Generalized Transceiver Beamforming for DFRC with MIMO Radar and MU-MIMO Communication

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***Abstract*—Spatial beamforming is an efficient way to realize dual-functional radar-communication (DFRC). In this paper, we study the DFRC design for a general scenario, where the dual-functional base station (BS) simultaneously detects the target as a multiple-input-multiple-output (MIMO) radar while communicating with multiple multi-antenna communication users (CUs). This necessitates a joint transceiver beamforming design for both MIMO radar and multi-user MIMO (MU-MIMO) communication. In order to characterize the performance tradeoff between MIMO radar and MU-MIMO communication, we first define the achievable performance region of the DFRC system. Then, both radar-centric and communication-centric optimizations are formulated to achieve the boundary of the performance region. For the radar-centric optimization, successive convex approximation (SCA) method is adopted to solve the non-convex constraint. For the communication-centric optimization, a solution based on weighted mean square error (MSE) criterion is obtained to solve the non-convex objective function. Furthermore, two low-complexity beamforming designs based on CU-selection and zero-forcing are proposed to avoid iteration, and the closed-form expressions of the low-complexity beamforming designs are derived. Simulation results are provided to verify the effectiveness of all proposed designs.**

***Index Terms*—Beamforming, multi-antenna, MU-MIMO, MIMO radar, performance region, transceiver design.**

I. Introduction

Due to the scarcity of the spectrum and the potential applications for beyond 5G network, integrated sensing and communication (ISAC) has recently drawn significant attention [1]. It is well-recognized that communication and radar signals have some common features in their waveforms. Although their purposes are different, it is feasible to use one signal for the purpose of the other's [2]. Nevertheless, the use of radar (communication) signals for communication (radar) functionalities leads to a number of challenges [3].

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Generally, there are two research directions of ISAC, i.e., coexisting radar and communication (CRC) and dual functional radar-communication (DFRC). CRC aims for the coexistence of separated radar and communication systems with mutual interference [4]. With the knowledge of the radar sampling scheme, the communication system can design its waveforms in an adaptive fashion to minimize the effective interference at the radar receiver [5]. Following the idea of cognitive radio, the authors in [6] proposed the spectrum sharing scheme of CRC, where the resulting interference of communication would not exceed the radar's tolerable level. To mitigate the radar interference to cellular systems, the work of [7] projected the radar waveform onto the null space of interference channel. The work of [8] exploited the constructive multi-user interference to achieve a better tradeoff between the performance of radar and communication. The time allocation was analyzed and optimized for an integrated bi-static radar and communications system in [9]. Compared to CRC, DFRC focuses on developing dual-functional systems that simultaneously perform radar and communication functions [10].

The simplest way to realize DFRC is based on multiple access technologies. The work of [11] utilized the pseudo-random sequences to realize both spread-spectrum communication and auto-correlation detection. In [12], the authors proposed an integrated orthogonal frequency-division multiplexing (OFDM) waveform to realize both high data transmission rate and low range sidelobes. By dynamically allocating time slots to radar function and communication function, the works in [13] optimized the corresponding tradeoff using sparse sensing techniques. The work of [14] further proposed a novel multiple access scheme named radar-aware carrier sense multiple access (RA-CSMA) to enable dual functions, which outperformed the above time division multiple access (TDMA) scheme. Based on IEEE 802.11ad wireless local area network (WLAN) protocol, joint waveform for automotive radar and a potential mmWave vehicular communication system was provided in [15]. The work of [16] further studied the feasibility of an opportunistic radar, which exploited the probing signals transmitted during the sector level sweep of the IEEE 802.11ad beamforming training protocol. Recently, the emerging orthogonal time frequency space (OTFS) modulation was also applied to DFRC system in dynamic mobile environments with high Doppler frequency [17].

Another way to achieve DFRC is to embed communication signals into radar pulses. Designing the transmit weight associated with the constellation, the information symbols were embedded into the radar pulses based on phase-modulation in [18]. The authors further proposed to embed information symbols in multiple simultaneously transmitted orthogonal waveforms through sidelobe control in [19]. Code shift keying based DFRC was proposed in [20], where each waveform was mapped to a code representing a specific information symbol. Based on the derived waveforms, the work of [21] provided a flexible tradeoff between radar and communications. In [22], the authors studied DFRC employing frequency-hopping code, where both multiple antennas and frequencies were used to generate the frequency-hopping waveform. Through antenna selection and waveform-antenna pairing, the work in [23] provided a method to embed communication information into the emission of multiple-input-multiple-output (MIMO) radar using sparse antenna array configurations.

Utilizing the spatial degrees of freedom, spatial beamforming of multi-antennas is an efficient way to realize DFRC. Considering both separated and shared antenna deployments, a series of optimization-based transmit beamforming approaches were studied in [24] for the DFRC system, where the communication signal was exploited for target detection. For probing multiple radar targets and communicating to multiple users simultaneously, the DFRC transmitter jointly precoded individual communication and radar waveforms in [25], which extended the MIMO radar waveform degrees of freedom (DoF) to its maximal value. To increase the spatial DoF, the work of [26] utilized both communication signals and dedicated radar signals to improve the sensing performance. Furthermore, it have been first proved in [27] that there is no need to add dedicated radar signals for line-of-sight communication if communication users (CUs) cannot cancel the interference of the dedicated radar signals. And dedicated radar signals are always beneficial if CUs have the capability of canceling the interference from the dedicated radar signals. Regarding the radar targets as potential eavesdroppers, beamforming with artificial noise was designed to enable communication-radar functions in [28]. In [29], we discussed a Pareto optimization framework of the DFRC system to provide an optimal performance tradeoff between radar and communication through beamforming design. Note that all these works only considered the communication of single-antenna CUs.

Recently, DFRC beamforming designs were extended to multi-antenna CU. In [30], the dual-functional base station (BS) was designed to serve a multi-antenna CU and detect targets simultaneously using a hybrid analog/digital beamforming technique. Based on mutual information, the authors in [31] provided an optimal spatio-temporal power mask design, where a typical packet-based signal structure including training and data symbols was studied. However, considering the communication of multiple multi-antenna CUs, the DFRC beamforming for both MIMO radar sensing and MU-MIMO communication has never been discussed to the best of our knowledge. It requires a complex joint transceiver beamforming for both MIMO radar and MU-MIMO communication compared to the existed DFRC beamforming with single-antenna CUs. The beamforming designs for MIMO radar [32]–[34] and MU-MIMO communication [35]–[37] have been extensively investigated in the recent literature independently, but they cannot be applied to the DFRC beamforming design directly due to the coupled communication and sensing performance of the DFRC system.

Motivated by the above observation, we propose a generalized beamforming design for the DFRC system, where the dual-functional BS simultaneously detects the target as a MIMO radar and communicates with multiple multi-antenna CUs. To characterize the performance tradeoff between MIMO radar and MU-MIMO communication, we first define the achievable performance region of the generalized DFRC system. In order to achieve the boundary of the performance region, we formulate the problem from both radar-centric and communication-centric viewpoints. For the radar-centric optimization, the problem is formulated to maximize the sensing signal-clutter-noise ratio (SCNR) of the MIMO radar under the sum rate constraint of the MU-MIMO communication. Due to its non-convex constraint, we provide an iterative solution based on successive convex approximation (SCA) method. For the communication-centric optimization, the problem is formulated to maximize the sum rate of the MU-MIMO communication given the sensing SCNR constraint of the MIMO radar. Due to its non-convex objective function, we provide an iterative solution based on the weighted mean square error (MSE) criterion. To further reduce the complexity of the iterative optimizations, we propose two low-complexity beamforming designs. One is based on the CU selection, where the optimal beamforming is given as semi-closed-form expressions for the single-CU scenario. The other is based on zero-forcing interference, where a block diagonalization beamforming is provided with the optimal power allocation. The main contributions of this work are summarized as follows.

- **Generalized DFRC beamforming model and performance.** The DFRC beamforming design is extended to a general scenario, where the dual-functional BS simultaneously detects the target as a MIMO radar and communicates with multiple multi-antenna CUs. Both radar-centric and communication-centric optimizations are formulated to achieve the boundary of the performance region for the generalized DFRC system.
- **Iterative solutions towards the optimal transceiver beamforming**. For the radar-centric formulation, we adopt SCA method to relax the non-convex constraint, where an iterative solution of the convex optimization is proposed. For the communication-centric formulation, the weighted MSE criterion is adopted to avoid the non-convex objective function, where the semi-closed-form expressions of the optimal transmit (receive) beamforming are derived with fixed receiver (transmitter).
- **Low-complexity beamforming designs for DFRC system**. Considering the high complexity of the above designs, we first simplify the multi-CU scenario to the single-CU scenario based on the CU selection, where the closed-form expressions of the optimal beamforming are provided for the transceiver. Then, based on zero-forcing interference between CUs, a block diagonalization beamforming is given, where the closed-form expressions of the optimal power allocation are derived.

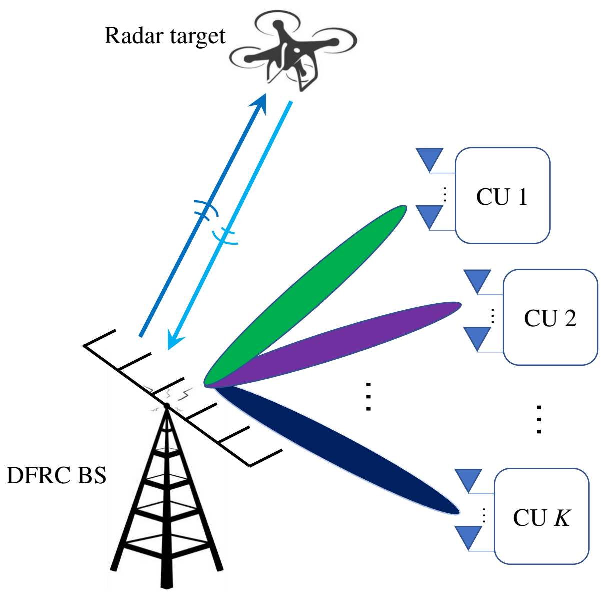


Figure 1. Generalized DFRC system model simultaneously detecting the target as a MIMO radar and communicating to multiple multi-antenna CUs.

The remainder of the paper is organized as follows. Section II presents the system model. Section III gives the DFRC beamforming designs to achieve the boundary of the performance region. We further provide two low-complexity beamforming designs without iteration in Section IV. Simulation results are provided in Section V, followed by concluding remarks in Section VI.

Notation: We use boldface lowercase letter to denote column vectors, and boldface uppercase letters to denote matrices. Superscripts $(\cdot)^H$ and $(\cdot)^T$ stand for Hermitian transpose and transpose, respectively. $\mathrm{tr}(\cdot)$, $\mathrm{diag}(\cdot)$, and $\mathrm{rank}(\cdot)$ represent the trace operation, the vector formed by the diagonal elements and the rank operator, respectively. $\det(\cdot)$ is the determinant of a matrix. $\mathcal{C}^{m\times n}$ is the set of complex-valued $m\times n$ matrices. $x \sim \mathcal{CN}(a,b)$ means that x obeys a complex Gaussian distribution with mean a and covariance b. $\mathrm{E}(\cdot)$ denotes the statistical expectation. $\|\mathbf{x}\|$ denotes the Euclidean norm of a complex vector $\mathbf{x}$.

II. System Model

As illustrated in Fig. 1, we consider a dual-functional MU-MIMO communication and MIMO radar sensing system. It has a multi-antenna dual-functional BS and K multi-antenna CUs indexed by $k\in\{1,\cdots,K\}$. The dual-functional BS has N_t transmit antennas and N_r receive antennas. The k-th CU has M_k antennas with $k\in\{1,\cdots,K\}$.

Due to the multiple antennas deployed at the CUs, the transmission of multiple data streams D_k for the k-th CU can be realized. The transmit signal of the dual-functional BS $\mathbf{x}\in\mathcal{C}^{N_t\times 1}$ is composed of the data signal for all CUs[1], i.e.,

$$\mathbf{x}=\sum_{k=1}^{K}\mathbf{B}_k\mathbf{d}_k, \tag{1}$$

[1]The symbol index is omitted for simplicity.

where $\mathbf{B}_k\in\mathcal{C}^{N_t\times D_k}$ and $\mathbf{d}_k\in\mathcal{C}^{D_k\times 1}$ are the transmit beamforming matrix and the data vector of the k-th CU with $D_k\le\min\{N_t,M_k\}$, respectively. $\mathbf{d}_k$ is assumed to be Gaussian distributed with $\mathbf{d}_k\sim\mathcal{CN}(0,\mathbf{I}_{D_k})$ and the data vectors of different CUs are assumed to be independent. Thus, the transmit power of the dual-functional BS can be calculated as

$$\mathrm{E}\left(\|\mathbf{x}\|^2\right)=\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{B}_k\mathbf{B}_k^H\right)\le P_0, \tag{2}$$

where P_0 is the transmit power constraint.

A. MU-MIMO Communication Performance

Given the transmit signal $\mathbf{x}$ in (1), the received signal of the k-th CU is expressed as

$$\begin{aligned}\mathbf{y}_k&=\mathbf{H}_k\mathbf{x}+\mathbf{z}_k\\&=\mathbf{H}_k\sum_{k=1}^{K}\mathbf{B}_k\mathbf{d}_k+\mathbf{z}_k,\end{aligned} \tag{3}$$

where $\mathbf{H}_k\in\mathcal{C}^{M_k\times N_t}$ is the channel matrix between the dual-functional BS and the CU, and $\mathbf{z}_k\in\mathcal{C}^{M_k\times 1}$ is the additive white Gaussian noise (AWGN) with $\mathbf{z}_k\sim\mathcal{CN}(0,\mathbf{I}_{M_k})$ including the reflected signal from the target and the clutter signal.

Based on the assumption of Gaussian distributed data vectors for each CU, the achievable rate of the k-th CU can be calculated as

$$C_k=\log\det\left(\mathbf{I}+\mathbf{B}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{B}_k\right),\forall k \tag{4}$$

where

$$\mathbf{R}_k=\mathbf{I}+\sum_{i=1,i\neq k}^{K}\mathbf{H}_k\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_k^H. \tag{5}$$

Then, the sum rate of the MU-MIMO communication can be given by

$$C=\sum_{k=1}^{K}\log\det\left(\mathbf{I}+\mathbf{B}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{B}_k\right). \tag{6}$$

B. MIMO Radar Sensing Performance

Given the transmit signal $\mathbf{x}$ in (1), the received signal of the radar receiver is

$$\begin{aligned}\mathbf{y}_0&=\alpha_0\mathbf{a}_r(\theta_0)\,\mathbf{a}_t^T(\theta_0)\,\mathbf{x}+\mathbf{c}+\mathbf{z}_0\\&=\alpha_0\mathbf{A}(\theta_0)\,\mathbf{x}+\mathbf{c}+\mathbf{z}_0\end{aligned}, \tag{7}$$

where a radar target is assumed to be located at angle θ_0, α_0 is the complex amplitude of the target, $\mathbf{c}\in\mathcal{C}^{N_r\times 1}$ is the clutter with $\mathbf{c}\sim\mathcal{CN}(0,\mathbf{R}_c)$ including the reflected signal from the CUs, $\mathbf{a}_t(\theta)=[1,\cdots,e^{-j2\pi(N_t-1)\Delta_t\sin\theta}]^T$ and $\mathbf{a}_r(\theta)=[1,\cdots,e^{-j2\pi(N_r-1)\Delta_r\sin\theta}]^T$ with Δ_t and Δ_r being the spacing between adjacent antennas normalized by the wavelength, and $\mathbf{z}_0\in\mathcal{C}^{N_r\times 1}$ is the AWGN satisfying $\mathcal{CN}(0,\mathbf{I}_{N_r})$.

Generally, the clutter $\mathbf{c}$ can be modeled to be signal-independent or signal-dependent. For the signal-independent clutter, its covariance matrix $\mathbf{R}_c$ is assumed to be constant. For the signal-dependent clutter, the clutter can be further modeled as $\mathbf{c} = \sum_{i=1}^{I} \alpha_i \mathbf{A}(\theta_i)\mathbf{x}$ with I clutter located at angle $\theta_i, i \in \{1, \cdots, I\}$. The corresponding covariance matrix can be calculated as $\mathbf{R}_c = \sum_{i=1}^{I} |\alpha_i|^2 \mathbf{A}(\theta_i)\left(\sum_{k=1}^{K} \mathbf{B}_k \mathbf{B}_k^H\right)\mathbf{A}^H(\theta_i)$ given $\mathbf{x}$ in (1). By adopting the classical iterative method in [38], $\mathbf{R}_c$ can be also regarded to be constant given the fixed beamforming matrix $\mathbf{B}_k$ in the last round of iteration. Thus, in the following discussion, we will focus on the constant covariance matrix of the clutter.

Then, the output of the radar receiver is

$$r = \mathbf{w}^H \mathbf{y}_0 = \alpha_0 \mathbf{w}^H \mathbf{A}(\theta_0)\mathbf{x} + \mathbf{w}^H \mathbf{c} + \mathbf{w}^H \mathbf{z}_0 \quad (8)$$

where $\mathbf{w} \in \mathcal{C}^{N_r \times 1}$ is the receive beamforming vector for SCNR maximization. The optimal $\mathbf{w}$ to maximize the SCNR of the MIMO radar can be given by

$$\begin{aligned} \mathbf{w}^* &= \arg\max \frac{\left|\mathbf{w}^H \mathbf{A}(\theta_0)\mathbf{x}\right|^2}{\mathbf{w}^H(\mathbf{R}_c + \mathbf{I})\mathbf{w}}, \\ &= \beta(\mathbf{R}_c + \mathbf{I})^{-1}\mathbf{A}(\theta_0)\mathbf{x} \end{aligned} \quad (9)$$

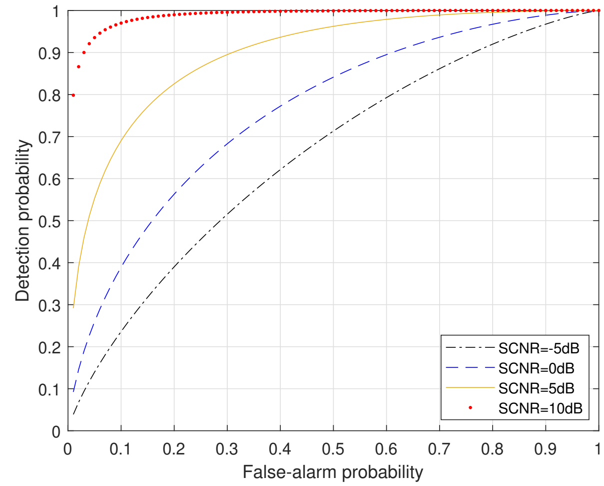
where β is an arbitrary constant. Its solution can be derived by solving the equivalent minimum variance distortionless response (MVDR) problem in [39]. Thus, the corresponding SCNR of the MIMO radar can be calculated as

$$\begin{aligned} \gamma &= \mathrm{E}\left[\frac{|\alpha_0|^2 \left|\mathbf{w}^H \mathbf{A}(\theta_0)\mathbf{x}\right|^2}{\mathbf{w}^H(\mathbf{R}_c + \mathbf{I})\mathbf{w}}\right] \\ &\overset{(a)}{=} \mathrm{E}\left[|\alpha_0|^2 \mathbf{x}^H \mathbf{A}^H(\theta_0)(\mathbf{R}_c + \mathbf{I})^{-1}\mathbf{A}(\theta_0)\mathbf{x}\right], \\ &\overset{(b)}{=} \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{\Phi}\mathbf{B}_k\mathbf{B}_k^H\right) \end{aligned} \quad (10)$$

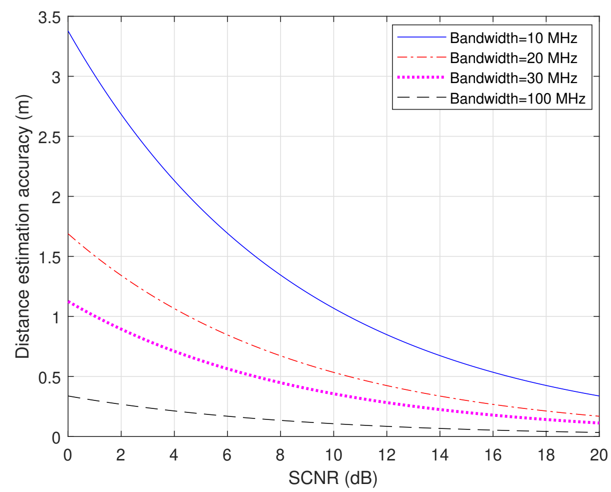
where the procedure (a) is due to the optimal receive beamforming vector $\mathbf{w}^*$ in (9), and the procedure (b) is due to $\mathrm{E}\left(\mathbf{x}\mathbf{x}^H\right) = \sum_{k=1}^{K} \mathbf{B}_k \mathbf{B}_k^H$, and

$$\mathbf{\Phi} = |\alpha_0|^2 \mathbf{A}^H(\theta_0)(\mathbf{R}_c + \mathbf{I})^{-1}\mathbf{A}(\theta_0). \quad (11)$$

Note that the sensing SCNR of the radar determines both the detection performance and the localization performance of the target. For example, considering an AWGN channel, the receiver operating characteristic (ROC) curve of the false-alarm probability and the detection probability is shown in Fig. 2(a) with different SCNR [40]. And the distance estimation accuracy versus different SCNR based on time-of-arrival (TOA) method is illustrated in Fig. 2(b) for different effective signal bandwidth [41].



(a) Illustration of the ROC curve with different SCNR.



(b) Distance estimation accuracy versus different SCNR.

Figure 2. The illustration of the detection performance and the location performance of the target versus the sensing SCNR of the radar.

III. Beamforming Design for Achievable Performance Region

Due to the coupled performance of radar and communication, we will first define the achievable performance region of the DFRC in Section III-A. Then, from the radar-centric viewpoint, we will formulate the problem to achieve the boundary of the performance region. An iterative solution based on SCA method will be further given in Section III-B. Finally, the problem to achieve the boundary of the performance region will be formulated from the communication-centric viewpoint. The weighted MSE criterion will be adopted to solve the problem in Section III-C.

A. Achievable Performance Region of DFRC

The beamforming design for the dual-functional BS has two distinct goals. One is to maximize the sum rate for the MU-MIMO communication given the channel matrix. The other is to maximize the sensing SCNR for the MIMO radar considering its detection probability. It brings up the question of what is the optimal beamforming design for the dual-functional BS? In order to shed lights on this question, we first define the achievable performance region of the DFRC system as follows.

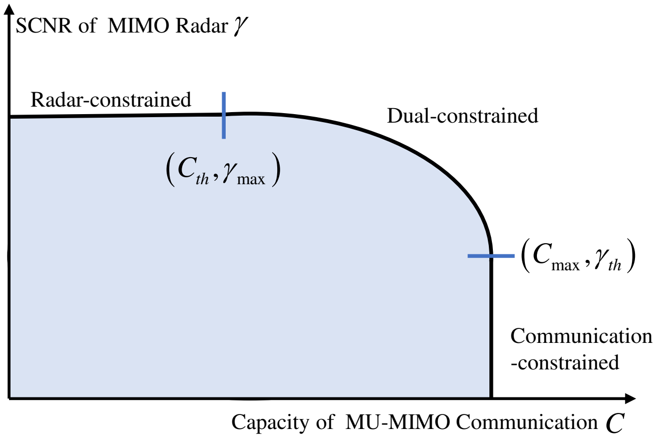


Figure 3. Illustration of achievable performance region of the DFRC Beamforming.

Definition 1. (**Achievable performance region of DFRC**) Considering the sum rate of the MU-MIMO communication and the sensing SCNR of the MIMO radar, the achievable performance region of the DFRC system under the transmit power constraint can be defined as follows.

$$\mathcal{R} = \left\{ (C, \gamma) : C \leq \sum_{k=1}^{K} \log\det\left(\mathbf{I} + \mathbf{B}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{B}_k\right), \right. \\ \left. \gamma \leq \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{\Phi} \mathbf{B}_k \mathbf{B}_k^H\right), \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{B}_k \mathbf{B}_k^H\right) \leq P_0 \right\}. \tag{12}$$

As illustrated in Fig.3, the boundary of the performance region can be divided into three sections, i.e., radar-constrained section, communication-constrained section and dual-constrained section, by two boundary points, i.e., $(C_{\text{th}}, \gamma_{\max})$ and $(C_{\max}, \gamma_{\text{th}})$. For radar-constrained section, the sum rate of the MU-MIMO communication satisfies $C \leq C_{\text{th}}$, and the sensing SCNR $\gamma_{\max}$ is determined by the traditional MIMO radar beamforming design without communication constraints. For communication-constrained section, the sensing SCNR of the MIMO radar satisfies $\gamma \leq \gamma_{\text{th}}$, and the sum rate $C_{\max}$ is determined by the traditional MU-MIMO beamforming design without radar constraints.

Note that it is non-obvious how to measure the DFRC system performance given the isolated radar and communication metrics. Given different performance metrics of the radar and the communication, e.g., Cramér-Rao bound (CRB), mutual information (MI), and so on, it will imply a different performance region with respect to the DFRC system. And even for the given performance metrics, the shape of performance region will be affected by the parameters of the radar and the communication, e.g., channel parameters, the number of CUs and so on.

There exists inherent conflict and tradeoffs for the DFRC system to optimize the multiple objectives simultaneously. And the optimal performance tradeoff between the MIMO radar and the MU-MIMO communication can be characterized by the boundary of the achievable performance region. In order to achieve the boundary, there are generally two equivalent ways to optimize the beamforming designs. One is radar-centric, where the performance of the MIMO radar is optimized under the performance constraint of the MU-MIMO communication. The other is communication-centric, where the performance of the MU-MIMO communication is maximized given the performance constraint of the MIMO radar. In the following discussion, we will discuss the radar-centric design and communication-centric design, respectively.

B. Radar-centric Beamforming

Specifically, the radar-centric beamforming maximizes the sensing SCNR of the MIMO radar given the sum rate constraint of the MU-MIMO communication, which can be formulated as follows.

$$\begin{aligned} \text{(P1)} \quad & \max_{\{\mathbf{B}_k\}} \quad \gamma = \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{\Phi} \mathbf{B}_k \mathbf{B}_k^H\right) \\ & \text{s.t.} \quad \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{B}_k \mathbf{B}_k^H\right) \leq P_0 \\ & \qquad \sum_{k=1}^{K} \log\det\left(\mathbf{I} + \mathbf{B}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{B}_k\right) \geq C_0 \end{aligned}, \tag{13}$$

where C_0 is the sum rate constraint of the MU-MIMO communication, and the boundary of the achievable performance region can be achieved by setting different C_0.

Compared to the existed waveform designs of the MIMO radar in [32]–[34], the formulated problem (P1) introduces an additional sum rate constraint of the MU-MIMO communication. Due to this additional constraint, the problem becomes non-convex. Thus, we resort to the SCA method based on sequential convex programming to provide a locally optimal solution.

Firstly, the achievable rate of the k-th CU in (4) can be rewritten as

$$\begin{aligned} C_k & \overset{(a)}{=} \log\det\left[\mathbf{R}_k^{-1}\left(\mathbf{R}_k + \mathbf{H}_k \mathbf{S}_k \mathbf{H}_k^H\right)\right] \\ & \overset{(b)}{=} \log\det\left(\mathbf{I} + \sum_{i=1}^{K} \mathbf{H}_k \mathbf{S}_i \mathbf{H}_k^H\right) - \log\det\left(\mathbf{R}_k\right) \end{aligned}, \tag{14}$$

where $\mathbf{S}_k = \mathbf{B}_k \mathbf{B}_k^H$ is the covariance matrix of the k-th CU, $\mathbf{R}_k$ in (5) can be rewritten as

$$\mathbf{R}_k = \mathbf{I} + \sum_{i=1, i \neq k}^{K} \mathbf{H}_k \mathbf{S}_i \mathbf{H}_k^H, \tag{15}$$

the procedure (a) is due to $\det\left(\mathbf{I} + \mathbf{AB}\right) = \det\left(\mathbf{I} + \mathbf{BA}\right)$, the procedure (b) is due to $\det\left(\mathbf{B}^{-1}\mathbf{A}\right) = \det\left(\mathbf{A}\right) / \det\left(\mathbf{B}\right)$.

Then, we adopt the first-order Taylor expansion to approximate C_k, where $\log\det\left(\mathbf{R}_k\right)$ is upper bounded by

$$\log\det\left(\mathbf{R}_k\right) \overset{(a)}{\leq} \log\det\left(\bar{\mathbf{R}}_k\right) + \mathrm{tr}\left(\bar{\mathbf{R}}_k^{-1}\sum_{i=1,i\neq k}^{K}\mathbf{H}_k\mathbf{S}_i\mathbf{H}_k^H\right) - \mathrm{tr}\left(\bar{\mathbf{R}}_k^{-1}\sum_{i=1,i\neq k}^{K}\mathbf{H}_k\bar{\mathbf{S}}_i\mathbf{H}_k^H\right), \tag{16}$$

where $\bar{\mathbf{S}}_k$ is the fixed covariance matrix of the k-th CU from the initialization or the last round of optimization, and $\bar{\mathbf{R}}_k = \mathbf{I} + \sum_{i=1,i\neq k}^{K}\mathbf{H}_k\bar{\mathbf{S}}_i\mathbf{H}_k^H$. The procedure (a) is due to the first-order approximation of $\log\det\left(\mathbf{I}+\mathbf{X}\right)$, i.e.,

$$\begin{aligned}\log\det\left(\mathbf{I}+\mathbf{X}\right) \leq & \log\det\left(\mathbf{I}+\mathbf{X}_0\right) \\ & + \mathrm{tr}\left[\left(\mathbf{I}+\mathbf{X}_0\right)^{-1}\left(\mathbf{X}-\mathbf{X}_0\right)\right]\end{aligned} \tag{17}$$

Thus, the achievable rate of the k-th CU in (14) is lower bounded by

$$\begin{aligned} C_k \geq & \log\det\left(\mathbf{I}+\sum_{i=1}^{K}\mathbf{H}_k\mathbf{S}_i\mathbf{H}_k^H\right) - \\ & \mathrm{tr}\left(\bar{\mathbf{R}}_k^{-1}\sum_{i=1,i\neq k}^{K}\mathbf{H}_k\mathbf{S}_i\mathbf{H}_k^H\right) + \nu_k \\ = & \log\det\left(\mathbf{I}+\mathbf{H}_k\mathbf{S}\mathbf{H}_k^H\right) - \mathrm{tr}\left(\bar{\mathbf{R}}_k^{-1}\mathbf{H}_k\mathbf{S}\mathbf{H}_k^H\right) + \\ & \mathrm{tr}\left(\bar{\mathbf{R}}_k^{-1}\mathbf{H}_k\mathbf{S}_k\mathbf{H}_k^H\right) + \nu_k \end{aligned}, \tag{18}$$

where $\mathbf{S} = \sum_{k=1}^{K}\mathbf{S}_k$ and

$$\nu_k = \mathrm{tr}\left(\bar{\mathbf{R}}_k^{-1}\sum_{i=1,i\neq k}^{K}\mathbf{H}_k\bar{\mathbf{S}}_i\mathbf{H}_k^H\right) - \log\det\left(\bar{\mathbf{R}}_k\right). \tag{19}$$

Since ν_k is constant, $\log\det(\mathbf{I}+\mathbf{H}_k\mathbf{S}\mathbf{H}_k^H)$ is concave, and both $\mathrm{tr}(\bar{\mathbf{R}}_k^{-1}\mathbf{H}_k\mathbf{S}\mathbf{H}_k^H)$ and $\mathrm{tr}(\bar{\mathbf{R}}_k^{-1}\mathbf{H}_k\mathbf{S}_k\mathbf{H}_k^H)$ are linear. C_k in (14) is lower bounded by a concave function.

Finally, the problem in each sequential optimization of SCA method for the problem (P1) can be formulated as

$$\begin{aligned} (\mathrm{P1.1}) \quad \max_{\{\mathbf{S}_k\}} \quad & \gamma = \mathrm{tr}\left(\mathbf{\Phi}\mathbf{S}\right) \\ \text{s.t.} \quad & \mathrm{tr}\left(\mathbf{S}\right) \leq P_0 \\ & \sum_{k=1}^{K}\left[\mathrm{tr}\left(\bar{\mathbf{R}}_k^{-1}\mathbf{H}_k\mathbf{S}_k\mathbf{H}_k^H\right) - \mathrm{tr}\left(\bar{\mathbf{R}}_k^{-1}\mathbf{H}_k\mathbf{S}\mathbf{H}_k^H\right)\right. \\ & \quad \left. + \log\det\left(\mathbf{I}+\mathbf{H}_k\mathbf{S}\mathbf{H}_k^H\right) + \nu_k\right] \geq C_0 \\ & \mathbf{S} = \sum_{k=1}^{K}\mathbf{S}_k, \mathbf{S}_k \succ 0, \forall k \end{aligned} \tag{20}$$

which is a convex problem and can be efficiently computed using standard convex optimization numerical techniques.

In the following, we provide an iterative algorithm based on SCA in Algorithm 1. In each iteration, the problem (P1.1) is repeatedly solved until the increase of the MIMO radar SCNR is lower than the defined threshold. The convergence

Algorithm 1 SCA method for radar-centric optimization

Initialize the convergence precision ε_γ;
Initialize the covariance matrix of the CUs $\mathbf{S}_k = \mathbf{0}, \forall k$;
repeat
  Initialize the covariance matrix of the CUs $\bar{\mathbf{S}}_k = \mathbf{S}_k, \forall k$.
  Compute the sensing SCNR of the MIMO radar $\bar{\gamma}$ based on $\bar{\mathbf{S}}_k$ according to (10).
  Compute $\bar{\mathbf{R}}_k$ and ν_k according to (14) and (19).
  Update $\mathbf{S}_k, \forall k$ through the problem (P1.1).
  Compute the updated sensing SCNR of the MIMO radar γ according to (10).
until $|\gamma - \bar{\gamma}| \leq \varepsilon_\gamma$

of the proposed algorithm can be guaranteed, because the MIMO radar SCNR is upper bounded and the iterations are monotonically non-decreasing.

There is no explicit rank constraint of the covariance matrix $\mathbf{S}_k, \forall k$ for the problem (P1.1). Thus, the number of data streams for the k-th CU, which corresponds to the rank of its covariance matrix $\mathbf{S}_k$, is determined by the optimization result. But the rank of the final solution $\mathbf{S}_k^*$ will not be larger than the number of antennas for the k-th CU M_k. It means that the k-th CU cannot receive more data streams than the number of its antennas. The worst case complexity of solving semidefinite programming (SDP) in the problem (P1.1) is $\mathcal{O}\left(\max\left\{N_t, M_1, \cdots, M_K\right\}^7\right)$. Assuming that the number of the iterations for SCA method in Algorithm 1 is L_0, the time complexity of Algorithm 1 can be given by $\mathcal{O}\left(L_0\max\left\{N_t, M_1, \cdots, M_K\right\}^7\right)$.

C. Communication-centric Beamforming

Besides the radar-centric design, the communication-centric design can also achieve the boundary of the performance region for the DFRC system. Specifically, the communication-centric beamforming maximizes the sum rate of the MU-MIMO communication subject to the sensing SCNR constraint of the MIMO radar, which can be formulated as

$$\begin{aligned} (\mathrm{P2}) \quad \max_{\{\mathbf{B}_k\}} \quad & C = \sum_{k=1}^{K}\log\det\left(\mathbf{I}+\mathbf{B}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{B}_k\right) \\ \text{s.t.} \quad & \sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{B}_k\mathbf{B}_k^H\right) \leq P_0 \\ & \sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{\Phi}\mathbf{B}_k\mathbf{B}_k^H\right) \geq \gamma_0 \end{aligned}, \tag{21}$$

where γ_0 is the sensing SCNR constraint of the MIMO radar, and the boundary of the achievable performance region can be achieved by setting different γ_0.

Due to the non-convex objective function, the problem (P2) is also non-convex. We resort to weighted MSE criterion to provide a sub-optimal solution based on alternating optimization. It is worth to mention that this argument is parallel to the one given for the MU-MIMO communication in interference channel [42] and full-duplex channel [43]. This work extends the above discussions from the MU-MIMO communication to the DFRC system with the additional sensing SCNR of the MIMO radar considered.

Given the received signal of the k-th CU in (3), the corresponding MSE matrix of the k-th CU can be calculated as

$$\begin{aligned}\mathbf{E}_k &= \mathrm{E}\left[\left(\mathbf{D}_k\mathbf{y}_k-\mathbf{d}_k\right)\left(\mathbf{D}_k\mathbf{y}_k-\mathbf{d}_k\right)^H\right]\\ &= \mathbf{D}_k\mathbf{H}_k\left(\sum_{i=1}^{K}\mathbf{B}_i\mathbf{B}_i^H\right)\mathbf{H}_k^H\mathbf{D}_k^H+\mathbf{D}_k\mathbf{D}_k^H-2\mathbf{D}_k\mathbf{H}_k\mathbf{B}_k+\mathbf{I}\end{aligned}, \tag{22}$$

where $\mathbf{D}_k \in \mathcal{C}^{D_k\times N_r}$ is the receive beamforming matrix of the k-th CU . Then, the sum of the weighted MSE can be defined as

$$E=\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{W}_k\mathbf{E}_k\right)-\log\det\left(\mathbf{W}_k\right), \tag{23}$$

where $\mathbf{W}_k \in \mathcal{C}^{D_k\times D_k}$ is the MSE-weight matrix of the k-th CU. The problem to minimize the sum of the weighted MSE under the transmit power constraint and the sensing SCNR constraint of the MIMO radar, can be formulated as

$$\begin{aligned}(\mathrm{P2.1})\quad &\min_{\{\mathbf{D}_k\}\{\mathbf{B}_k\}} && E=\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{W}_k\mathbf{E}_k\right)-\log\det\left(\mathbf{W}_k\right)\\ &\text{s.t.} && \sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{B}_k\mathbf{B}_k^H\right)\le P_0\\ & && \gamma=\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{\Phi}\mathbf{B}_k\mathbf{B}_k^H\right)\ge\gamma_0\end{aligned}. \tag{24}$$

Furthermore, the relationship between the original problem (P2) and the above formulated problem (P2.1) can be provided as follows.

Proposition 1. (**Relationship between problems**) In the sense that the Karush-Kuhn-Tucker (KKT) conditions and the optimal solutions are identical, the problem (P2) to maximize the sum rate is equivalent to the problem (P2.1) to minimize the weighted-sum MSE under both the transmit power constraint and the sensing SCNR constraint of the MIMO radar for the DFRC system, when the weight matrix satisfies

$$\mathbf{W}_k=\frac{\mathbf{E}_k^{-1}}{\log 2},\forall k. \tag{25}$$

Proof. The proof is given in Appendix A. □

Although we have established the equivalence between the problem (P2) and the problem (P2.1) for the DFRC system, the problem (P2.1) is still not jointly convex on $\{\mathbf{D}_k\}$ and $\{\mathbf{B}_k\}$ and no standard convex optimization method can be used to find the optimal solution. Note that the sum of the weighted MSE is convex on $\{\mathbf{B}_k\}$ with fixed $\{\mathbf{D}_k\}$, and vice versa, we can adopt the classical alternating optimization to find efficient sub-optimal solutions based on the following proposition.

Proposition 2. (**Alternating beamforming design**) For the DFRC system, we can provide the following alternating optimization solutions to minimize the sum of the weighted MSE of the MU-MIMO communication under the transmit power constraint and the sensing SCNR constraint of the MIMO radar. With fixed transmit beamforming of the dual-functional BS $\{\mathbf{B}_k\}$, the optimal receive beamforming of the CU is

$$\mathbf{D}_k^*=\mathbf{B}_k^H\mathbf{H}_k^H\left(\mathbf{H}_k\mathbf{B}_k\mathbf{B}_k^H\mathbf{H}_k^H+\mathbf{R}_k\right)^{-1},\forall k. \tag{26}$$

Then, with fixed receive beamforming of the CU $\{\mathbf{D}_k\}$, the optimal transmit beamforming of the dual-functional BS is

$$\mathbf{B}_k^*=\left(\mathbf{H}_k^H\mathbf{D}_k^H\mathbf{W}_k\mathbf{D}_k\mathbf{H}_k+\lambda\mathbf{I}-\mu\mathbf{\Phi}\right)^{-1}\mathbf{H}_k^H\mathbf{D}_k^H\mathbf{W}_k, \tag{27}$$

where $\lambda>0$ and $\mu>0$ are dual variables of the transmit power constraint and the sensing SCNR constraint of MIMO radar, respectively.

Proof. The proof is given in Appendix B. □

Algorithm 2 Bisection search of dual variables

```
Initialize the convergence precision ε_λ and ε_μ ;
Initialize the range of dual variables [λ_min, λ_max] and [μ_min, μ_max];
repeat
    Set λ = (λ_min + λ_max)/2;
    Set μ = (μ_max + μ_min)/2;
    Calculate B_k^* according to (27);
    if Σ_{k=1}^K tr(Φ B_k B_k^H) < γ_0
        μ_max = μ;
    else
        μ_min = μ;
    endif
    if Σ_{k=1}^K tr(B_k B_k^H) < P_0
        λ_max = λ;
    else
        λ_min = λ;
    endif
until |λ_max − λ_min| ≤ ε_λ and |μ_max − μ_min| ≤ ε_μ;
```

Algorithm 3 Weighted MSE iteration for communication-centric optimization

```
Initialize t = 0, ε;
Initialize the {B_k^(0)} satisfying the constraints of problem (P0);
Calculate {D_k^(0)} and {W_k^(0)} according to (26) and (25), respectively.
Calculate {E_k^(0)} and the corresponding E^(0) according to (22) and (23).
repeat
    t = t + 1;
    Calculate the corresponding {B_k^(t)} according to (27) and Algorithm 2;
    Calculate {D_k^(t)} and {W_k^(t)} according to (26) and (25), respectively;
    Calculate {E_k^(t)} and the corresponding E^(t) according to (22) and (23);
until |E^(t) − E^(t−1)| ≤ ε.
```

According to (27), the optimal transmit beamforming given fixed receive beamforming has a semi-closed-form expression in terms of the dual variables λ and μ, which can be determined by the two-dimensional bisection search in Algorithm 2. The initial value of $\lambda_{\min} = 0$, and the initial value of $\lambda_{\max}$ should satisfy the transmit power constraint. By approximating $\mathbf{B}_k^*$ in (27) to $\tilde{\mathbf{B}}_k = \mathbf{H}_k^H \mathbf{D}_k^H \mathbf{W}_k$, the corresponding $\lambda_{\max}$ can be given by $\lambda_{\max}^2 = \sum_{k=1}^K \mathrm{tr}\left(\mathbf{H}_k^H \mathbf{D}_k^H \mathbf{W}_k \mathbf{W}_k \mathbf{D}_k \mathbf{H}_k\right) \Big/ P_0$. Furthermore, the initial value of $\mu_{\min}$ and $\mu_{\max}$ can be set in a similar way. Above all, we propose an alternating optimization solution to solve problem (P2), as illustrated in Algorithm 3. It is composed of three steps. First, the transmit beamforming $\{\mathbf{B}_k\}$ is updated with fixed receive beamforming $\{\mathbf{D}_k\}$ and weight matrix $\{\mathbf{W}_k\}$ according to (27), where the dual variables is calculated according to Algorithm 2. Then, the receive beamforming $\{\mathbf{D}_k\}$ and the weight matrix $\{\mathbf{W}_k\}$ are updated with fixed transmit beamforming $\{\mathbf{B}_k\}$ according to (26) and (25), respectively. Finally, the MSE matrix $\{\mathbf{E}_k\}$ is updated with fixed transmit beamforming $\{\mathbf{B}_k\}$ and receive beamforming $\{\mathbf{D}_k\}$ according to (22).

The iteration in Algorithm 3 makes the weighted MSE of the MU-MIMO communication decrease monotonically, which is lower bounded. Thus, the convergence of the iteration can be guaranteed. Furthermore, the iteration number of bisection search for Algorithm 2 is determined by $\max\{L_1, L_2\}$, where $L_1 = \log(\lambda_{\max} - \lambda_{\min}) - \log \varepsilon_\lambda$ and $L_2 = \log(\mu_{\max} - \mu_{\min}) - \log \varepsilon_\mu$. The time complexity of $\mathbf{B}_k^*$ computation in (27), $\mathbf{D}_k^*$ computation in (26) and $\mathbf{E}_k^*$ computation in (22) are $O\left(N_t^3\right)$, $O\left(M_k^3\right)$ and $O\left(D_k^3\right)$, respectively. Assuming the number of the iterations in Algorithm 3 is L_3, the time complexity of Algorithm 3 can be given by $O\left(\max\{L_1, L_2\} L_3 K (\max\{N_t, M_1, \cdots, M_k\})^3\right)$.

IV. Low-Complexity Beamforming Designs

Both radar-centric and communication-centric formulations require iterative optimizations with high computational complexity. In order to avoid iteration, we will provide two low-complexity beamforming designs in this section. First, we will simplify the multi-CU scenario to the single-CU scenario based on CU selection. And closed-form expressions of the optimal beamforming will be derived in Section IV-A. Then, based on interference zero-forcing, a block diagonalization beamforming will be designed for the DFRC system. The optimal power allocation will be derived in Section IV-B.

A. Single-CU Scenario Beamforming

When multiple CUs are scheduled based on round robin or opportunistic scheduling, multi-CU scenario degrades to single-CU scenario. Round robin is a fair scheduling algorithm, where each CU is assigned a fixed time slot in a cyclic way. And opportunistic scheduling achieves better performance utilizing multi-user diversity gain. It chooses a CU with the highest communication rate in each time slot. For the single-CU scenario, the received signal of the CU with M receive antennas in (3) can be rewritten as

$$\mathbf{y} = \mathbf{HBd} + \mathbf{z}, \tag{28}$$

where $\mathbf{H} \in \mathcal{C}^{M \times N_t}$ is the channel matrix between the dual-functional BS and the CU, $\mathbf{B} \in \mathcal{C}^{N_t \times D}$ is the transmit beamforming matrix of the dual-functional BS, $\mathbf{d} \in \mathcal{C}^{D \times 1}$ is the CU's data vector with $D \leq \min\{N_t, M\}$ data streams distributed as $\mathcal{CN}(0, \mathbf{I}_D)$, and $\mathbf{z} \in \mathcal{C}^{M \times 1}$ is the AWGN with $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{I}_M)$. Then the rate of the CU communication can be calculated as

$$C = \log\det\left(\mathbf{I} + \mathbf{HBB}^H\mathbf{H}^H\right). \tag{29}$$

Furthermore, for the single-CU scenario, the received signal of the radar receiver in (7) can be rewritten as

$$\mathbf{y}_0 = \alpha_0 \mathbf{A}(\theta_0)\mathbf{Bd} + \mathbf{c} + \mathbf{z}_0. \tag{30}$$

Given the optimal receive beamforming vector in (9), the sensing SCNR of the MIMO radar can be given by

$$\gamma = \mathrm{tr}\left(\mathbf{\Phi}\mathbf{BB}^H\right), \tag{31}$$

where $\mathbf{\Phi}$ is given in (11).

Thus, the original problem (P2) to achieve the boundary of the achievable performance region of the DFRC system can be also rewritten as

$$\begin{aligned} \text{(P3)} \quad & \max_{\mathbf{B}} \quad C = \log\det\left(\mathbf{I} + \mathbf{HBB}^H\mathbf{H}^H\right) \\ & \text{s.t.} \quad \mathrm{tr}\left(\mathbf{BB}^H\right) \leq P_0 \\ & \qquad\;\; \gamma = \mathrm{tr}\left(\mathbf{\Phi}\mathbf{BB}^H\right) \geq \gamma_0 \end{aligned} . \tag{32}$$

Proposition 3. (Optimal beamforming for single-CU scenario) The optimal transmit beamforming of the dual-functional BS for the single-CU scenario can be given by

$$\mathbf{B}^* = (\lambda\mathbf{I} - \mu\mathbf{\Phi})^{-1/2}\tilde{\mathbf{V}}\tilde{\mathbf{\Lambda}}^{1/2}, \tag{33}$$

where $\tilde{\mathbf{V}} \in \mathcal{C}^{M \times L}$ is the right singular matrix of $\mathbf{H}(\lambda\mathbf{I} - \mu\mathbf{\Phi})^{-1/2}$, i.e., $\tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H = \mathbf{H}(\lambda\mathbf{I} - \mu\mathbf{\Phi})^{-1/2}$, $\tilde{\mathbf{\Lambda}} = \mathrm{diag}\{\tilde{p}_1, \cdots, \tilde{p}_M\}$ with $\tilde{p}_l = \left(1 - 1/\tilde{h}_l\right)^+, \forall l$ and $\tilde{\mathbf{\Sigma}} = \mathrm{diag}\{\tilde{h}_1, \cdots, \tilde{h}_L\}$, λ and μ are dual variables associated to the transmit power constraint and the sensing SCNR constraint of the MIMO radar. Then the optimal receive beamforming for the CU is

$$\mathbf{D}^* = \mathbf{B}^{*H}\mathbf{H}^H\left(\mathbf{HB}^*\mathbf{B}^{*H}\mathbf{H}^H + \mathbf{I}\right)^{-1}, \tag{34}$$

where $\mathbf{B}^*$ is given in (33).

Proof. The proof is given in Appendix C. □

It can be seen that the optimal transmit beamforming has a semi-closed-form expression in terms of the dual variables λ and μ, which can be also achieved by the two-dimensional bisection search in Algorithm 2. Then, the optimal receive beamforming in (34) can be calculated from the optimal transmit beamforming. It avoids the iterative solution for transceiver design, which reduces the complexity. The iteration numbers of the dual variables bisection search for Algorithm 2 is determined by $\max\{L_1, L_2\}$, where $L_1 = \log(\lambda_{\max} - \lambda_{\min}) - \log \varepsilon_\lambda$ and $L_2 = \log(\mu_{\max} - \mu_{\min}) -$

$\log \varepsilon_\mu$. The time complexity of $\mathbf{B}^*$ computation in (33) and $\mathbf{D}^*$ computation in (34) are $O\left(N_t^3\right)$ and $O\left(M^3\right)$, respectively. Thus, the algorithm complexity of the DFRC transmit beamforming design for the single-CU scenario is given by $O\left(\max\left\{L_1, L_2\right\}\left(\max\left\{N_t, M\right\}\right)^3\right)$. Compared to the problem (P2), the algorithm complexity is reduced by a factor of L_3, i.e., the number of transceiver iterations.

B. Block Diagonalization Design

The key idea of block diagonalization is to design the transmit beamforming to eliminate the interference between different CUs. Thus, we have to impose additional constraints as follows.

$$\mathbf{H}_i \mathbf{B}_k = \mathbf{0}, \text{for} \quad i \neq k, \forall i, k \in \{1, \cdots, K\}. \tag{35}$$

In order to satisfy the above constraints, the transmit beamforming of the k-th CU should be in the null space of the channel matrix of the other CUs, i.e.,

$$\begin{aligned} \hat{\mathbf{H}}_k &= \left[\mathbf{H}_1^T, \cdots, \mathbf{H}_{k-1}^T, \mathbf{H}_{k+1}^T, \cdots, \mathbf{H}_K^T\right]^T \\ &\overset{(a)}{=} \hat{\mathbf{U}}_k \hat{\mathbf{\Sigma}}_k \left[\hat{\mathbf{V}}_k^{(1)}, \hat{\mathbf{V}}_k^{(0)}\right]^H \end{aligned}, \tag{36}$$

where the procedure (a) is the Singular Value Decomposition (SVD) of $\hat{\mathbf{H}}_k$, and $\hat{\mathbf{V}}_k^{(0)}$ forms an orthogonal basis for the null space of $\hat{\mathbf{H}}_k$, which includes the last $N_t - \mathrm{rank}(\hat{\mathbf{H}}_k)$ right singular vectors.

Furthermore, in order to formulate block channel for each CUs, the modified channel matrix of the k-th CU can be given by

$$\mathbf{H}_k \hat{\mathbf{V}}_k^{(0)} = \bar{\mathbf{U}}_k \bar{\mathbf{\Sigma}}_k \bar{\mathbf{V}}_k^H, \tag{37}$$

where $\bar{\mathbf{\Sigma}}_k = \mathrm{diag}\left\{h_{k,1}, \cdots, h_{k,T}\right\}$. Then, the block diagonalization transmit beamforming and receive beamforming can be designed as

$$\mathbf{B}_k^{(\mathrm{BD})} = \hat{\mathbf{V}}_k^{(0)} \bar{\mathbf{V}}_k \mathbf{\Lambda}_k^{\frac{1}{2}}, \tag{38}$$

$$\mathbf{D}_k^{(\mathrm{BD})} = \bar{\mathbf{U}}_k^H, \tag{39}$$

where $\mathbf{\Lambda}_k = \mathrm{diag}\left\{p_{k,1}, \cdots, p_{k,T}\right\}$. Thus, the received signal processed by the receive beamforming of the k-th CU is given by

$$\begin{aligned} \hat{\mathbf{d}}_k &= \mathbf{D}_k^{(\mathrm{BD})}\left(\mathbf{H}_k \sum_{k=1}^{K} \mathbf{B}_k^{(\mathrm{BD})} \mathbf{d}_k + \mathbf{z}_k\right) \\ &= \bar{\mathbf{\Sigma}}_k \mathbf{\Lambda}_k^{\frac{1}{2}} \mathbf{d}_k + \bar{\mathbf{U}}_k^H \mathbf{z}_k \end{aligned}, \tag{40}$$

and the corresponding sum rate of the MU-MIMO communication in (6) can be rewritten as

$$C = \sum_{k=1}^{K} \sum_{i=1}^{T} \log_2\left(1 + \frac{h_{k,i}^2 p_{k,i}}{\sigma_k^2}\right). \tag{41}$$

Then, with the block diagonalization transmit beamforming, the sensing SCNR of the MIMO radar in (10) can be rewritten as

$$\begin{aligned} \gamma &= \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{\Phi} \mathbf{B}_k \mathbf{B}_k^H\right) \\ &= \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{\Phi} \hat{\mathbf{V}}_k^{(0)} \bar{\mathbf{V}}_k \mathbf{\Lambda}_k \bar{\mathbf{V}}_k^H \hat{\mathbf{V}}_k^{(0)H}\right) \\ &= \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{\Lambda}_k \mathbf{\Upsilon}_k\right) \\ &= \sum_{k=1}^{K} \sum_{i=1}^{T} p_{k,i} \kappa_{k,i} \end{aligned}, \tag{42}$$

where $\mathbf{\Upsilon}_k = \bar{\mathbf{V}}_k^H \hat{\mathbf{V}}_k^{(0)H} \mathbf{\Phi} \hat{\mathbf{V}}_k^{(0)} \bar{\mathbf{V}}_k$ and $\kappa_{k,i}$ is the i-th diagonal element of $\mathbf{\Upsilon}_k$.

Thus, the original problem (P2) to achieve the boundary of the achievable region of DFRC can be rewritten as

$$\begin{aligned} \text{(P4)} \quad \max_{\{p_{k,i}\}} \quad & C = \sum_{k=1}^{K} \sum_{i=1}^{T} \log_2\left(1 + \frac{h_{k,i}^2 p_{k,i}}{\sigma_k^2}\right) \\ \text{s.t.} \quad & \gamma = \sum_{k=1}^{K} \sum_{i=1}^{T} p_{k,i} \kappa_{k,i} \geq \gamma_0 \\ & \sum_{k=1}^{K} \sum_{i=1}^{T} p_{k,i} \leq P_0 \end{aligned}. \tag{43}$$

The transmit beamforming design of the dual-functional BS in the problem (P2) becomes to the power allocation problem in the problem (P4) based on block diagonalization. The corresponding optimal power allocation can be given as follows.

Proposition 4. (**Optimal power allocation based on block diagonalization**) The optimal power allocation of the dual-functional BS to the problem (P4) based on block diagonalization can be given by

$$p_{k,i} = \left[\frac{1}{\log 2\left(\lambda - \mu \kappa_{k,i}\right)} - \frac{\sigma_k^2}{h_{k,i}^2}\right]^+, \tag{44}$$

where λ and μ are dual variables associated to the transmit power constraint and the sensing SCNR constraint of the MIMO radar.

Proof. The proof is similar to that of the conventional water-filling power allocation, which is omitted for conciseness. □

It can be seen that the transmit beamforming design of the dual-functional BS based on block diagonalization is determined by the SVD of the modified channel matrix in (37). The power allocation has a semi-closed-form expression in terms of the dual variables λ and μ, which can be also achieved by the two-dimensional bisection search in Algorithm 2. The iteration numbers of the dual-variable bisection search for Algorithm 2 is determined by $\max\left\{L_1, L_2\right\}$, where $L_1 = \log\left(\lambda_{\max} - \lambda_{\min}\right) - \log \varepsilon_\lambda$ and $L_2 = \log\left(\mu_{\max} - \mu_{\min}\right) - \log \varepsilon_\mu$. The time complexity of SVD for the modified channel matrix of the k-th CU is $O\left(\max\left\{N_t, M_k\right\}^3\right)$. Thus, the time complexity of the transceiver beamforming design for the DFRC based on block diagonalization can be given by $O\left(\max\left\{L_1, L_2\right\} K \max\left\{N_t, M_1, \cdots, M_k\right\}^3\right)$. Compared to

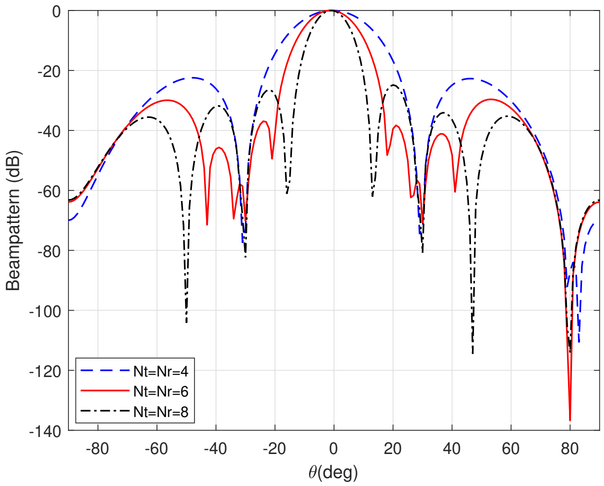


Figure 4. The optimized beampatterns with different numbers of the radar antennas, $N_t = N_r = 4, 6, 8$.

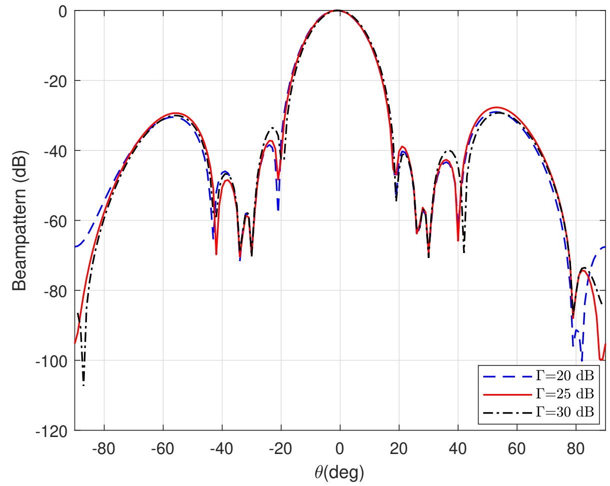


Figure 5. The optimized beampatterns with different CU's SCNR constraint, $\Gamma = 20, 25, 30$ dB.

the problem (P2), the algorithm complexity is also reduced by a factor of L_3, which is the number of transceiver iterations.

V. Simulation Results and Discussion

In this section, we evaluate the performance of our proposed design via simulation. The dual-functional BS and the radar receiver are assumed to be equipped with uniform linear arrays (ULAs) with the same number of elements. And the interval between adjacent antennas of the dual-functional BS and the radar receiver is assumed to be half-wavelength. The parameter settings are provided as follows unless specified otherwise. The transmit signal-power-to-noise ratio is set as $P_0 = 30$ dB. The numbers of antennas for the dual-functional BS, the radar receiver and the CU are $N_t = 6$, $N_r = 6$, and $M_k = 2, \forall k$, respectively. The communication channels are assumed to be Rayleigh fading, and the elements of the channel matrices are as i.i.d. complex Gaussian random variables $\mathcal{CN}(0, 1)$. A target is located at the spatial angle $\theta_0 = 0°$ with channel power gain $|\alpha_0|^2 = 10$ dB, and two fixed clutter signals are located at the spatial angels $\theta_1 = -30°$ and $\theta_2 = 30°$, which are both distributed as $\mathcal{CN}(0, \mathbf{I})$. Different beamforming designs are evaluated, where "SCA" refers to the iterative beamforming design based on SCA method, "MSE" refers to the iterative beamforming design based on weighted MSE criterion, "BLK" is the low-complexity beamforming design based on block diagonalization, "SUR" and "SUO" are the low-complexity beamforming designs for single-CU scenario based on round-robin scheduling and opportunistic scheduling, respectively. All simulation results are obtained using 10^3 Monte-Carlo simulations.

In Fig. 4, the optimized beampatterns for different numbers of the radar antenna are shown under the constraint of the CU's SCNR $\Gamma = 20$ dB. The performance of the MU-MIMO communication is determined by the constraint of the CU's SCNR. We define the optimized beampattern as $P(\theta) = |\mathbf{w}^{*H}\mathbf{A}(\theta)\mathbf{x}|^2$, where $\mathbf{w}^*$ is the receive beamforming vector of the MIMO radar for SCNR maximization in (9). The main

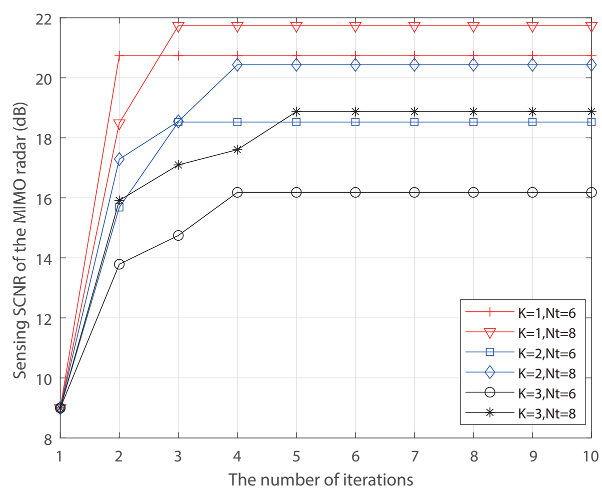


Figure 6. Convergence performance of the proposed SCA algorithm with different numbers of the CUs $K = 1, 2, 3$ and different numbers of the radar antennas $N_t = N_r = 6, 8$.

beam is located at the target's spatial angle $\theta_0 = 0°$, and the nulls are placed at the clutters' spatial angles $\theta_1 = -30°$ and $\theta_2 = 30°$. When the number of the radar antennas increases, the performance of beampattern becomes better from the radar's viewpoint. Specifically, the maximum peak to sidelobe ratio decreases with the increase of the number of the radar antennas. And the main beam width also decreases with the increase of the number of the radar antennas. Then, the optimized beampatterns under different CU's SCNR constraint are illustrated in Fig. 5. When the SCNR constraint of the CU increases, the shapes of the beampatterns are almost the same. Particularly, both the peak-to-sidelobe ratio and the main bean width are unchanged. Thus, the impact of the MU-MIMO communication to the MIMO radar is trivial in terms of the beampattern, which means that we can still push information with an ideal radar beampattern.

We evaluate the convergence performance of the proposed

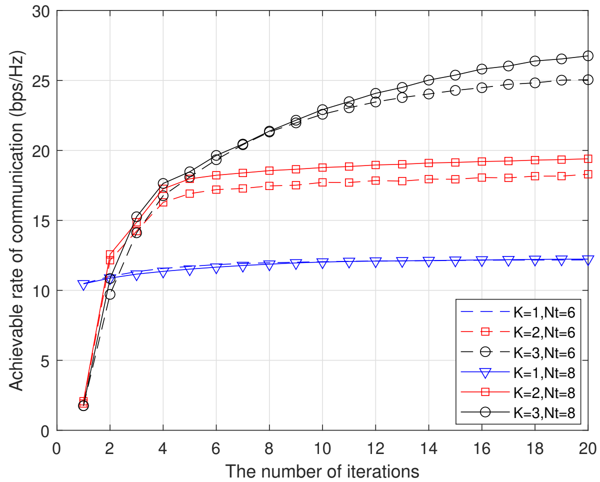


Figure 7. Convergence performance of the proposed weighted MSE algorithm with different numbers of the CUs $K = 1, 2, 3$ and different numbers of the radar antennas $N_t = N_r = 6, 8$.

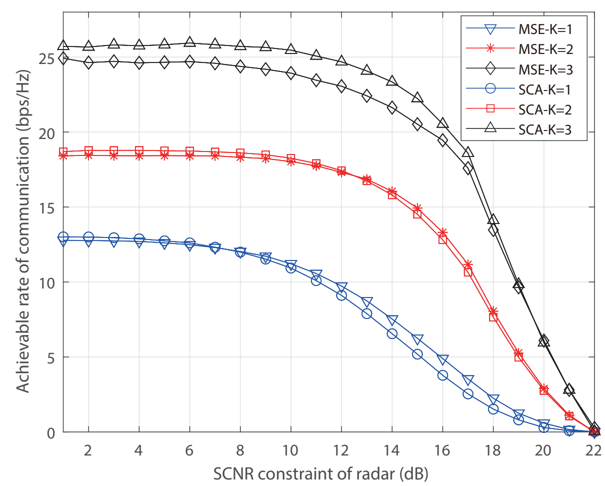


Figure 8. Tradeoff between the achievable rate of the communication and the SCNR constraint of the MIMO radar with different numbers of the CUs $K = 1, 2, 3$.

SCA algorithm and the weighted MSE algorithm for different numbers of the CUs and different numbers of the radar antennas in Fig. 6 and Fig. 7, respectively. For the proposed SCA method in Algorithm 1, the sensing SCNR of the MIMO radar versus the number of iterations is provided. It can be seen that sensing SCNR of the MIMO radar will always increase during the iteration. This guarantees the convergence of the proposed SCA algorithm. For the proposed weighted MSE criterion in Algorithm 3, the achievable rate of the communication will always increase during the iteration. Thus, the convergence of the proposed weighted MSE algorithm is also guaranteed. It can be also seen that the number of iterations increases with the increase of the number of CUs and the number of the dual-functional BS antennas for both the SCA algorithm and weighted MSE algorithm. Specifically, for the single-CU scenario, i.e., $K = 1$, the achievable rate of the communication vibrates about 6 iterations and converges to a fixed value. When the number of the CUs increases to $K = 3$, the number of iterations increases to about 20. That is because the searching space of the optimization becomes larger with the increase of the number of CUs or the number of the dual-functional BS antennas.

In Fig. 8, the tradeoff between the achievable rate of the communication and the SCNR constraint of the MIMO radar is illustrated for different numbers of the CUs, where both the proposed SCA algorithm and weighted MSE algorithm are considered. The performances of these two algorithms are almost the same, when the number of CUs is $K = 1, 2$. When the number of the CUs is $K = 3$, the proposed SCA algorithm outperforms the proposed weighted MSE algorithm at the price of high algorithm complexity. It is easy to identify the boundary point to divide the tradeoff boundary into dual-constrained and communication-constrained sections. Specifically, when the SCNR constraint of the MIMO radar is below 4 dB, this constraint does not affect the achievable rate of the communication, which is fixed at about 13 bps/Hz for the

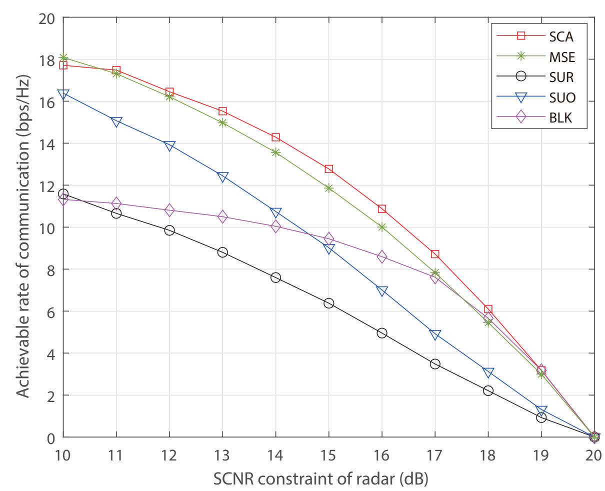


Figure 9. Tradeoff between the achievable rate of the MU-MIMO communication and the SCNR constraint of the MIMO radar with different beamforming designs.

single-CU scenario. For the $K = 2$ scenario, when the SCNR constraint of the MIMO radar is below 8 dB, the achievable rate of the communication is fixed at about 18 bps/Hz, where the performance region boundary is in the communication-constrained section. When the SCNR constraint of the MIMO radar is above 20 dB, the corresponding problem becomes infeasible. Thus, the achievable rate of the communication are all 0 with different numbers of the CUs. Note that the boundary point to divide the tradeoff boundary into dual-constrained and radar-constrained sections is not clear. That is because the simulation results are obtained by using 10^3 Monte-Carlo simulations.

We evaluate the the tradeoff between the achievable rate of the communication and the SCNR constraint of the MIMO radar with different beamforming designs in Fig. 9 for the $K = 2$ scenario. It can be seen that "SCA" design achieves

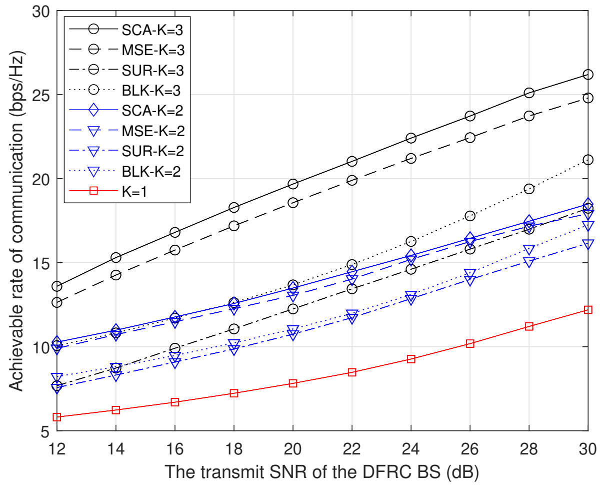


Figure 10. Achievable rate of the communication versus different transmit SNR $10 \sim 30$ dB with different number of CUs $K = 1, 2, 3$ and different beamforming designs.

the best performance compared to the other designs at the price of the high complexity. The performance of "MSE" design is almost the same as that of "SCA" design. The performance of "SUO" design is better than that of "SUR" design at the cost of CUs scheduling complexity. The performance of "BLK" design is worse than that of "SCA" design and "MSE" design, because it only allocates the power of different antennas with zero-forcing inter-CU interference. It can be also seen that the "BLK" design performs better than "SUO" design when the SCNR constraint of the MIMO radar is large. When the SCNR constraint of the MIMO radar is small, the performance of "SUO" design is better than that of "BLK" design.

In Fig. 10, the achievable rate of the communication is provided versus different transmit SNR $10 \sim 30$ dB of the dual-functional BS for different number of CUs $K = 1, 2, 3$ and different beamforming designs. It can be seen that the achievable rate of the communication almost linearly increases with the transmit SNR of the dual-functional BS. The increasing rate increases with the number of the CUs. It is can be seen that the increasing rate of $K = 3$ scenario is larger than that of $K = 1, 2$ scenarios. The low-complexity beamforming designs, i.e., "BLK", "SUO" and "SUR", will incur an almost constant performance loss compared to the iterative beamforming designs, i.e., "SCA" and "MSE". And the increasing rate with different transmit SNR is almost the same for different beamforming designs with the same number of CUs.

VI. Conclusion

In this paper, we have extended the DFRC beamforming designs to a general scenario, where it simultaneously detects the target as a MIMO radar and communicates to multiple multi-antenna CUs. In order to achieve the boundary of the achievable performance region for the DFRC system, both radar-centric and communication-centric optimizations have been formulated. For the radar-centric formulation, an iterative solution based on SCA method has been proposed. For the communication-centric formulation, the weighted MSE criterion has been adopted to solve the non-convex problem. To avoid the complex iterations, we have further proposed two low-complexity beamforming designs. Through the CU selection, we have derived the semi-closed-form expression of the optimal beamforming for the single-CU scenario. Furthermore, by zero-forcing multi-CU interference, a block diagonalization beamforming design and the corresponding optimal power allocation have been provided.

Appendix A
Proof of Proposition 1

Following the argument for the MU-MIMO communication in interference channel [42] and full-duplex channel [43], the Lagrangian of the problem (P2) and the problem (P2.1) can be given by

$$\mathcal{L}_0\left(\mathbf{B}_k,\lambda,\mu\right)=-C+\lambda\left(\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{B}_k\mathbf{B}_k^H\right)-P_0\right)-\mu\left(\gamma-\gamma_0\right), \tag{45}$$

and

$$\mathcal{L}_1\left(\mathbf{B}_k,\lambda,\mu\right)=E+\lambda\left(\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{B}_k\mathbf{B}_k^H\right)-P_0\right)-\mu\left(\gamma-\gamma_0\right), \tag{46}$$

respectively, where $\lambda > 0$ and $\mu > 0$ are dual variables of the transmit power constraint and the sensing SCNR constraint of the MIMO radar, respectively. According to C in (6), E in (23) and γ in (10), one has that

$$\frac{\partial\mathcal{L}_0\left(\mathbf{B}_k,\lambda,\mu\right)}{\partial\mathbf{B}_k}=\frac{\partial\mathcal{L}_1\left(\mathbf{B}_k,\lambda,\mu\right)}{\partial\mathbf{B}_k},\forall k, \tag{47}$$

if the weight matrix satisfies

$$\mathbf{W}_k=\frac{\mathbf{E}_k^{-1}}{\log 2},\forall k. \tag{48}$$

Note that

$$\begin{array}{c}\dfrac{\partial\mathcal{L}_0\left(\mathbf{B}_k,\lambda,\mu\right)}{\partial\lambda}=\dfrac{\partial\mathcal{L}_1\left(\mathbf{B}_k,\lambda,\mu\right)}{\partial\lambda}\\ \dfrac{\partial\mathcal{L}_0\left(\mathbf{B}_k,\lambda,\mu\right)}{\partial\mu}=\dfrac{\partial\mathcal{L}_1\left(\mathbf{B}_k,\lambda,\mu\right)}{\partial\mu}\end{array}, \tag{49}$$

the KKT conditions of the problem (P2) and the problem (P2.1) are identical.

Furthermore, substituting (48) into the problem (P2.1) and ignoring the constant term, it can be rewritten as

$$\begin{array}{lll}(\text{P2.2}) & \min\limits_{\{\mathbf{D}_k\}\{\mathbf{B}_k\}} & -\log\det\left(\mathbf{E}_k^{-1}\right)\\ & \text{s.t.} & \sum\limits_{k=1}^{K}\mathrm{tr}\left(\mathbf{B}_k\mathbf{B}_k^H\right)\leq P_0\\ & & \gamma=\sum\limits_{k=1}^{K}\mathrm{tr}\left(\mathbf{\Phi}_0\mathbf{B}_k\mathbf{B}_k^H\right)\geq\gamma_0\end{array}. \tag{50}$$

The optimal receiver beamforming of the k-th CU with fixed transmit beamforming can be calculated as $\partial E/\partial \mathbf{D}_k$=$\partial \mathrm{tr}\left(\mathbf{W}_k \mathbf{E}_k\right)/\partial \mathbf{D}_k$=0. Thus, one has that

$$\mathbf{D}_k^* = \mathbf{B}_k^H \mathbf{H}_k^H \left(\mathbf{H}_k \mathbf{B}_k \mathbf{B}_k^H \mathbf{H}_k^H + \mathbf{R}_k\right)^{-1}, \tag{51}$$

and the corresponding MSE matrix can be given by

$$\mathbf{E}_k = \left(\mathbf{I} + \mathbf{B}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{B}_k\right)^{-1}. \tag{52}$$

and then the sum rate in (6) can be rewritten as

$$C = \sum_{k=1}^{K} \log\det\left(\mathbf{E}_k^{-1}\right). \tag{53}$$

Thus, the problem (P2) is identical to the problem (P2.1) if the weight matrix satisfies (48), which completes the proof.

Appendix B
Proof of Proposition 2

With fixed $\{\mathbf{B}_k\}$, the problem (P2.1) is convex over $\{\mathbf{D}_k\}$ and the optimal $\{\mathbf{D}_k\}$ can be derived by calculating $\partial E/\partial \mathbf{D}_k = \mathbf{0}$, i.e.,

$$\begin{aligned}\frac{\partial E}{\partial \mathbf{D}_k} &= \frac{\partial \mathrm{tr}\left(\mathbf{W}_k \mathbf{E}_k\right)}{\partial \mathbf{D}_k} \\ &= 2\mathbf{W}_k \left[\mathbf{H}_k \left(\sum_{i=1}^{K} \mathbf{B}_i \mathbf{B}_i^H\right) \mathbf{H}_k^H \mathbf{D}_k + \mathbf{D}_k - \mathbf{B}_k^H \mathbf{H}_k^H\right]. \\ &= \mathbf{0}\end{aligned} \tag{54}$$

Thus, one has that

$$\begin{aligned}\mathbf{D}_k^* &= \mathbf{B}_k^H \mathbf{H}_k^H \left(\mathbf{H}_k \left(\sum_{i=1}^{K} \mathbf{B}_i \mathbf{B}_i^H\right) \mathbf{H}_k^H + \mathbf{I}\right), \\ &= \mathbf{B}_k^H \mathbf{H}_k^H \left(\mathbf{H}_k \mathbf{B}_k \mathbf{B}_k^H \mathbf{H}_k^H + \mathbf{R}_k\right)\end{aligned} \tag{55}$$

given $\mathbf{R}_k$ in (5). Also, with fixed $\{\mathbf{D}_k\}$, the problem (P1) is convex over $\{\mathbf{B}_k\}$. Then, the Lagrangian of problem (P2.1) can be given by

$$\begin{aligned}&\mathcal{L}_1\left(\mathbf{B}_k, \lambda, \mu\right) \\ &= E + \lambda \left(\sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{B}_k \mathbf{B}_k^H\right) - P_0\right) - \mu \left(\sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{\Phi} \mathbf{B}_k \mathbf{B}_k^H\right) - \gamma_0\right), \\ &= \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{\Psi}_k \mathbf{B}_k \mathbf{B}_k^H\right) - 2\mathrm{tr}\left(\mathbf{W}_k \mathbf{D}_k \mathbf{H}_k \mathbf{B}_k\right) - \lambda P_0 + \mu \gamma_0\end{aligned} \tag{56}$$

where

$$\mathbf{\Psi}_k = \mathbf{H}_k^H \mathbf{D}_k^H \mathbf{W}_k \mathbf{D}_k \mathbf{H}_k + \lambda \mathbf{I} - \mu \mathbf{\Phi}, \tag{57}$$

and $\lambda > 0$ and $\mu > 0$ are dual variables of the power constraint and the SCNR constraint of MIMO radar, respectively. Letting $\partial \mathcal{L}_1\left(\mathbf{B}_k, \lambda, \mu\right)/\partial \mathbf{B}_k = 0$, one has that

$$\begin{aligned}\mathbf{B}_k^* &= {\mathbf{\Psi}_k}^{-1} \mathbf{H}_k^H \mathbf{D}_k^H \mathbf{W}_k \\ &= \left(\mathbf{H}_k^H \mathbf{D}_k^H \mathbf{W}_k \mathbf{D}_k \mathbf{H}_k + \lambda \mathbf{I} - \mu \mathbf{\Phi}_0\right)^{-1} \mathbf{H}_k^H \mathbf{D}_k^H \mathbf{W}_k,\end{aligned} \tag{58}$$

which completes the proof.

Appendix C
Proof of Proposition 3

Assuming $\mathbf{F} = \mathbf{B}\mathbf{B}^H$, the problem (P3) can be rewritten as

$$\begin{aligned}(\text{P3.1}) \quad \max_{\mathbf{F}} \quad & C = \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{F}\mathbf{H}^H\right) \\ \text{s.t.} \quad & \mathrm{tr}\left(\mathbf{F}\right) \le P_0 \\ & \gamma = \mathrm{tr}\left(\mathbf{\Phi}\mathbf{F}\right) \ge \gamma_0\end{aligned} \quad . \tag{59}$$

The problem (P3.1) is a convex optimization problem, since the objective function is concave over $\mathbf{F}$ and its feasible set defined by the constraints a also convex set of $\mathbf{F}$. The Lagrangian of (P3.1) can be written as

$$\begin{aligned}\mathcal{L}' = &\log\det\left(\mathbf{I} + \mathbf{H}\mathbf{F}\mathbf{H}^H\right) - \lambda\left(\mathrm{tr}\left(\mathbf{F}\right) - P_0\right) \\ &+ \mu\left(\mathrm{tr}\left(\mathbf{\Phi}\mathbf{F}\right) - \gamma_0\right),\end{aligned} \tag{60}$$

where λ and μ are dual variables associated to the power and the radar SCNR constraints. Ignoring the constants, the Lagrangian of (P3.1) is equivalent to

$$\mathcal{L} = \log\det\left(\mathbf{I} + \mathbf{H}\mathbf{F}\mathbf{H}^H\right) - \mathrm{tr}\left(\mathbf{\Omega}\mathbf{F}\right), \tag{61}$$

where $\mathbf{\Omega} = \lambda \mathbf{I} - \mu \mathbf{\Phi}$. Since the optimal value of the problem (P3.1) is lower-bounded, it can be proved that $\mathbf{\Omega} \succeq 0$. Then the Lagrangian of (P3.1) can be rewritten as

$$\mathcal{L} = \log\det\left(\mathbf{I} + \tilde{\mathbf{H}}\tilde{\mathbf{F}}\tilde{\mathbf{H}}^H\right) - \mathrm{tr}\left(\tilde{\mathbf{F}}\right), \tag{62}$$

where

$$\tilde{\mathbf{H}} = \mathbf{H}\mathbf{\Omega}^{-1/2} = \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H, \tag{63}$$

and

$$\tilde{\mathbf{F}} = \tilde{\mathbf{V}}\mathrm{diag}\left\{\tilde{p}_1, \cdots, \tilde{p}_M\right\}\tilde{\mathbf{V}}^H = \tilde{\mathbf{V}}\tilde{\mathbf{\Lambda}}\tilde{\mathbf{V}}^H \tag{64}$$

with $\tilde{\mathbf{U}} \in \mathcal{C}^{N_r \times L}, \tilde{\mathbf{V}} \in \mathcal{C}^{M \times L}, \tilde{\mathbf{\Sigma}} = \mathrm{diag}\{\tilde{h}_1, \cdots, \tilde{h}_L\}$, and $\tilde{p}_l = \left(1 - 1/\tilde{h}_l\right)^+, \forall l$. Then the water-filling solution can be given by

$$\mathbf{B}^* = \mathbf{\Omega}^{-1/2}\tilde{\mathbf{V}}\tilde{\mathbf{\Lambda}}^{1/2} = \left(\lambda \mathbf{I} - \mu \mathbf{\Phi}\right)^{-1/2}\tilde{\mathbf{V}}\tilde{\mathbf{\Lambda}}^{1/2}, \tag{65}$$

and the corresponding receive beamforming for the CU can be calculated according to (26) with $\mathbf{R}_k = \mathbf{I}$, which completes the proof.

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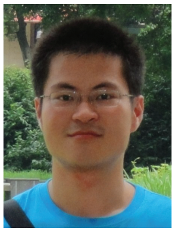

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