Generating Ekpyrotic Curvature Perturbations Before the Big Bang

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Abstract

We analyze a general mechanism for producing a nearly scale-invariant spectrum of cosmological curvature perturbations during a contracting phase preceding a big bang, that can be entirely described using 4d effective field theory. The mechanism, based on first producing entropic perturbations and then converting them to curvature perturbations, can be naturally incorporated in cyclic and ekpyrotic models in which the big bang is modelled as a brane collision, as well as other types of cosmological models with a pre-big bang phase. We show that the correct perturbation amplitude can be obtained and that the spectral tilt n_s tends to range from slightly blue to red, with $0.97 < n_s < 1.02$ for the simplest models, a range compatible with current observations but shifted by a few per cent towards the blue compared to the prediction of the simplest, large-field inflationary models.

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1 Introduction

The primordial perturbations so dramatically imaged by WMAP and other cosmological probes offer some of our best clues as to the fundamental physics underlying the hot big bang. The observed perturbations seem extremely simple in character: small-amplitude, nearly Gaussian, growing-mode perturbations that are approximately scale-invariant and adiabatic in form. Any proposed cosmological model has to explain these basic features, and any predicted deviations from Gaussianity, scale-invariance or adiabaticity are likely to be critical in allowing us to discriminate between models.

For over two decades, it has been well-understood that simple scalar field models of inflation can, with some degree of fine-tuning, produce density perturbations of the required form. More recently, an alternative set of cosmological models has been proposed – the ekpyrotic and cyclic models [1, 2] – which, it has been argued, also solve the standard horizon and flatness puzzles before the big bang and generate a nearly scale-invariant spectrum of realistic cosmological perturbations.

The ekpyrotic mechanism for generating cosmological perturbations has been the subject of considerable debate due to a key conceptual hurdle. In most cosmological models of interest, including the ekpyrotic and cyclic models, all or most of the contraction phase prior to the big bang can be described approximately using 4d effective field theory. In this description, the growing-mode, adiabatic perturbations in the contracting, pre-bang phase have a geometrical character that is very different from the growing-mode adiabatic perturbations in the expanding, post-bang phase. In an expanding phase, the growing adiabatic mode is a local dilatation (or curvature perturbation) of the constant density hypersurfaces. But by time-reversal invariance, this mode is generically the *decaying* mode in the corresponding contracting universe. Instead, the adiabatic growing mode in a contracting universe is a time-delay perturbation to the big crunch which, under time-reversal, maps to a decaying mode perturbation in an expanding universe. Consequently, any pre-big bang scenario for generating the primordial perturbations must explain how growing mode time-delay perturbations before the bang convert into growing mode curvature perturbations after the bang. This issue has been clearly analyzed by Creminelli *et al.* [3], for example.

One possibility is that it is precisely the breakdown of the 4d effective theory near the crunch/bang transition that allows the growing time-delay mode to transform naturally into a growing mode curvature perturbation. This possibility has been studied extensively,

for example, in the Randall-Sundrum model [4, 5], where it was shown that, near the bang (the collision between branes), the warping of the 5d bulk causes the 4d effective theory to fail at order $(V/c)^2$, where V is the brane collision speed. This breakdown allows for mixing between the growing and decaying modes, which is normally prohibited within a purely 4d context. To date, the analysis has relied on matching a 4d effective description far from the bang with a 5d description near the collision, which is not completely satisfying. Ultimately, the goal is to describe this mechanism entirely in 5d, but this requires including the interbrane potential within a fully 5d calculation, which has not yet been achieved. But even if the 5d mechanism is proven rigorously to work, one may inquire whether these inherently 5d effects are the only approach for obtaining nearly scale-invariant fluctuations during a contracting phase.

In this paper, we show the answer is no: we analyze here an alternative "entropic" mechanism – described entirely in terms of 4d effective field theory – that converts growing mode scale-invariant perturbations developed in a contracting phase into scale-invariant curvature perturbations just *before* the big crunch/big bang transition. This mechanism, which has been suggested previously by Notari and Riotto [6] (see also Tsujikawa *et al.* [7]), uses elements that are familiar to many cosmologists who study entropic perturbations in inflationary cosmology. Here we analyze the predictions for the spectral amplitude and tilt and show that the ingredients needed to generate entropic perturbations and convert them into curvature perturbations before the big bang arise naturally in cosmological models like the ekpyrotic and cyclic picture. The mechanism can also be applied to other models with a transition from a contracting to an expanding phase, including those which do not have a higher dimensional realization. We understand that, following conference presentations of the work described here by one of us [8], Buchbinder *et al.* [9] and Creminelli *et al.* [10] have been applying similar ideas to ekpyrotic models with a non-singular bounce.

In this paper, we shall use as an example heterotic M-theory [11, 12, 13, 14], which underlies the ekpyrotic and cyclic models, and show that it contains in its simplest consistent truncation all the necessary ingredients. First, there is not just one scalar modulus, but two. It is quite plausible for *both* of these fields to develop scale-invariant perturbations in the contracting phase. Since both scalars are perturbed independently, there is naturally produced a spectrum of scale-invariant *entropy* perturbations in the pre-bang phase. Second, as has recently been understood [15], there are generically one or more "hard boundaries" in moduli space, at which the 4d scalar field trajectory bounces suddenly

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before the collision between branes (the big crunch). The resulting bounce from these boundaries turns out to be just what is required to convert scale-invariant entropy perturbations into scale-invariant curvature perturbations. Neglecting any higher-dimensional corrections, the matching prescription of Ref. [4] then implies that these curvature perturbations generated just before the bounce propagate across the crunch/bang transition and into growing mode curvature perturbations in the ensuing expanding phase. Under the assumption that all scalar moduli, with the possible exception of the radion, are frozen well after the big bang, the isocurvature perturbations naturally die away, leaving only the curvature perturbation to seed structure in the late universe.

The outline of this paper is as follows. Sec. 2 reviews the difference between time-delay and curvature perturbations and explains why the former naturally dominate in contracting universes. Sec. 3 reviews the ekpyrotic mechanism for generating scale-invariant perturbations in scalar fields, pointing out its wide generality and the connection with classical scale-invariance. The analysis is first done without including gravity in order to emphasize that, unlike the case of inflation, the ekpyrotic mechanism is essentially non-gravitational. Next, we extend this analysis to the case of two or more fields and show that it generates an additional "entropic" or isocurvature growing mode. We show that the perturbations in the absence of gravity generically have a red tilt of a few percent away from scale-invariant. We then compute the spectrum of entropy perturbations in the presence of gravity, showing that there is an additional Planck-scale suppressed, blue correction to the spectral tilt. Sec. 4 reviews how, if one of these scalar fields undergoes a sudden change in its evolution before the bang, it converts the entropic perturbation to a curvature perturbation. We discuss how these sudden changes occur naturally in heterotic M-theory due to various types of "hard boundaries" in scalar field space, in particular the one due to the bounce of the negative tension brane in heterotic M-theory [15, 16]. We compute the curvature perturbation amplitude on long wavelengths and show that it can match the observed value; for example, if the potential minimum and other characteristic parameters are set near the unification scale, the density fluctuation amplitude is $\mathcal{O}(10^{-5})$. Sec. 5 focuses on the spectral index. We present various equivalent expressions for the spectral index in terms of: (1) the equation of state and its time-variation; (2) the scalar field potential and its derivatives; and, (3) the conventional parameters $\bar{\epsilon}$ and $\bar{\eta}$ [17]. Using both model-independent and model-dependent analyses, we show that the predicted range of n_s for the simplest models is between 0.97 and 1.02, a few per cent bluer than the analogous range for inflationary models or for ekpyrotic models in which

the curvature perturbations are produced through 5d effects [4, 5]. Sec. 6 discusses the implications of these results.

2 The Problem of Generating Curvature Perturbations in a Contracting Universe

To understand the basic difference between the time-delay and curvature modes, it is helpful to use the following mathematical trick for generating long-wavelength solutions to the linearized Einstein equations. One starts by choosing a completely gauge-fixed gauge, such as conformal Newtonian gauge, according to which scalar perturbations of a flat FRW line element take the form

$$ds^2 = -(1 + 2\Phi)\,dt^2 + a^2(t)(1 - 2\Psi)\,d\mathbf{x}^2. \tag{2.1}$$

One way to find the spatial perturbation modes is to begin with a general linear combination of pure homogeneous gauge transformations (which, by definition, precisely satisfy Einstein's equations) and then promote the coefficients to functions of space. This construction ensures that the combination automatically satisfies the Einstein equations up to spatial gradients, which is what we seek.

Gauge transformations, with spatial gradients neglected, of a homogenous, isotropic background do not generate anisotropic stress at linear order. In conformal Newtonian gauge, however, as is well known, the absence of anisotropic stress implies $\Psi = \Phi$. It is then straightforward to check that the most general gauge transformation preserving (2.1) up to spatial gradient terms is

$$\delta t = \frac{\alpha_1(\mathbf{x})}{a} - \frac{\alpha_2(\mathbf{x})}{a} \int^t dt'\, a(t'), \qquad \delta x^i = \alpha_2(\mathbf{x})\, x^i, \tag{2.2}$$

where the constant coefficients have been replaced by functions of space. Here, $\alpha_1(\mathbf{x})$ is a local time delay to or from the singularity and $\alpha_2(\mathbf{x})$ describes a local dilatation (or curvature perturbation). The resulting Newtonian potential is

$$\Phi = \alpha_1(\mathbf{x})\frac{\dot{a}}{a^2} + \alpha_2(\mathbf{x})\left(1 - \frac{\dot{a}}{a^2}\int^t dt'\, a(t')\right), \tag{2.3}$$

where dots denote t derivatives.

In an expanding universe, the former is the decaying mode and the latter is the growing mode. In a contracting universe, these roles are reversed. Hence, the two modes have very

different geometrical characters and, at least within the context of 4d effective theory, it is not obvious how scale-invariant growing perturbations developed in a collapsing phase might match across a crunch/bang transition to growing mode perturbations of the type required in the ensuing expanding phase. This problem has been the focus of some concern amongst those using purely 4d effective theory to analyze pre-big bang and ekpyrotic perturbations; see, for example, Ref. [3].

As noted in the introduction, one solution to the problem that has been pursued for ekpyrotic and cyclic models has been the inclusion of intrinsically 5d effects that have no simple analogue in 4d effective theory [4, 5]. The remainder of this paper, though, considers a second possibility, an entropic mechanism that may be fully described within 4d effective theory, that can be applied both to ekpyrotic/cyclic models and to more general cosmological models with a contracting phase.

3   Ekpyrotic Perturbations

The ekpyrotic mechanism for generating perturbations is essentially non-gravitational. It relies on the quantum fluctuations that naturally occur when scalar fields roll down steep scalar field potentials, even in the absence of gravity. The relevant potentials are similar in form to those which occur naturally in many string theory and supergravity models. Hence it provides an interesting alternative to the usual inflationary mechanism. In this section, we first analyze the generation of ekpyrotic perturbations of a single scalar field in the absence of gravity, based on a combination of classical scale-invariance and simple quantum physics. Then, we extend the mechanism to the case of multiple fields and point out the production of entropy perturbations. Finally, we consider the gravitational corrections.

3.1   Ekpyrotic Perturbations without Gravity

Consider a scalar field ϕ in Minkowski spacetime, with action

$$\mathcal{S} = \int \mathrm{d}^4 x \left(-\frac{1}{2}(\partial\phi)^2 + V_0 e^{-c\phi} \right), \tag{3.1}$$

where we use signature $-+++$. We consider the case where V_0 is positive so the potential energy is formally unbounded below and the scalar field runs to $-\infty$ in a finite time. Note that the actual value of V_0 is not a physical parameter, since by shifting the field ϕ one

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can alter the value of V_0 arbitrarily. Furthermore, in the ekpyrotic or cyclic models, the potential is expected to turn up towards zero at large negative ϕ (because this corresponds to the limit in heterotic M-theory in which the string coupling approaches zero and the potential disappears [2]), but this detail is irrelevant to the generation of perturbations on long wavelengths.

We now argue that, as the background scalar field rolls down the exponential potential towards $-\infty$, then, to leading order in $\hbar$, its quantum fluctuations acquire a scale-invariant spectrum as the result of three features. First, the action (3.1) is classically scale-invariant. Second, by re-scaling $\phi \to \phi/c$ and re-defining V_0, the constant c can be brought out in front of the action and absorbed into Planck's constant $\hbar$, *i.e.* $\hbar \to \hbar/c^2$, in the expression $i\mathcal{S}/\hbar$ governing the quantum theory. Finally, it shall be important that ϕ has dimensions of mass in four spacetime dimensions.

To see the classical scale-invariance, note that shifting the field $\phi \to \phi + \epsilon$, and re-scaling coordinates $x^\mu \to x^\mu e^{c\epsilon/2}$, just re-scales the action by $e^{c\epsilon}$ and hence is a symmetry of the space of solutions of the classical field equations. Now we consider a spatially homogeneous background solution corresponding to zero energy density in the scalar field. This "zero energy" condition is a reasonable initial state to assume for analyzing perturbations in the cyclic and ekpyrotic models because the phase in which the perturbations are generated is preceded by a very low energy density phase with an extended period of accelerated expansion, like that of today's universe, which drives the universe into a very low energy, homogeneous state. No new energy scale enters and the solution for the scalar field is then determined (up to a constant) by the scaling symmetry: $\phi_b = (2/c)\ln(-At)$. Next we consider quantum fluctuations $\delta\phi$ in this background. The classical equations are time-translation invariant, so a spatially homogeneous time-delay is an allowed perturbation, $\phi = (2/c)\ln(-A(t + \delta t)) \Rightarrow \delta\phi \propto t^{-1}$. On long wavelengths, for modes whose evolution is effectively frozen by causality, *i.e.* $|kt| \ll 1$, we can expect the perturbations to follow this behavior. Hence, the quantum variance in the scalar field,

$$\langle \delta\phi^2 \rangle \propto \hbar t^{-2} \tag{3.2}$$

Restoring c via $\phi \to c\phi$ and $\hbar \to c^2\hbar$ leaves the result unchanged. However, since $\delta\phi$ has the same dimensions as t^{-1} in four spacetime dimensions, it follows that the constant of proportionality in (3.2) is dimensionless, and therefore that $\delta\phi$ must have a scale-invariant spectrum of spatial fluctuations.

It is straightforward to check this in detail. Setting $\phi = \phi_b(t) + \delta\phi(t, \mathbf{x})$, to linear order

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in $\delta\phi$ the field equation reads

$$\ddot{\delta\phi} = -V_{,\phi\phi}\,\delta\phi + \nabla^2\delta\phi \tag{3.3}$$

Using the zero energy condition for the classical background, we obtain $V_{,\phi\phi} = c^2V = -c^2\dot{\phi_b}^2/2 = -2/t^2$. Next we set $\delta\phi(t,\mathbf{x}) = \sum_{\mathbf{k}}(a_{\mathbf{k}}\chi_k(t)e^{i\mathbf{k}\cdot\mathbf{x}}+h.c.)$ with $a_{\mathbf{k}}$ the annihilation operator and $\chi_k(t)$ the normalized positive frequency modes. The mode functions χ_k obey

$$\ddot{\delta\chi_k} = \frac{2}{t^2}\,\chi_k - k^2\chi_k, \tag{3.4}$$

and the incoming Minkowski vacuum state corresponds to $\chi_k = e^{-ikt}\left(1 - i/(kt)\right)/\sqrt{2k}$. For large $|kt|$, this solution tends to the usual Minkowski positive-frequency mode. But as $|kt|$ tends to zero, each mode enters the growing time-delay solution described above, with $\chi_k \propto t^{-1}$. Computing the variance of the quantum fluctuation and subtracting the usual Minkowski spacetime divergence, we obtain

$$\langle\delta\phi^2\rangle = \hbar\int\frac{k^2\mathrm{d}k}{4\pi^2}\frac{1}{k^3t^2}, \tag{3.5}$$

where the integral is taken over k modes which have "frozen in" to follow the time-delay mode. In agreement with the general argument above, we have obtained scale-invariant spectrum of growing scalar field perturbations. To recap, classical scale-invariance determines the t dependence of the perturbations, and dimensional analysis then gives a scale-invariant spectrum in k, in three space dimensions.

Let us now generalize the discussion to a theory which is only *approximately* scale-invariant. As we shall see, the main change is to alter the coefficient 2 on the right hand side of (3.4) by a small multiplicative correction. We replace the potential in (3.1) by

$$V = -V_0 e^{-\int\mathrm{d}\phi\,c(\phi)}, \tag{3.6}$$

with $c(\phi)$ a slowly-varying function of ϕ. From the zero-energy condition, we find the background solution obeys

$$\sqrt{2V_0}(-t) = \int\mathrm{d}\phi\,e^{\int\mathrm{d}\phi\,c/2}. \tag{3.7}$$

We now express the integral as an expansion in derivatives of c with respect to ϕ, by writing $e^{\int\mathrm{d}\phi c/2} = (2/c)(\mathrm{d}/\mathrm{d}\phi)\,e^{\int\mathrm{d}\phi c/2}$ and integrating by parts, twice:

$$\int e^{\int c/2} = \frac{2}{c}e^{\int c/2}\left(1 + 2\frac{c_{,\phi}}{c^2}\right) - \int 4\left(\frac{c_{,\phi}}{c^3}\right)_{,\phi}e^{\int c/2}, \tag{3.8}$$

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and so on. From (3.6) we have $V_{,\phi\phi} = (c^2 - c_{,\phi})V$. Hence, combining (3.7) and (3.8), we find

$$V_{,\phi\phi} \approx -\frac{2}{t^2}\left(1 + 3\frac{c_{,\phi}}{c^2}\right), \qquad (3.9)$$

plus corrections involving higher numbers of ϕ derivatives. (By dimensions, each derivative is accompanied by an additional power of c^{-1}). Note that our sign convention is that ϕ is rolling towards $-\infty$, so a positive $c_{,\phi}$ means that c decreases as the contracting phase proceeds. This is what occurs naturally in models like the cyclic universe, where the steeply decreasing potential eventually bottoms out before the bang.

The correction $3c_{,\phi}/c^2$ in (3.9) may be treated in the first approximation as a constant, and one can then compute the correction to the spectral index as follows. The normalized positive frequency solution of (3.4), with 2 replaced by $2\left(1 + 3(c_{,\phi}/c^2)\right)$, is, up to a constant, the Hankel function $H_\nu^{(2)}(-kt)$, with $\nu = \frac{3}{2}\left(1 + \frac{4}{3}c_{,\phi}/c^2\right)$. Using the small-argument expansion of the Hankel function, the term k^{-3} in (3.5) becomes instead $k^{-3(1+(4/3)c_{,\phi}/c^2)}$. Hence to leading order in derivatives of c, the deviation of the spectral index from scale-invariance is

$$n_s - 1 = -4\frac{c_{,\phi}}{c^2}. \qquad (3.10)$$

The spectrum is red for positive $c_{,\phi}$, the natural case in the cyclic model, for example. This index characterizes is the non-gravitational contribution to the fluctuation spectrum.

3.2 Ekpyrotic Perturbations with Two Fields

The preceding discussion without gravity included is easily generalized to two or more fields. For example, consider two decoupled fields with a combined scalar potential

$$V_{tot} = -V_1 e^{-\int c_1 \mathrm{d}\phi_1} - V_2 e^{-\int c_2 \mathrm{d}\phi_2}, \qquad (3.11)$$

where $c_1 = c_1(\phi_1)$, $c_2 = c_2(\phi_2)$, and V_1 and V_2 are positive constants. We consider potentials in which the c_i are slowly varying and hence the potentials are locally exponential in form. Furthermore, for simplicity, we focus on scaling background solutions in which both fields *simultaneously* diverge to $-\infty$.

We are specifically interested in the entropy perturbation, namely the relative fluctuation in the two fields, defined as follows

$$\delta s \equiv (\dot\phi_1\,\delta\phi_2 - \dot\phi_2\,\delta\phi_1)/\sqrt{\dot\phi_1^2 + \dot\phi_2^2}. \qquad (3.12)$$

Since this quantity is gauge-invariant under linearized coordinate transformations, it can be expected to survive with only Planck-scale suppressed corrections even when gravity is turned on.

In the absence of gravity, the equation of motion of δs is (see e.g. [18])

$$\ddot{\delta s} + \left(k^2 + V_{ss} + 3\dot{\theta}^2 \right) \delta s = 0, \tag{3.13}$$

where

$$V_{ss} = \frac{\dot{\phi_2}^2 V_{,\phi_1\phi_1} - 2\dot{\phi_1}\dot{\phi_2}V_{,\phi_1\phi_2} + \dot{\phi_1}^2 V_{,\phi_2\phi_2}}{\dot{\phi_1}^2 + \dot{\phi_2}^2}, \tag{3.14}$$

$$\dot{\theta} = \frac{\dot{\phi_2}V_{,\phi_1} - \dot{\phi_1}V_{,\phi_2}}{\dot{\phi_1}^2 + \dot{\phi_2}^2}. \tag{3.15}$$

Here, $\dot{\theta}$ measures the bending of the trajectory in scalar field space, (ϕ_1, ϕ_2). We want to consider background solutions where both fields run to large negative values, with $\dot{\phi_2}$ and $\dot{\phi_1}$ remaining comparable in magnitude. For this to be true, the trajectory should not bend too strongly.

For simplicity we shall only study the easiest case, $\dot{\theta} = 0$, for which the background scalar field trajectory is a straight line. In this case, we have

$$\dot{\phi_2} = \gamma\dot{\phi_1}, \tag{3.16}$$

with γ an arbitrary constant. From the definition (3.15), the two scalar field potentials are related, and, by integrating, we find that up to an irrelevant constant, we must have

$$V_{tot} = V(\phi_1) + \gamma^2 V(\phi_2/\gamma), \tag{3.17}$$

for some function $V(\phi)$. The equation of motion (3.13) now becomes

$$\ddot{\delta s} + (k^2 + V_{,\phi\phi}) \, \delta s = 0, \tag{3.18}$$

where we have set $\phi = \phi_1$. This is exactly the same equation as that governing the fluctuations of a single field (3.3), and it is straightforward to check that the background solution is also just that for the field ϕ in the potential $V(\phi)$. Hence, the analysis of the previous section may be applied without change, with $c(\phi) \equiv -(\ln V)_{,\phi}$. Furthermore, since δs is a canonically normalized field according to its definition in (3.12), the power spectrum generated from quantum fluctuations is given, in the scale-invariant case, by

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the same expression as that for a scalar field, namely (3.5). Let us emphasize that (3.16) is *not* an attractor solution of the background equations. In fact, it is precisely the instability of (3.16) which generates approximately scale-invariant perturbations, according to (3.18). The question of how the system enters the background solution (3.16) is of course important, but shall not be addressed in this paper.

In the next subsection, we will consider the gravitational corrections to the entropy perturbation spectrum, which are suppressed by inverse powers of the Planck mass M_{Pl}. Since the only relevant physical parameter in the actions we have so far discussed is c, which has inverse mass dimensions, we can expect the gravitational effects to be small when $c^{-1}/M_{Pl} \ll 1$, and then the estimates obtained above should be accurate.

3.3 Ekpyrotic Perturbations including Gravity

Next we turn our attention to the more realistic setting where gravity is included. We consider the action for N decoupled fields interacting only through gravity:

$$\int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\sum_{i=1}^{N}(\partial\phi_i)^2 - \sum_{i=1}^{N}V_i(\phi_i)\right), \tag{3.19}$$

where we have chosen units in which $8\pi G \equiv M_{Pl}^{-2} = 1$. In a flat Friedmann-Robertson-Walker background with line element $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2$, the scalar field and Friedmann equations are given by

$$\ddot{\phi}_i + 3H\dot{\phi}_i + V_{i,\phi_i} = 0 \tag{3.20}$$

and

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\sum_i \dot{\phi}_i^{\;2} + \sum_i V_i(\phi_i)\right], \tag{3.21}$$

where $H = \dot{a}/a$ and $V_{i,\phi_i} = (\partial V_i/\partial\phi_i)$ with no summation implied. Another useful relation is

$$\dot{H} = -\frac{1}{2}\sum_i \dot{\phi}_i^{\;2}. \tag{3.22}$$

If all the fields have negative exponential potentials $V_i(\phi_i) = -V_i\, e^{-c_i\phi_i}$ then as is well-known, the Einstein-scalar equations admit the scaling solution

$$a = (-t)^p, \qquad \phi_i = \frac{2}{c_i}\ln(-A_i t), \qquad V_i = \frac{2A_i^2}{c_i^2}, \qquad p = \sum_i \frac{2}{c_i^2}. \tag{3.23}$$

Thus, if $c_i \gg 1$ for all i, we have a very slowly contracting universe with $p \ll 1$.

As before, we focus on the entropy perturbation since this is a local, gauge-invariant quantity, and on the case of only two scalar fields. The entropy perturbation equation (3.13) in flat spacetime is replaced (see e.g. [18]) by

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{ss} + 3\dot{\theta}^2\right)\delta s = \frac{4k^2\dot{\theta}}{a^2\sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}}\Phi. \tag{3.24}$$

Again, for simplicity we will focus attention on straight line trajectories in scalar field space. Since $\dot{\theta} = 0$, the entropy perturbation is not sourced by the Newtonian potential Φ and we can solve the equations rather simply. We shall assume, as before, that the background solution obeys scaling symmetry so that $\dot{\phi}_2 = \gamma\dot{\phi}_1$.

It is convenient at this point to continue the analysis in terms of conformal time τ. Denoting τ derivatives with primes, and introducing the re-scaled entropy field

$$\delta S = a(\tau)\,\delta s, \tag{3.25}$$

Eq. (3.24) becomes

$$\delta S'' + \left(k^2 - \frac{a''}{a} + a^2 V_{,\phi\phi}\right)\delta S = 0. \tag{3.26}$$

The crucial term governing the spectrum of the perturbations is then

$$\tau^2\left(\frac{a''}{a} - V_{,\phi\phi}\,a^2\right). \tag{3.27}$$

When this quantity is approximately 2, we will again get nearly scale-invariant perturbations.

It is customary to define the quantity

$$\epsilon \equiv \frac{3}{2}(1+w) \equiv \frac{\dot{\phi}_1^2 + \dot{\phi}_2^2}{2H^2} = \frac{(1+\gamma^2)\dot{\phi}^2}{2H^2}. \tag{3.28}$$

In the background scaling solution,

$$\epsilon = \frac{c^2}{2(1+\gamma^2)}. \tag{3.29}$$

We proceed by evaluating the quantity in (3.27) in an expansion in inverse powers of ϵ and its derivatives with respect to N, where $N = \ln(a/a_{end})$, where a_{end} is the value of a at the end of the ekpyrotic phase. Note that N decreases as the fields roll downhill and the contracting ekpyrotic phase proceeds.

We obtain the first term in (3.27) by differentiating (3.22), obtaining

$$\frac{a''}{a} = 2H^2 a^2\left(1 - \frac{1}{2}\epsilon\right). \tag{3.30}$$

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The second term in (3.27) is found by differentiating (3.28) twice with respect to time and using the background equations and the definition of N. We obtain

$$a^2 V_{,\phi\phi} = -a^2 H^2 \left(2\epsilon^2 - 6\epsilon - \frac{5}{2} \epsilon_{,N} \right) + O(\epsilon^0). \tag{3.31}$$

Finally, need to express $\mathcal{H} \equiv (a'/a) = aH$ in terms of the conformal time τ. From (3.30) we obtain

$$\mathcal{H}' = \mathcal{H}^2 (1 - \epsilon), \tag{3.32}$$

which integrates to

$$\mathcal{H}^{-1} = \int_0^\tau d\tau (\epsilon - 1). \tag{3.33}$$

Now, inserting $1 = d(\tau)/d\tau$ under the integral and using integration by parts we can re-write this as

$$\mathcal{H}^{-1} = \epsilon\tau \left(1 - \frac{1}{\epsilon} - (\epsilon\tau)^{-1} \int_0^\tau \epsilon'\tau d\tau \right). \tag{3.34}$$

Using the same procedure once more, the integral in this expression can be written as

$$(\epsilon\tau)^{-1} \int_0^\tau \epsilon'\tau d\tau = \frac{\epsilon'\tau}{\epsilon} - (\epsilon\tau)^{-1} \int_0^\tau \frac{d}{d\tau}(\epsilon'\tau)\,\tau d\tau. \tag{3.35}$$

Now using the fact that $\epsilon' = \mathcal{H}\epsilon_{,N}$, and that to leading order in $1/\epsilon$, $\mathcal{H}$ can be replaced by its value in the scaling solution (with constant ϵ), $\mathcal{H}\tau = \epsilon^{-1}$, we can re-write the second term on the right-hand side as

$$-(\epsilon\tau)^{-1} \int_0^\tau \frac{d}{d\tau}(\epsilon'\tau)\,\tau d\tau = -(\epsilon\tau)^{-1} \int_0^\tau d\tau \frac{1}{\epsilon} \left(\frac{\epsilon_{,N}}{\epsilon} \right)_{,N}, \tag{3.36}$$

which shows that this term is of order $1/\epsilon^2$ and can thus be neglected. Altogether we obtain

$$\mathcal{H}^{-1} = \int_0^\tau d\tau (\epsilon - 1) \approx \epsilon\tau \left(1 - \frac{1}{\epsilon} - \frac{\epsilon_{,N}}{\epsilon^2} \right). \tag{3.37}$$

Using (3.30) and (3.31) with (3.37) we can calculate the crucial term entering the entropy perturbation equation,

$$\tau^2 \left(\frac{a''}{a} - V_{,\phi\phi} a^2 \right) = 2 \left(1 - \frac{3}{2\epsilon} + \frac{3}{4} \frac{\epsilon_{,N}}{\epsilon^2} \right) \tag{3.38}$$

As explained in the discussion preceding equation (3.10), the deviation from scale-invariance in the spectral index of the entropy perturbation is then given by

$$n_s - 1 = \frac{2}{\epsilon} - \frac{\epsilon_{,N}}{\epsilon^2}. \tag{3.39}$$

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The first term on the right-hand side is the gravitational contribution, which, being positive, tends to make the spectrum blue. The second term is the non-gravitational contribution, which tends to make the spectrum red. We will return to this expression in Sec. 5, after explaining how these entropic perturbations are naturally converted to curvature perturbations.

4   Converting Entropy to Curvature Perturbations

We have shown how an approximately scale-invariant spectrum of entropy perturbations may be generated by scalar fields in a contracting universe. In this section, we will discuss how these perturbations may be converted to curvature perturbations if the scalar field undergoes a sudden acceleration, and we will estimate the curvature perturbation amplitude.

As an example, we will consider the common case where the scalar field trajectory encounters a boundary in moduli space and bounces off it. Such a bounce was recently found in heterotic M-theory [15, 16], when the negative tension brane bounces off the zero of the bulk warp factor just before the positive and negative tension branes collide. We refer to those papers for further details: for the purpose of this paper, all we need to know is that in the 4d effective description there are two scalar field moduli, ϕ_1 and ϕ_2, living on the half-plane $-\infty < \phi_1 < \infty$, $-\infty < \phi_2 < 0$. Furthermore, the cosmological solution of interest is one in which ϕ_2 encounters the boundary $\phi_2 = 0$, and reflects off it elastically.

Let us assume that an approximately scale-invariant entropy perturbation δs has been generated, as described in previous sections, in a contracting phase of the universe in which both ϕ_1 and ϕ_2 run down steep negative potentials. The question we want to address is whether this entropy perturbation may be converted into a curvature perturbation on large scales, within the realm of validity of 4d effective theory. It is not hard to see how a scalar field "bounce" off a boundary in moduli space readily achieves this feat. As is well known (see e.g. Ref. [18]), defining $\mathcal{R}$ to be the curvature perturbation on comoving spatial slices, for N scalar fields with general Kähler metric $g_{ij}(\phi)$ on scalar field space, the linearized Einstein-scalar field equations lead to

$$\dot{\mathcal{R}} = -\frac{H}{\dot{H}} \left(g_{ij} \frac{D^2 \phi^i}{Dt^2} s^j - \frac{k^2}{a^2} \Psi \right), \tag{4.1}$$

where the $N - 1$ entropy perturbations

$$s^i = \delta\phi^i - \dot\phi^i \, \frac{g_{jk}(\phi) \, \dot\phi^j \delta\phi^k}{g_{lm}(\phi) \, \dot\phi^l \dot\phi^m} \tag{4.2}$$

are just the components of $\delta\phi^i$ orthogonal to the background trajectory, and the operator D^2/Dt^2 is just the geodesic operator on scalar field space. In our case, things simplify because the scalar field space is flat, so the metric is $g_{ij} = \delta_{ij}$, and D/Dt reduces to an ordinary time derivative. Considering only two scalar fields, we have

$$s^1 = -\dot\phi_2 \, \delta s/\sqrt{\dot\phi_1^2 + \dot\phi_2^2}, \qquad s^2 = +\dot\phi_1 \, \delta s/\sqrt{\dot\phi_1^2 + \dot\phi_2^2}. \tag{4.3}$$

For a straight line trajectory in field space, the right-hand side of (4.2) vanishes even if the entropy perturbation is nonzero. However, if there is a departure from geodesic motion, the entropy perturbation directly sources the curvature perturbation. This can happen when scalar field potentials are present, and we would expect it generically to occur when the ekpyrotic potentials turn off, around $t_{end} < t_b$, but this contribution due to the resulting bending of the scalar field trajectory will in general be very model-dependent. In contrast, a simple reflection of one of the two fields (say ϕ_2) off a boundary (in this case at $\phi_2 = 0$), at some time t_b, results in a model-independent contribution which is easily computed.

For simplicity, we assume that the scalar field bounce occurs after the ekpyrotic potentials are turned off, so that the universe is kinetic-dominated from the 4d point-of-view. The scalar field trajectory is $\dot\phi_2 = -\tilde\gamma\dot\phi_1$, for $t < t_b$, and $\dot\phi_2 = \tilde\gamma\dot\phi_1$, for $t > t_b$, with $\dot\phi_1$ constant and negative in the vicinity of the bounce. The bounce leads to a delta function on the right-hand side of (4.1),

$$\frac{D^2\phi_2}{Dt^2} = \delta(t - t_b) \, 2\dot\phi_2(t_b^+), \tag{4.4}$$

where t_b is the time of the bounce of the negative-tension brane. As can be readily seen from (4.1), if the entropy perturbations already have acquired a scale-invariant spectrum by the time t_b, then the bounce leads to their instantaneous conversion into curvature perturbations with precisely the same long wavelength spectrum.

We can estimate the amplitude of the resulting curvature perturbation by integrating equation (4.2) using (4.4). Since we have assumed the universe is kinetic-dominated at this time, $H = 1/(3t)$. As pointed out earlier, since the entropy perturbation (3.12) is canonically normalized, its spectrum is given by (3.5) up to non-scale-invariant corrections. This expression only holds as long as the ekpyrotic behavior is still underway: the

ekpyrotic phase ends at a time t_{end} approximately given by $|V_{min}| = 2/(c^2 t_{end}^2)$. After t_{end}, the entropy perturbation obeys $\ddot{\delta s} + t^{-1}\dot{\delta s} = 0$, which has the solution $\delta s = A + B \ln(-t)$. Matching this solution to the growing mode solution t^{-1} in the ekpyrotic phase, one finds that by t_b the entropy grows by an additional factor of $1 + \ln(t_{end}/t_b)$. Employing the Friedmann equation to relate $\dot{\phi}_2 = \tilde{\gamma}\dot{\phi}_1$ to H, putting everything together and restoring the Planck mass, we find for the variance of the spatial curvature perturbation in the scale-invariant case,

$$\langle \mathcal{R}^2 \rangle = \hbar \frac{c^2 |V_{min}|}{3\pi^2 M_{Pl}^2} \frac{\tilde{\gamma}^2}{(1 + \tilde{\gamma}^2)^2} \left(1 + \ln(t_{end}/t_b)\right)^2 \int \frac{\mathrm{d}k}{k} \equiv \int \frac{\mathrm{d}k}{k} \Delta_{\mathcal{R}}^2(k) \qquad (4.5)$$

for the perfectly scale-invariant case. Notice that the result depends only logarithmically on t_b: the main dependence is on the minimum value of the effective potential and the parameter c. Observations on the current Hubble horizon indicate $\Delta_{\mathcal{R}}^2(k) \approx 2.2 \times 10^{-9}$. Ignoring the logarithm in (4.5), this requires $c|V_{min}|^{\frac{1}{2}} \approx 10^{-3} M_{Pl}$, or approximately the GUT scale. This is of course entirely consistent with the heterotic M-theory setting [19]. Having shown it is straightforward to obtain the right amplitude, we next consider the spectral index.

5 Comparing Predictions for the Spectral Index

If the entropic perturbations are suddenly converted to curvature perturbations, as in the example considered in the previous section, the curvature perturbations inherit the spectral tilt given in (3.39). In this section, we analyze this relation using several techniques and compare the prediction to the those for curvature perturbations in inflation and for a cyclic model in which the time-delay fluctuations are converted to curvature perturbations before the bang.

As a first approach, let us consider the model-independent estimating procedure used in Ref. [20]. We begin by re-expressing Eq. (3.39) in terms of $\mathcal{N}$, the number of e-folds before the end of the ekpyrotic phase (where $\mathrm{d}\mathcal{N} = (\epsilon - 1)N$ and $\epsilon \gg 1$):

$$n_s - 1 = \frac{2}{\epsilon} - \frac{\mathrm{d}\ln\epsilon}{\mathrm{d}\mathcal{N}}. \qquad (5.1)$$

This expression is identical to the case of the Newtonian potential perturbations derived in [20], except that the first term has the opposite sign. In this expression, $\epsilon(\mathcal{N})$ measures the equation of state during the ekpyrotic phase, which must decrease from a value much

greater than unity to a value of order unity in the last $\mathcal{N}$ e-folds. If we estimate $\epsilon \approx \mathcal{N}^\alpha$, then the spectral tilt is

$$n_s - 1 \approx \frac{2}{\mathcal{N}^\alpha} - \frac{\alpha}{\mathcal{N}}. \tag{5.2}$$

Here we see that the sign of the tilt is sensitive to α. For nearly exponential potentials ($\alpha \approx 1$), the spectral tilt is $n_s \approx 1 + 1/N \approx 1.02$, slightly blue, because the first term dominates. However, there are well-motivated examples (see below) in which the equation of state does not decrease linearly with $\mathcal{N}$. We have introduced α to parameterize these cases. If $\alpha > 0.14$, the spectral tilt is red. For example, $n_s = 0.97$ for $\alpha \approx 2$. These examples represent the range that can be achieved for the entropically-induced curvature perturbations in the simplest models, roughly $0.97 < n_s < 1.02$.

For comparison, if we use the same estimating procedure for the Newtonian potential fluctuations in the cyclic model (assuming they converted to curvature fluctuations before the bounce through 5d effects), we obtain $0.95 < n_s < 0.97$. This range agrees with the estimate obtained by an independent analysis based on studying inflaton potentials directly [21]. Furthermore, as shown in Ref. [20], the same range is obtained for time-delay (Newtonian potential) perturbations in the cyclic model, due to a "duality" in the perturbations equations [22]. Hence, all estimates are consistent with one another, and we can conclude that the range of spectral tilt obtained from entropically-induced curvature perturbations is typically bluer by a few percent.

We note that, in the cyclic model, say, it is possible that curvature perturbations are created both by the entropic mechanism and by converting Newtonian potential perturbations into curvature perturbations through 5d effects. In this case, the cosmologically relevant contribution is the one with the bigger amplitude. In particular, the conversion of Newtonian potential perturbations is sensitive to the brane collision velocity V, [4, 5], whereas the entropic mechanism is not. So, conceivably, either contribution could dominate.

A second way of analyzing the spectral tilt is to assume a form for the scalar field potential. Consider the case where the two fields have steep potentials that can be modelled as $V(\phi_1) = -V_0 \, e^{-\int c \, \mathrm{d}\phi}$ and $\dot{\phi}_2 = \gamma \dot{\phi}_1$. Then Eq. (3.39) becomes

$$n_s - 1 = \frac{4(1 + \gamma^2)}{c^2 M_{Pl}^2} - \frac{4c_{,\phi}}{c^2}, \tag{5.3}$$

where we have used the fact that $c(\phi)$ has the dimensions of inverse mass and restored the factors of Planck mass. The presence of M_{Pl} clearly indicates that the first term on the right is a gravitational term. It is also the piece that makes a blue contribution to the

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spectral tilt. The second term is the non-gravitational term and agrees precisely with the flat space-time result (3.10), although the agreement is not at all obvious at intermediate steps of the calculation.

For a pure exponential potential, which has $c_{,\phi} = 0$, the non-gravitational contribution is zero, and the spectrum is slightly blue, as our model-independent analysis suggested. For plausible values of $c = 20$ and $\gamma = 1/2$, say, the gravitational piece is about one percent and the spectral tilt is $n_s \approx 1.01$, also consistent with our earlier estimate. However, this case with $c_{,\phi}$ precisely equal to zero is unrealistic. In the cyclic model, for example, the steepness of the potential must decrease as the field rolls downhill in order that the ekpyrotic phase comes to an end, which corresponds to $c_{,\phi} > 0$. If $c(\phi)$ changes from some initial value $\bar{c} \gg 1$ to some value of order unity at the end of the ekpyrotic phase after ϕ changes by an amount $\Delta\phi$, then $c_{,\phi} \sim \bar{c}/\Delta\phi$. When c is large, the non-gravitational term in Eq. (5.3) typically dominates and the spectral tilt is a few per cent towards the red.

For example, suppose $c \propto \phi^\beta$ and $\int c(\phi)\, d\phi \approx 125$; then, the spectral tilt is

$$n_s - 1 = -0.03 \frac{\beta}{1+\beta}, \tag{5.4}$$

which corresponds to $.97 < n_s < 1$ for positive $0 < \beta < \infty$, in agreement with our earlier estimate. We note that negative potentials of this type with very large values of c have been argued to arise naturally in string theory. For example, in the work of Conlon and Quevedo [23], the potential is of the form we require with $c(\phi) \sim C\phi^{1/3}$, where the constant C is very large. For this potential, one finds $n_s \approx 0.99$.

Our expression for the spectral tilt of the entropically induced curvature spectrum can also be expressed in terms of the customary "fast-roll" parameters [17]

$$\bar{\epsilon} \equiv \left(\frac{V}{V_{,\phi}}\right)^2 = \frac{1}{c^2} \qquad \bar{\eta} \equiv \left(\frac{V}{V_{,\phi}}\right)_{,\phi}. \tag{5.5}$$

Note that $\bar{\epsilon} = 1/(2(1+\gamma^2)\epsilon)$. Then, the spectral tilt is

$$n_s - 1 = \frac{4(1+\gamma^2)}{M_{Pl}^2}\bar{\epsilon} - 4\bar{\eta}. \tag{5.6}$$

This result can be compared with the spectral index of the time-delay (Newtonian potential) perturbation obtained in earlier work [17], where the corresponding formula is

$$n_s - 1 = -\frac{4}{M_{Pl}^2}\bar{\epsilon} - 4\bar{\eta}. \tag{5.7}$$

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Here, the first term is again gravitational, but it has the opposite sign of the gravitational contribution to the entropically induced fluctuation spectrum. So, the tilt is typically a few per cent redder.

Finally, for inflation, the spectral tilt is

$$n_s - 1 = -6\epsilon + 2\eta \tag{5.8}$$

where the result is expressed in terms of the slow-roll parameters $\epsilon \equiv (1/2)(M_{Pl}V_{,\phi}/V)^2$ and $\eta \equiv M_{Pl}^2 V_{,\phi\phi}/V$. Here we have revealed the factors of M_{Pl} to illustrate that both inflationary contributions are gravitational in origin. This gives the same range for n_s as the Newtonian potential perturbations in the cyclic model.

6   Conclusions

The entropic mechanism for generating approximately scale-invariant curvature perturbations in a contracting universe has two appealing features. First, it can be analyzed entirely within the context of 4d effective theory. For those who were skeptical about the ekpyrotic and cyclic models because of their apparent reliance on 5d effects to create curvature perturbations, this work shows that there is another, more prosaic mechanism that can be totally understood in familiar terms. This should terminate the debate on whether it is possible, in principle, to generate curvature perturbations in a pre-big bang phase.

The second attractive feature is that the essential elements occur quite naturally in extra-dimensional theories like string and M-theory. There is no shortage of scalar field moduli, and, quite generically, these fields can possess negative and steeply decreasing potentials of the ekpyrotic form. In this situation, approximate scaling solutions exist in which several fields undergo ekpyrosis simultaneously so that nearly scale-invariant entropy perturbations are naturally generated. Furthermore, if the relevant scalar field trajectory encounters a boundary in moduli space (like that described in Ref. [15]), then as the trajectory reflects off the boundary, entropy perturbations are naturally converted into curvature perturbations with the identical large-scale power spectrum.

We hasten to add that, although we have only presented here the concrete example of heterotic M-theory, it is clear that the present formalism is generic and can be applied to other types of pre-big bang models, including those that do not rely on there being extra dimensions.

We have also seen that the entropic mechanism has an interesting signature. Because of the gravitational contribution to the spectral tilt of the entropically-induced perturbations, the spectrum is typically a few per cent bluer than the time-delay (Newtonian potential) perturbations or the density perturbation in inflation. To push the inflationary perturbations into this bluer range requires adding extra degrees of otherwise unnecessary fine-tuning, as delineated in Ref. [21]. In particular, Ref. [21] shows that the natural range for inflationary models is $0.93 < n_s < 0.97$, whereas entropically-induced spectra tend to lie in a range that is a few per cent bluer, roughly $0.97 < n_s < 1.02$ by our estimates. Hence, a highly precise measure of the spectral tilt at the one per cent level or better could serve as an indicator of which mechanism is responsible. For example, a value of $n_s = 0.99$ is awkward to obtain with inflation but right in the middle of the predicted range for pre-big bang entropically-induced perturbations.

$$***$$

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