Generating All Circular Shifts by Context-Free Grammars in Chomsky Normal Form

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Abstract — Let $\{a_1, a_2, \ldots, a_n\}$ be an alphabet of n symbols and let C_n be the language of circular shifts of the word $a_1a_2\cdots a_n$; so $C_n = \{a_1a_2\cdots a_{n-1}a_n, a_2a_3\cdots a_na_1, \ldots, a_na_1\cdots a_{n-2}a_{n-1}\}$. We discuss a few families of context-free grammars G_n ($n \geq 1$) in Chomsky normal form such that G_n generates C_n. The grammars in these families are investigated with respect to their descriptional complexity, i.e., we determine the number of nonterminal symbols $\nu(n)$ and the number of rules $\pi(n)$ of G_n as functions of n. These ν and π happen to be functions bounded by low-degree polynomials, particularly when we focus our attention to unambiguous grammars. Finally, we introduce a family of minimal unambiguous grammars for which ν and π are linear.

Keywords: context-free grammar, Chomsky normal form, permutation, circular shift, descriptional complexity, unambiguous grammar.