right]^{1/2} e^{-y/2} y^{1/4} I_{\lambda+1/2} \left((2\nu C y)^{1/2} \right), \quad (30)$$

where I is the modified Bessel function.

3.3 Example III: The squeeze algebra.

We now consider a symmetry system which does not have as its origin a Hamiltonian system. We consider the $\text{su}(1,1)$ symmetry of Eqs. (8, 9). Our ladder-operator squeezed states are thus the solutions to

$$\left[\left(\frac{1+s}{2} \right) aa + \left(\frac{1-s}{2} \right) a^\dagger a^\dagger \right] \psi_{ss} = \beta^2 \psi_{ss}, \quad (31)$$

where the analogue of B is s and the role of C is taken by β^2 . Using the differential representations of the ladder operators, Eq. (31) can be written in the form

$$\left[\frac{d^2}{dy^2} + 2ys \frac{d}{dy} + y^2 + (s - 2\beta^2) \right] \psi_{s,s} = 0. \quad (32)$$

Observe that the ladder operators raise and lower the number states by two units. Therefore, there will be two solutions to this equation, one containing only even number states and one containing only odd number states. We will designate these as $\psi_{E,s,s}$ and $\psi_{O,s,s}$. These solutions are

$$\psi_{E,s,s} = N_E \exp \left[-\frac{y^2}{2}(s + \sqrt{s^2 - 1}) \right] \Phi \left(\left[\frac{1}{4} + \frac{\beta^2}{2\sqrt{s^2 - 1}} \right], \frac{1}{2}; y^2 \sqrt{s^2 - 1} \right). \quad (33)$$

$$\psi_{O,s,s} = N_O y \exp \left[-\frac{y^2}{2}(s + \sqrt{s^2 - 1}) \right] \Phi \left(\left[\frac{3}{4} + \frac{\beta^2}{2\sqrt{s^2 - 1}} \right], \frac{3}{2}; y^2 \sqrt{s^2 - 1} \right). \quad (34)$$

In the limit $s \rightarrow 1$, these become the even and odd coherent states:

$$\psi_{Ecs} = \left[\frac{e^{-\beta^2}}{\pi^{1/2} \cosh |\beta|^2} \right]^{1/2} \exp \left[-\frac{1}{2} y^2 \right] \cosh(\sqrt{2} \beta y), \quad (35)$$

$$\psi_{Ocs} = \left[\frac{e^{-\beta^2}}{\pi^{1/2} \sinh |\beta|^2} \right]^{1/2} \exp \left[-\frac{1}{2} y^2 \right] \sinh(\sqrt{2} \beta y). \quad (36)$$

Using generating formulae, these can be written in the number-state basis as

$$\psi_{Ecs} = [\cosh |\beta|^2]^{-1/2} \sum_{n=0}^{\infty} \frac{\beta^{2n}}{\sqrt{(2n)!}} |2n\rangle, \quad (37)$$

$$\psi_{Ocs} = [\sinh |\beta|^2]^{-1/2} \sum_{n=0}^{\infty} \frac{\beta^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle. \quad (38)$$

Up to the normalization, these are the "even and odd coherent states" previously found in Ref. [23]. Although this system did not come from a Hamiltonian, one could

have used a minimum-uncertainty principle to obtain the same states by starting with the commutation relation $[K_+, K_-] = -2K_0$. However, one does not obtain the same coherent states from the displacement-operator method. Those coherent states, defined by $S(z)|0\rangle$, are the squeezed-vacuum Gaussian of Eq. (24) with $x_0 = p_0 = 0$.

The above three examples have all been cases where $A_- = (A_+)^{\dagger}$. Sometimes that is not the case, as in certain potential systems whose eigenenergies are not equally spaced [20, 21]. Then, as in Eq. (26), one should use the operator form for “ n ”: $A_n \rightarrow A_{n(H)}$, to connect to the minimum-uncertainty method. In these cases, the ladder-operator coherent and squeezed states can be different than, though related to, their minimum-uncertainty counterparts.

4 Future: Generalized Displacement-Operator Squeezed States?

Although the displacement-operator method is the natural one for defining coherent states of Lie groups, there is as yet no well-known general extension of this method to define general displacement-operator squeezed states. It has basically only been applied to harmonic-oscillator-like systems [9, 10].

This has been touched upon in discussions [24] about higher-order generalizations of the “squeeze operator,” $S(z)$. In particular, although harmonic-oscillator-like systems admit squeeze operators (or Bogoliubov transformations) connecting the displacement-operator and ladder-operator methods [25, 26], the appropriate generalization of these squeeze operators have not been found. Therefore, for now, the ladder-operator method is generally connected only to the minimum-uncertainty method.

Note also that for finite-dimensional representations, such as for angular momentum coherent states, the ladder-operator method does not allow a solution for coherent states, although the displacement-operator method does [25]. Contrariwise, for squeezed states, we observe that the opposite is true.

It thus remains for the future to find generalized squeezed states from the displacement operator method. I have been given some helpful advice by a number of people at this Symposium, including John Klauder and Arthur Wightman, but the solution remains to be found. Hopefully, at the next Symposium something further can be said.

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