Generalizing a Closed-Form Correlation Model of Oriented Bandpass Natural Images

ZEINA SINNO – ALAN BOVIK

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Outline

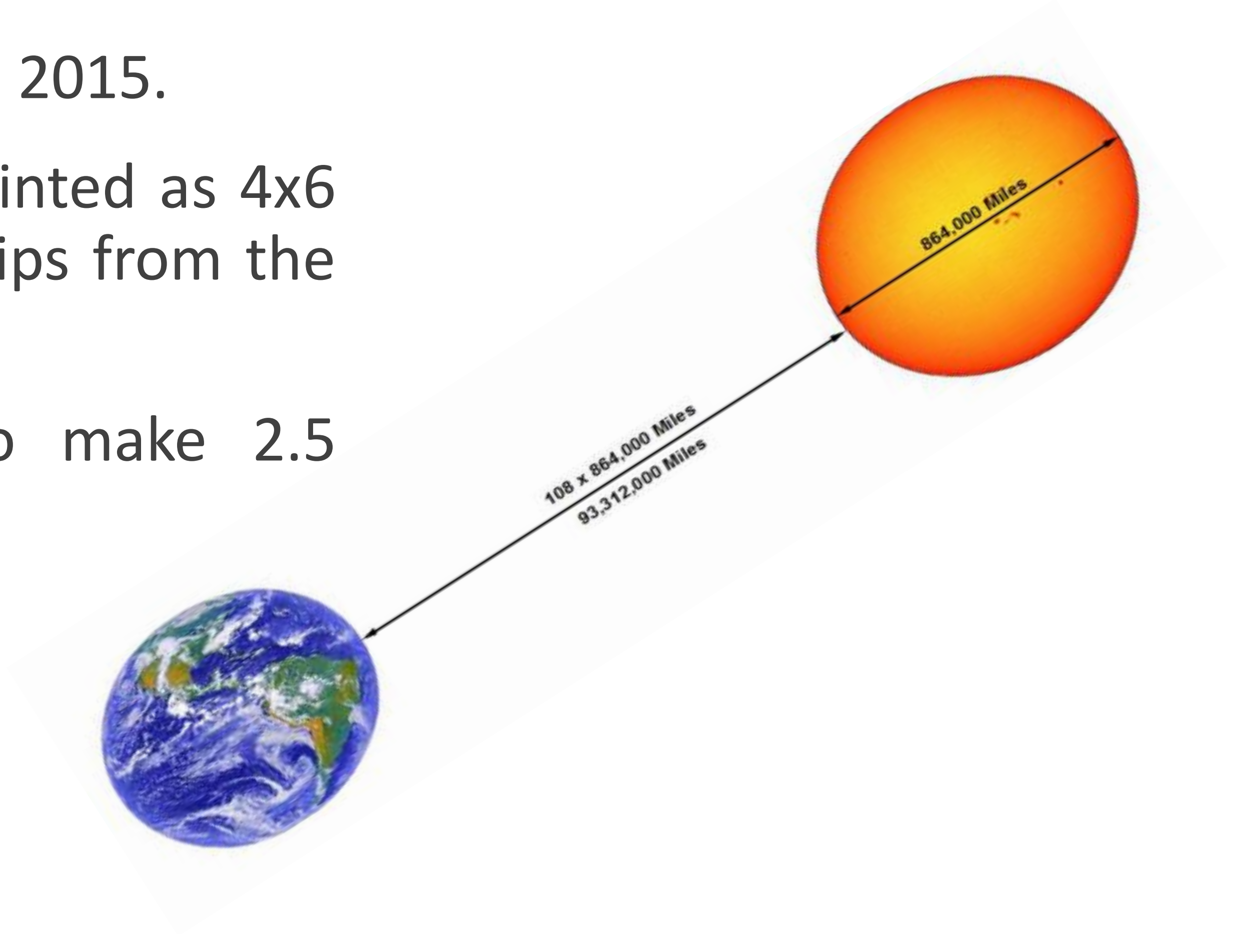
- Background
- The Basic Image Model
- The Generalized Model
- Validation of the Model
- Conclusion and Future Work

Photos to the Sun and Back

- **One trillion pictures** were captured in 2015.

- If the stored photos in a year are printed as 4x6 inches they would stretch 1.1 roundtrips from the Earth to the Sun.

- In 2017, the trail will stretch to make 2.5 roundtrips. [1]



[1]: Infotrends 2014 Worldwide Image Capture Forecast.

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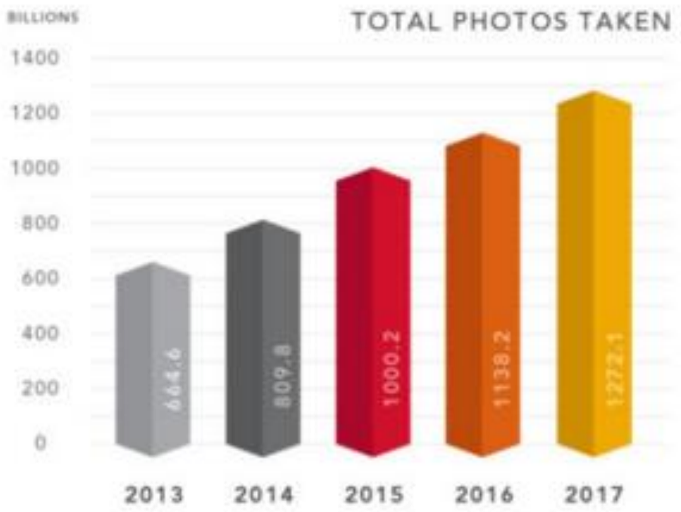


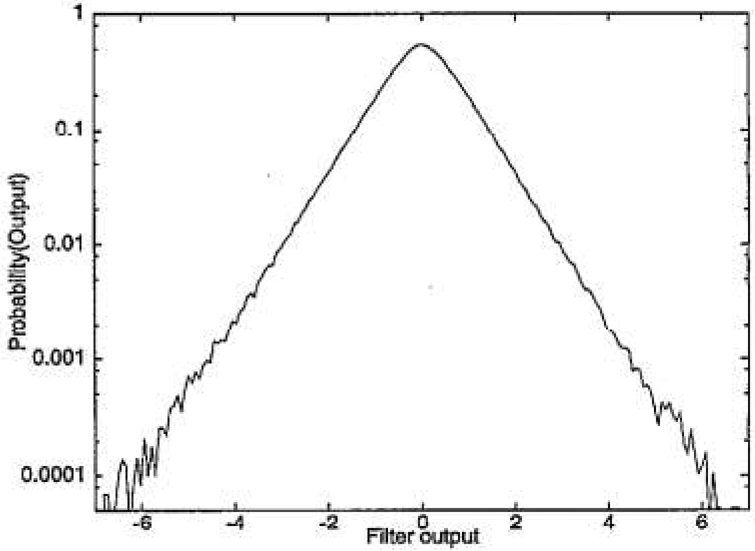
Understanding the **Natural Scene Statistics** is of a great importance.

[1]: Infotrends 2014 Worldwide Image Capture Forecast.

Natural Scene Statistics (NSS)

- Ruderman applied a simple form of "predictive coding" filter to a large numbers of images and observed that images follow an empirical probability distribution function.

- NSS follow **well behaved** and **universal models**.
- Combining NSS with Human Visual System provides **powerful tools**:
  - Image Denoising [2]
  - Image and Video Quality Assessment [3]
  - Image Defocus [4]
  - …



[1]: D.L., "The Statistics of Natural Images, *Nature*, 1994.

[2]: J. Perry and W. S. Geisler, "Natural scene statistics for image denoising," Center for Perceptual Systems, The University of Texas at Austin, Tech. Rep., 2013

[3]: Z. Wang, A. C. Bovik, H. Sheikh, and E. Simoncelli, "Image quality assessment: From error visibility to structural similarity," *IEEE Trans. on Imag. Proc.*, vol. 13, no. 4, pp. 600–612, 2004.

[4]: J. Burge and W. S. Geisler, "Optimal defocus estimation in individual natural images,"Proc. of the *Nat. Acad. of Sci*.", vol. 108, no. 40, pp. 16 849–16 854, 2011

Natural Scene Statistics (NSS)

Extending NSS to understand the relationship between two pixels, referred as Bivariate NSS and more pixels is **useful** for:

- Understanding correlated noise ➡ better denoising algorithms
- Estimating the depth from single images
- Modeling textures
- Estimating the quality of images more closely to human judgement
- …

Background

- Previous work on NSS focused mainly on Univariate NSS.

- Random processes were used to model Bivariate NSS.

➡ **No closed-form models!**

- Su *et. al* [1] modeled spatially adjacent bandpass image responses.

- Model proved to be useful for:
  - Depth Estimation from Single Images [2]
  - Stereoscopic Image quality Prediction [3]

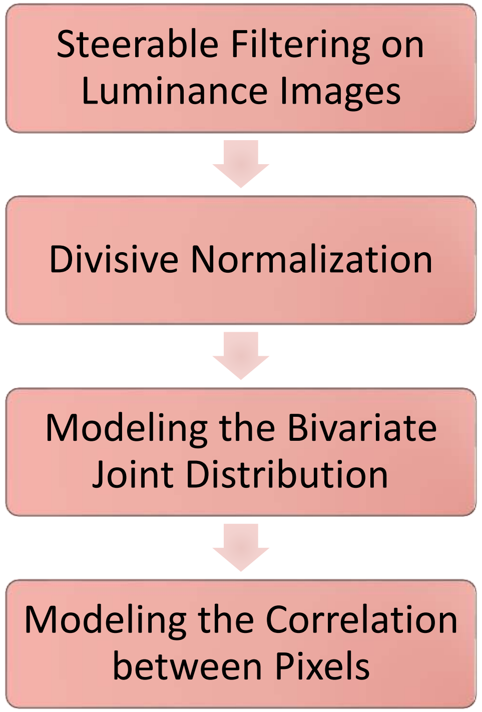
Goal: To extend this model over more pixels and more scales.

[1]: C. C. Su, L. Cormack, and A. C. Bovik, "Closed-form correlation model of oriented bandpass natural images," *Signal Processing Lett. IEEE*, vol. 22, no. 1, pp. 21–25, Jan 2015.
[2]: C. C. Su, L. K. Cormack, and A. C. Bovik, "Bivariate statistical modeling of color and range in natural scenes," in Proc. SPIE, *Human Vis. Electron. Imag. XIX*, vol. 9014, Feb. 2014.
[3]: C. Su, L. Cormack, and A. Bovik, "Oriented correlation models of distorted natural images with application to natural stereopair quality evaluation," *IEEE Trans. Image Process.*, vol. 24, no. 5, pp. 1685–1699, May 2015.

The Basic Image Model

Model constructed using 100 pristine images:

- 29 images from the LIVE Database [1]
- 71 images from the Berkeley Segmentation Database [2]



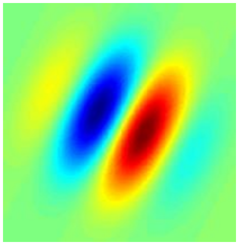
[1]: H. Sheikh, M. Sabir, and A. Bovik, “A statistical evaluation of recent full reference image quality assessment algorithms,”*IEEE Trans. Image Processing*, vol. 15, no. 11, pp. 3440–3451, Nov 2006

[2]: D. Martin, C. Fowlkes, D. Tal, and J. Malik, “A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics,” in *Int. Conf. Comput. Vision*, vol. 2. IEEE, 2001, pp. 416–423.

The Basic Image Model

- Luminance images considered as they relate better to the human perception of colors.

- Steerable filters were applied as a simple model of simple cells in the primary visual cortex (area V1).



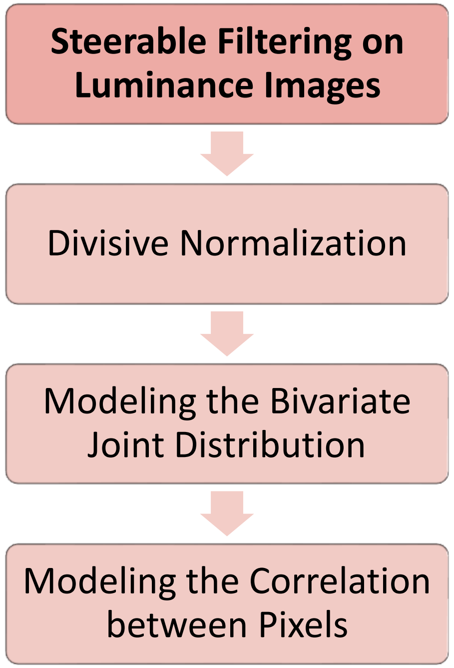
Simple Cells

- Steerable filters at an orientation θ_1 :

$$F(\theta_1) = cos(\theta_1)F_x + sin(\theta_1)F_y$$

F_x and F_y : Gradient of a 2D Gaussian of variance σ w.r.t to the x and y axis.

- Considered 15 different θ_1 spanning $[0,\pi)$ with steps of $\frac{\pi}{15}$.



The Basic Image Model

- Divisive Normalization was applied to model the non-linear adaptive gain control of neuronal responses in the area V1.

- It is represented as:

$$u(x_i, y_i) = \frac{w(x_i, y_i)}{\sqrt{s + \sum_j g(x_j, y_j) w(x_j, y_j)^2}}$$

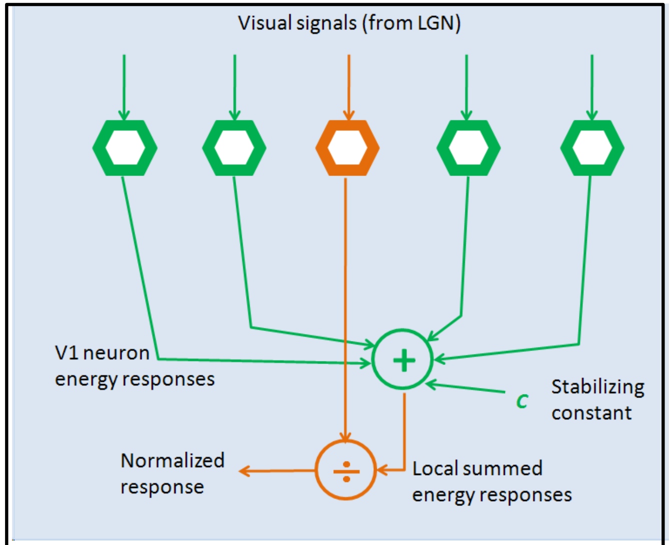
(x_i, y_i) : spatial coordinates

$u(x_i, y_i)$: coefficient after divisive normalization

$g(x_j, y_j)$: Gaussian weighting function

$w(x_i, y_i)$: steerable filters coefficients

s : stabilization constant



Divisive Normalization

Steerable Filtering on Luminance Images

Divisive Normalization

Modeling the Bivariate Joint Distribution

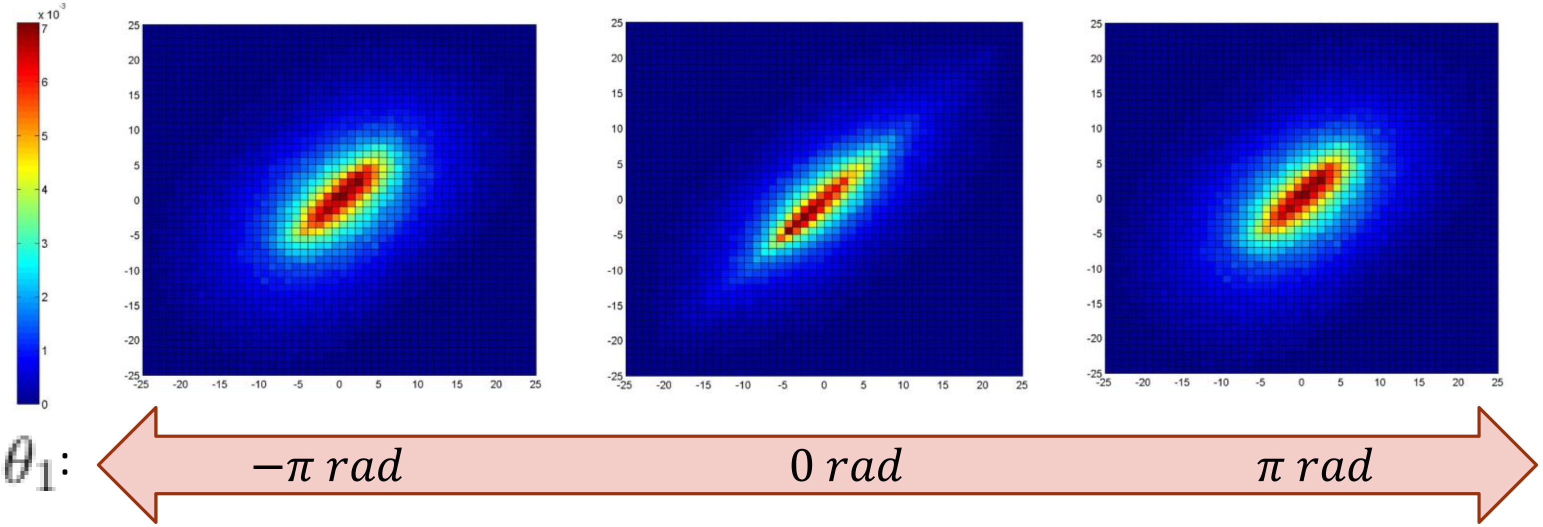
Modeling the Correlation between Pixels

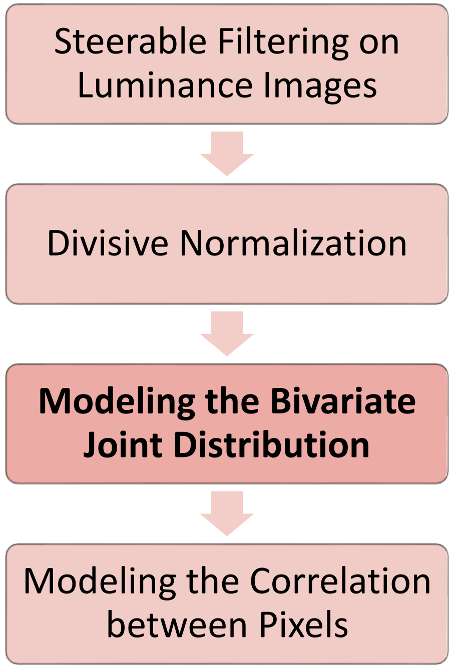
The Basic Image Model

- The bivariate joint distribution characterizes pixels between distance 1 to 10, at a spatial orientation θ_2.

- Define the relative orientation as: $\theta_2 - \theta_1$

$\theta_2 - \theta_1$:





The Basic Image Model

- This is modeled using a Bivariate Generalized Gaussian Distribution (BGGD):

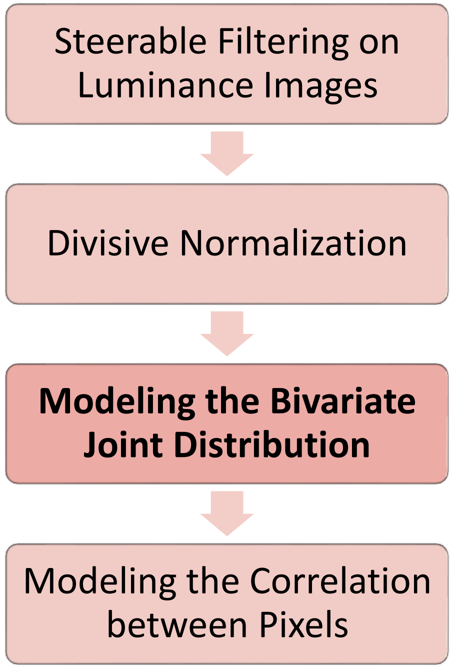
$$p(x; M, \alpha, \beta) = \frac{1}{|M|^{\frac{1}{2}}} g_{\alpha,\beta}(x^T M^{-1} x)$$

x : Pixels to be correlated

M : Scatter Matrix

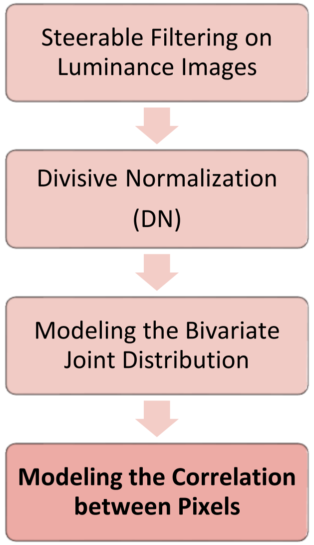
α, β : Shape Parameters

$g_{\alpha,\beta}$: Density generator function



The Generalized Image Model

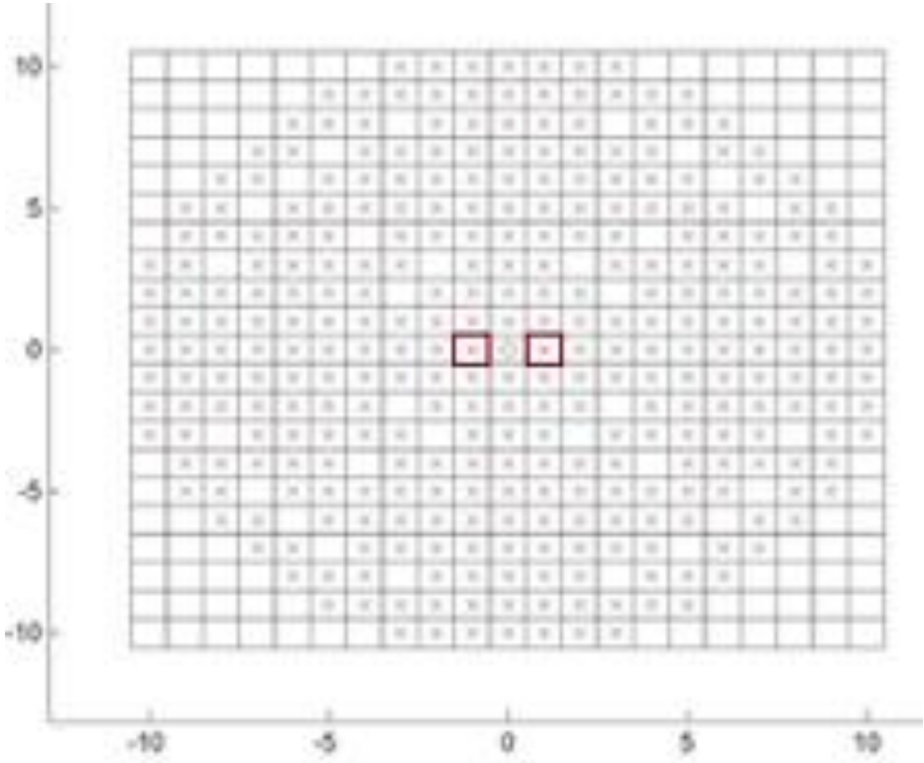
- Take the correlation between the pixels after the DN for:
  - a spatial separation in [0,10]
  - at all possible digital angles
  - for scales σ in [1,4] with steps of 0.5 (7 scales in total)
- Su *et. al* [1] considered horizontally adjacent pixels only and 5 scales.



[1]: C. C. Su, L. Cormack, and A. C. Bovik, "Closed-form correlation model of oriented bandpass natural images," *Signal Processing Lett. IEEE*, vol. 22, no. 1, pp. 21–25, Jan 2015.

The Generalized Image Model

- Su *et. al* 's Model [1]



[1]: C. C. Su, L. Cormack, and A. C. Bovik, “Closed-form correlation model of oriented bandpass natural images,” *Signal Processing Lett. IEEE*, vol. 22, no. 1, pp. 21–25, Jan 2015.

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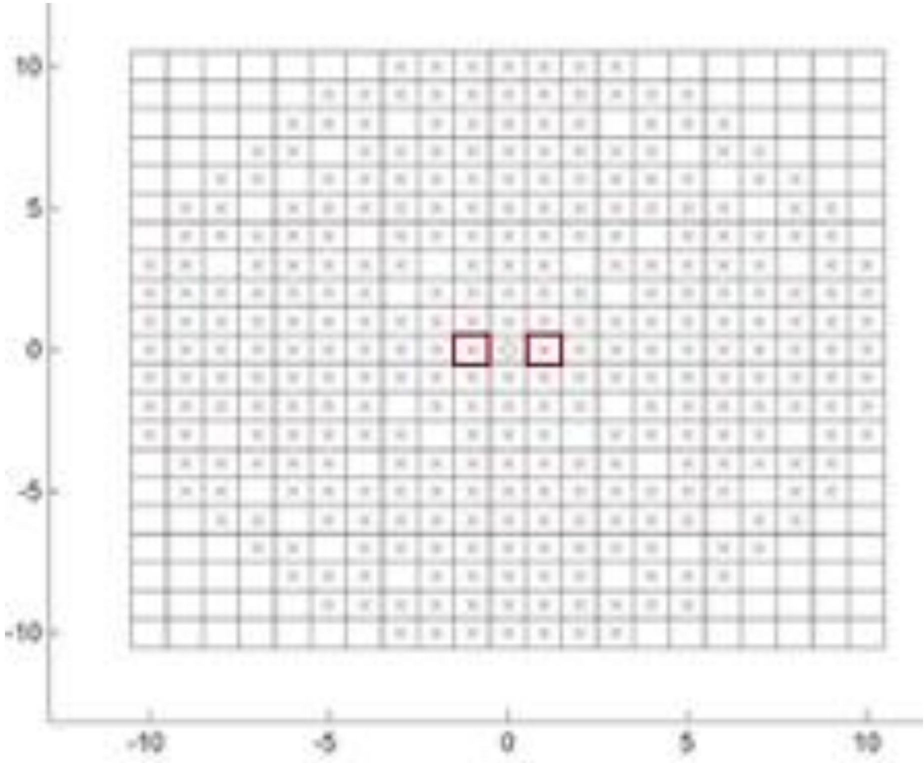

A More General Model

[1]: C. C. Su, L. Cormack, and A. C. Bovik, "Closed-form correlation model of oriented bandpass natural images," *Signal Processing Lett. IEEE*, vol. 22, no. 1, pp. 21–25, Jan 2015.

The Generalized Model

- Model the correlation as a function of $\theta_2 - \theta_1$:

$$\rho = A cos(\theta_2 - \theta_1) \quad + c$$

versus Su *et. al* 's model [1] :

$$\rho = A cos(\theta_2 - \theta_1)^{2\gamma} + c$$

The Generalized Model

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versus Su *et. al* 's model [1] :

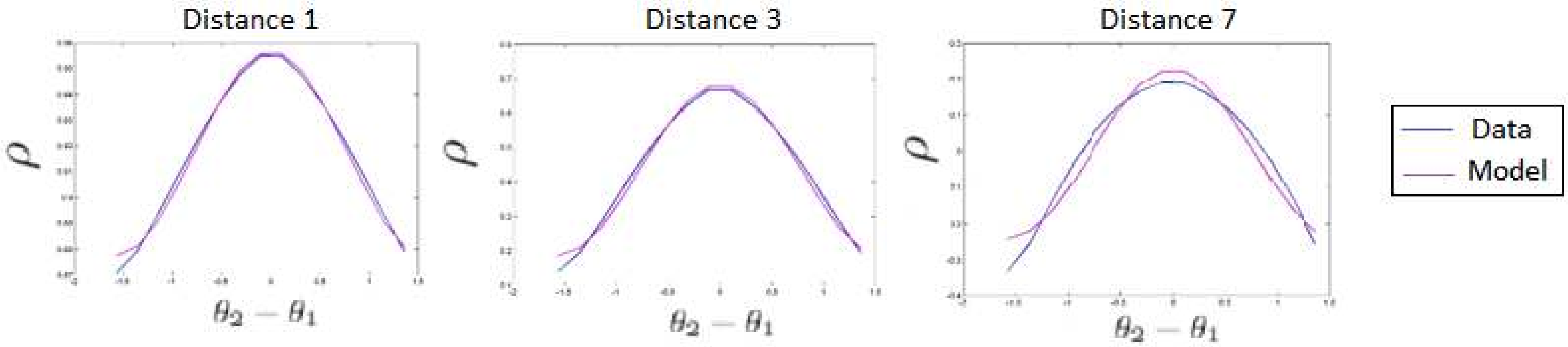
$$\rho = Acos(\theta_2 - \theta_1)^{2\gamma} + c$$

A Simpler Model

The Generalized Model

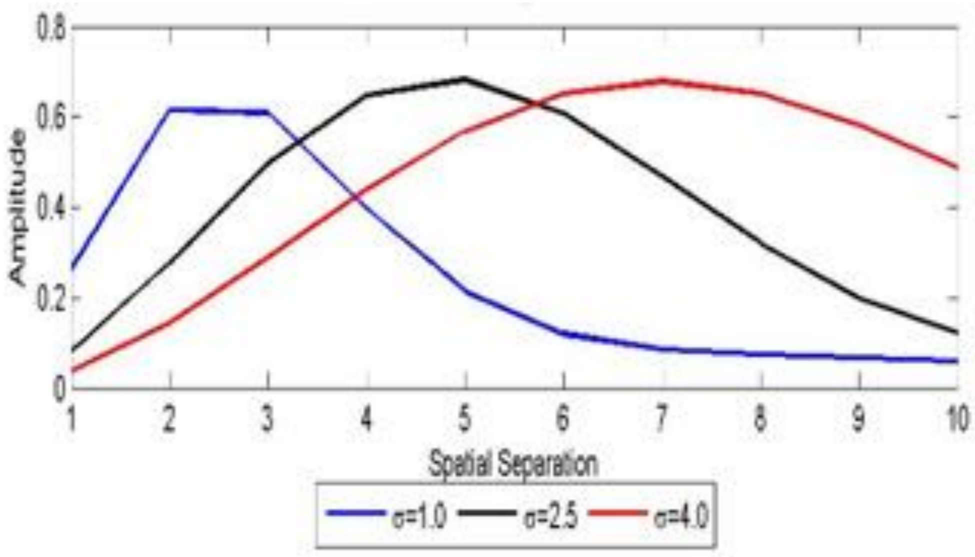
- Consistent model over all the scales and distances:

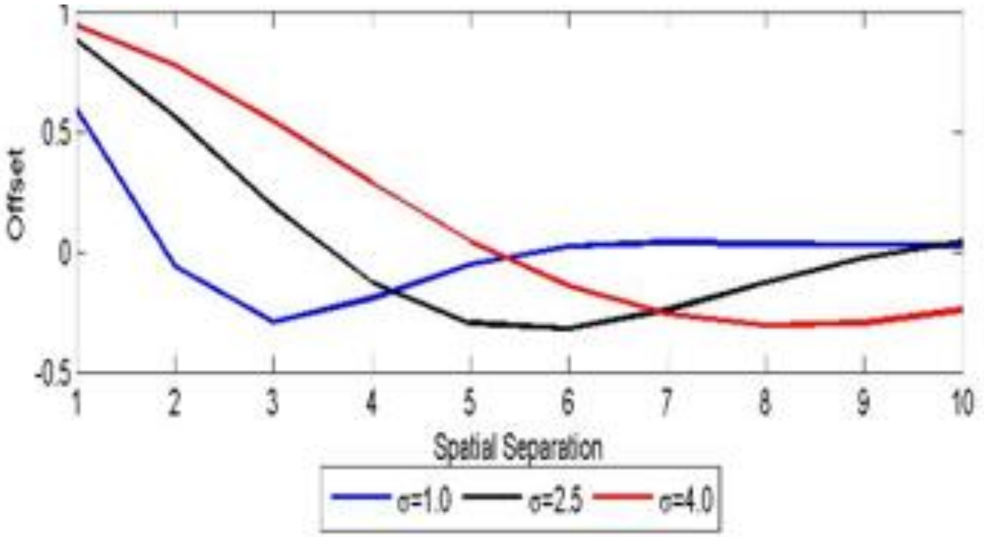
For $\sigma = 2.5$ and $\theta_2 = \frac{\pi}{2}$:



The Generalized Model

- Amplitude (A) and Offset (c) over multiple scales:





Validation of the Model

Model validated using Mean Squared Error (MSE) and Pearson's χ^2 test:

	MSE training	χ^2 training	MSE testing	χ^2 testing
Distance 1	0.004178	0.459442	0.001053	0.11582
Distance 3	0.166615	48.77902	0.038803	11.43254
Distance 5	0.364689	-27.6250	0.070773	-0.46758
Distance 7	0.323636	290.3075	0.057003	47.40307

Training Data: the 100 images used to build the model

Testing Data: the 23 pristine images of the VCL@FER database[1]

[1]: A. Zaric, N. Tatalovic, N. Brajkovic, H. Hlevnjak, M. Loncaric, E. Dumic, and S. Grgic, "Vcl@ fer image quality assessment database," *AUTOMATIKA*, vol. 53, no. 4, pp. 344–354, 2012.

Future Work

- Find a closed form for the amplitude and offset.

- Extend the model over more scales.

- Apply the model to problems including:
  - texture modeling
  - image quality prediction
  - image interpolation
  - …

Thank you

Backup Slide

Density Generator Function:

$$g_{\alpha,\beta}(y) = \frac{\beta\Gamma(\frac{N}{2})}{(2^{\frac{1}{\beta}}\pi\alpha)^{\frac{N}{2}}\Gamma(\frac{N}{2\beta})} e^{-\frac{1}{2}(\frac{y}{\alpha})^{\beta}}$$

Γ : the Digamma function

$y \in \mathbb{R}^{+}$

α, β : Shape Parameters