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# Generating fiber orientation maps in human heart models using Poisson interpolation

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## Abstract

Smoothly varying muscle fiber orientations in the heart are critical to its electrical and mechanical function. From detailed histological studies and diffusion tensor imaging, we now know that fiber orientations in humans vary gradually from approximately −70° in the outer wall to +80° in the inner wall. However, the creation of fiber orientation maps for computational analyses remains one of the most challenging problems in cardiac electrophysiology and cardiac mechanics. Here we show that Poisson interpolation generates smoothly varying vector fields that satisfy a set of user-defined constraints in arbitrary domains. Specifically, we enforce the Poisson interpolation in the weak sense using a standard linear finite element algorithm for scalar-valued second-order boundary value problems and introduce the feature to be interpolated as a global unknown. User-defined constraints are then simply enforced in the strong sense as Dirichlet boundary conditions. We demonstrate that the proposed concept is capable of generating smoothly varying fiber orientations, quickly, efficiently, and robustly, both in a generic bi-ventricular model and in a patient-specific human heart. Sensitivity analyses demonstrate that the underlying algorithm is conceptually able to handle both arbitrarily and uniformly distributed user-defined constraints; however the quality of the interpolation is best for uniformly distributed constraints. We anticipate our algorithm to be immediately transformative to experimental and clinical settings, where it will allow us to quickly and reliably create smooth interpolations of arbitrary fields from data sets, which are sparse but uniformly distributed.

### Keywords

feature-based interpolation; Poisson equation; finite element method; anisotropy; fiber orientation; cardiac electrophysiology; cardiac mechanics

## 1. Introduction

The structural architecture of the heart is one of the most important factors to healthy pump function (Kumar 2005). While many geometric factors such as shape, thickness, and size play a critical role in disease diagnosis, the correct alignment of cardiac muscle fibers is crucial for proper choreography of cardiac muscle contraction (Opie 2003). In fact, many cardiac disorders or traumatic events induce cardiac fiber misalignment, which may cyclically reduce cardiac function and lead to further disorganization (Libby et al. 2007). For example, in response to cardiac infarctions, residual fibers are secreted to structurally



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stabilize the heart. However, these fibers lose organization and may actually form nodes of orthogonal myofiber intersection or contact that impede proper cardiac function (Sosnovik et al. 2009). Other pathologies such as myocardial fibrosis are also known to modify organized fiber orientation, ultimately impairing both electrical and mechanical function (Schmitt et al. 2009). Therefore, it is imperative for computational models of the heart to include proper fiber orientation distributions to accurately predict the electrical and mechanical function of the heart in health and disease.

Figure 1 illustrates the characteristic microstructure of the heart by means of the fiber direction $\boldsymbol{f}$, the sheet plane normal $\boldsymbol{s}$ and their normal $\boldsymbol{m}$. Fortunately, the overall structural alignment of the human heart is well characterized in the literature (Streeter & Hanna 1973), where fibers have been reported to vary transmurally from approximately −70° in the epicardium, the outer wall, to +80° in the endocardium, the inner wall (Arts et al. 2001; Geerts et al. 2002). However, along the septal wall, the right endocardium displays a fiber orientation of approximately −70°. Although there is a wide agreement on these fiber orientations in the epicardial and endocardial walls, there is no generally accepted concept to assign these fiber orientations to realistic anatomical computer models of the human heart. Simplified attempts represent the two ventricles as nested, truncated ellipsoids and allow for an analytical characterization of the fiber orientation distribution (Göktepe & Kuhl 2010). Other techniques rely on complex projections and use least squares fitting to generate a continuum description of fiber orientations (Toussaint et al. 2010). Recent attempts have adopted a geometrically-based approach to mitigate some of the difficulties in determining approximate fiber orientations in irregular geometries (Takayama et al. 2008).

When heart fiber orientations were first characterized, the goal was to identify an analytical description for a single ellipsoid representing the left ventricle (Nielsen et al. 1991). Initially, Hammer projections served to map a real heart geometry onto an idealized prolate spheroidal ellipsoid to create a mathematical model of the fiber directions in the heart. A more recent variation of this technique has been applied to diffusion tensor image interpolation (Toussaint et al. 2010). The irregular geometry is first projected to a prolate spheriod coordinate system, and then smoothed using an averaging kernel and least squares approach to generate an interpolation in the idealized domain. In addition, some groups use least squares techniques to fit the experimentally measured fiber directions to an approximated geometrical mesh (Costa et al. 1996; Toussaint et al. 2010). The fiber directions from a least squares approach can then be interpolated onto element centroids, nodes, or integration points of the finite element mesh by using the corresponding finite element shape functions. For more complex interpolating schemes, different weights can be assigned to the individual interpolants. Most recently, a geometrical-based approach was proposed to provide sketch-based methods for generating layered fiber structures for the heart (Takayama et al. 2008).

Unfortunately, there are several essential drawbacks in using ellipsoid, projected, or least squares techniques. For example, some difficulties may arise when trying to determine proper projections for mapping a non-ellipsoidal geometry onto an idealized ellipsoidal representation of the heart. With the relative accessibility of different three-dimensional imaging modalities today, we can attain patient-specific heart geometries relatively easily (Kotikanyadanam et al. 2010). Generally these geometries do not look like ellipsoids, and there may be inherent singularities when mapping each ventricle onto an idealized ellipsoid, in particular in the septal region and close to the apex (Toussaint et al. 2010a). Of all existing techniques, least squares techniques might therefore be best suited to fit experimentally measured data. However, they may require some fine tuning to create suffiently continuous fiber orientations. In addition, they typically require a sufficiently complete set of experimental data.

Overall, the challenges above mainly add complexity to designating fiber orientation maps throughout the heart. Here we propose a novel fiber interpolation concept, which significantly reduces the complexity of generating fiber orientation distributions within arbitrary geometries. In addition, it allows for more flexibility with regards to mesh refinement and is easily accessible and intuitive to the field of computational biomechanics. The underlying approach is adopted from feature-based interpolation techniques (Yang et al. 2012) used in computer graphics (Fisher et al. 2007), and is conceptually similar to sketch-based methods (Takayama et al. 2008). However, our approach is finite-element based and provides a straightforward refinement to properly describe both ventricles of the heart, while maintaining a continuous fiber orientation distribution. The key to reducing complexity, ironically, is to reformulate the orientation of the heart from an explicit representation of measured or mathematically parametrized orientations to a more implicit one. This eliminates having to make difficult and subjective assumptions by simply calculating the fiber orientation as the solution to a boundary value problem. While the concept is inherently applicable to different types of boundary value problems, here we use the Poisson equation as an interpolation method to solve for our designated fiber angle orientation distribution.

This paper is organized into four additional sections. First, in Section 2, we introduce the Poisson interpolation, the generation of local cardiac coordinate systems, and the creation of Dirichlet boundary conditions, which serve as the basis for the fiber interpolation. Then, in Section 3, we illustrate the concept of the Poisson interpolation on an idealized geometry to quantitatively compare it to an analytical fiber orientation and on a patient-specific geometry to demonstrate the generality of the underlying scheme. We also perform two sensitivity analyses to identify the amount of information required to generate smooth fiber orientation maps. This mimics the potential to interpolate fiber orientations distributions using experimentally acquired data. Last, in Section 4, we discuss the merits and limitations of our algorithm and highlight future possible avenues.

## 2. Methods

### 2.1 Poisson interpolation of scalar-valued feature θ

The key idea of our fiber orientation algorithm is to generate a smooth coordinate-free fiber interpolation using algorithms from computer graphics motivated by feature-based interpolation. In essence, we solve the following Poisson equation for the scalar-valued feature $\theta$,

$$\mathrm{div}\,(\boldsymbol{K} \cdot \nabla \theta) = 0 \quad \text{in} \quad \mathscr{B} \qquad (1)$$

for given Dirichlet boundary conditions

$$\theta = \bar{\theta} \quad \text{on} \quad \partial \mathscr{B}_{\theta}. \qquad (2)$$

For isotropic constant diffusion coefficients, $\boldsymbol{K} = K\,\boldsymbol{I}$, the Poisson equation reduces to the homogeneous Laplacian, $\Delta\theta = 0$, generating a smooth linear interpolation of the feature $\theta$ across the cardiac domain $\mathscr{B}$. To solve the underlying partial differential equation (1), we transform it into its weak form, integrate it over the domain $\mathscr{B}$, discretize it with linear tetrahedral elements, and solve the resulting system with the corresponding Dirichlet boundary conditions (2). This implies that we solve the fiber interpolation (1) in a weak sense, while we enforce the set of user-defined constraints (2) in a strong sense. For the finite element solution, we can adopt any standard Finite Element program for linear diffusion or thermal problems, and interpret the scalar-valued feature $\theta$ as the primary unknown.

### 2.2 Local cardiac coordinate system {n, c, z}

To establish a local cardiac coordinate system $\{\boldsymbol{n}, \boldsymbol{c}, \boldsymbol{z}\}$, we identify the normal $\boldsymbol{n}$, circumferential $\boldsymbol{c}$, and longitudinal $\boldsymbol{z}$ directions at each node (Tsamis et al. 2011). First, we assign the vector $\boldsymbol{z}$ pointing along the long axis of the heart. Second, we calculate the nodal normals $\boldsymbol{n}$ using an area-weighted face-normal averaging algorithm commonly used in computer graphics (Max 1999). We calculate the facet normals to each surface triangle by taking the cross product of two edge vectors of the corresponding facet. We then calculate the nodal normal $\boldsymbol{n}$ as average over all facet normals connected at the particular node. Since the magnitude of the cross product is proportional to the area of the corresponding facet, our nodal normal is automatically area averaged. Third, we calculate the circumferential direction $\boldsymbol{c}$ as the weighted cross product of the longitudinal and normal vectors $\boldsymbol{z}$ and $\boldsymbol{n}$ (Tsamis et al. 2012),

$$\boldsymbol{c}=\varphi\,[\boldsymbol{z}\times\boldsymbol{n}], \quad (3)$$

To ensure a proper circumferential orientation we weight the cross product with the orientation index $\varphi$,

$$\varphi=\overline{\varphi}=\begin{cases} +1 & \text{on} \quad \partial\mathscr{B}_{\varphi^+} \\ -1 & \text{on} \quad \partial\mathscr{B}_{\varphi^-} \end{cases} \quad (4)$$

which is positive on the epicardium and the right septal endocardium $\partial\mathscr{B}_{\varphi^+}$ and negative on the left endocardium and right free wall endocardium $\partial\mathscr{B}_{\varphi^-}$.

### 2.3 Nodal fiber and sheet orientations f and s

We can now assign the fiber and sheet orientations $\boldsymbol{f}$ and $\boldsymbol{s}$ as indicated in Figure 1. In particular, we assume that fibers $\boldsymbol{f}$ at the inner and outer surfaces of the heart lie within the tangent plane to the corresponding surface. To begin, we calculate the outward pointing normal $\boldsymbol{n}_{\mathrm{cz}}$ with respect to the local cz-plane,

$$\boldsymbol{n}_{\mathrm{cz}}=\boldsymbol{c}\times\boldsymbol{z}. \quad (5)$$

Although we could, in principle, assign any given sheet angle $\beta$ to define the sheet direction, here, without loss of generality, we select the sheet normal $\boldsymbol{s}$ to be aligned with the outward facing nodal normal,

$$\boldsymbol{s}=\mathrm{sign}(\boldsymbol{n}_{\mathbf{cz}}\cdot\boldsymbol{n})\,\boldsymbol{n}. \quad (6)$$

To specify the fiber angle with respect to the horizontal plane, we assume that all fibers within the epicardium and right septal endocardium are inclined with −70° and all fibers within the left endocardium and right free wall endocardium are inclined with 80° (Nielsen et al. 1991),

$$\alpha = \begin{cases} -70^\circ & \text{on} \quad \partial\mathscr{B}_{\varphi^+} \\ +80^\circ & \text{on} \quad \partial\mathscr{B}_{\varphi^-} \end{cases} . \quad (7)$$

We then construct the projection $\boldsymbol{p}$ of the fiber direction $\boldsymbol{f}$ on the cz-plane corresponding to the fiber orientation angle $\alpha$ with respect to the circumferential direction $\boldsymbol{c}$,

$$\boldsymbol{p} = \text{proj}(\boldsymbol{f}) = \cos(\alpha)\, \boldsymbol{c} + \sin(\alpha)\, \boldsymbol{z}. \quad (8)$$

Finally, we rotate the fiber direction $\boldsymbol{p}$ from the cz-plane back into the sheet plane tangent to the surface,

$$\boldsymbol{f} = [-\boldsymbol{p} \cdot \boldsymbol{s}]\, \boldsymbol{n}_{\mathbf{cz}} + [\boldsymbol{n}_{\mathbf{cz}} \cdot \boldsymbol{s}]\, \boldsymbol{p}. \quad (9)$$

At this point, we have assigned fiber and sheet directions $\boldsymbol{f}$ and $\boldsymbol{s}$ to each surface node on $\partial \mathcal{B}_{\phi^+}$ and $\partial \mathcal{B}_{\phi^-}$, which we can then normalize for the sake of convenience. To obtain smooth fiber distributions, we interpolate $\boldsymbol{f}$ and $\boldsymbol{s}$ into the cardiac domain $\mathcal{B}$ by interpreting each vector component as scalar-valued feature $\theta$ and solve the corresponding Poisson problem (1) a total of six times. Theoretically, to increase computational efficiency, we could interpolate the two first components of $\boldsymbol{f}$ and $\boldsymbol{s}$, e.g., the components in the $x$- and $y$-directions, and then determine the third vector component, the component in the $z$-direction, through normalization. This would reduce the computational cost to solving the Poisson problem a total of four times, plus calculating the third vector component for both $\boldsymbol{f}$ and $\boldsymbol{s}$. Table 1 summarizes the fiber and sheet interpolation algorithm for a given heart mesh and given fiber angles $\alpha$ on the cardiac surfaces.

**Remark 1—**If we preferred the sheet directions to be orthonormal, we could first interpolate the normal directions $\boldsymbol{n}$ using the Poisson interpolation. Then, we would create sheet normals $\tilde{\boldsymbol{s}}$ orthogonal to the fiber direction $\boldsymbol{f}$.

$$\tilde{\boldsymbol{s}} = [\boldsymbol{f} \times \boldsymbol{s}] \times \boldsymbol{f} \quad (10)$$

We recommend reprojecting the sheet normal directions $\boldsymbol{s}$ instead of the fiber directions $\boldsymbol{f}$ because of the no-contact assumption.

**Remark 2—**Our algorithm is designed to interpolate sparse vector fields across a given domain. The user-defined constraints, which serve as Dirichlet boundary conditions, do not necessarily have to be prescribed at the epicardial and endocardial surfaces $\partial \mathcal{B}_{\phi^+}$ and $\partial \mathcal{B}_{\phi^-}$, but could potentially also be prescribed anywhere inside the domain.

## 3. Results

To illustrate our algorithm, we analyze four benchmark problems. First, we use an idealized geometry compatible with prolate spheroid coordinate systems to compare our method against an analytically assigned fiber orientation. Second, we utilize a human heart mesh to demonstrate how our algorithm works on a real patient-specific geometry. Third, we analyze the sensitivity of the fiber distribution with respect to randomly distributed boundary conditions. Fourth, we perform a second sensitivity analysis, however now with respect to uniformly distributed boundary conditions.

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### 3.1 Generic bi-ventricular heart model

Figure 2 shows the Poisson interpolation providing a smooth continuum description across the predefined boundary sets. We observe a spiral-like fiber orientation in the left endocardium, which contributes to the characteristic torsional motion in the left ventricle.

Figure 3 shows the fibers and sheet plane normals at slices towards the top, middle, and bottom of the ventricles. Even the fiber orientation around the apex, a challenging region for fiber assignment, seems reasonable and smooth. In summary, our feature-based Poisson interpolation agrees excellently with the analytically assigned fiber orientations of the generic bi-ventricular heart model (Göktepe & Kuhl 2010).

### 3.2 Patient-specific human heart model

Figure 4 shows the results for the patient-specific heart model generated from magnetic resonance images (Kotikanyadanam et al. 2010; Wong et al. 2012). Similar to the generic idealized heart, the Poisson interpolation provides a smooth continuum description across the predefined boundary sets.

Figure 5 illustrates the resulting fiber and sheet interpolations. The solution of the Poisson problem creates a smooth interpolation through the thickness of the myocardium for both fiber directions and sheet plane normals. Again, we can observe the characteristic torsion-inducing fiber orientation in both ventricles. Like the generic model, the fiber distribution near the apex is smooth and continuous. In summary, the algorithm is capable of creating smooth fiber and sheet interpolations on arbitrary patient-specific geometries, even in geometrically challenging regions such as the apex.

### 3.3 Sensitivity analysis for randomly distributed boundary conditions

Since our fiber interpolation algorithm is framed as a boundary value problem with user-defined constraints, we next perform numerical tests to determine the relative sensitivity of the overall fiber distribution with respect to changes in the boundary conditions. In our first set of tests, we select random subsets of our boundary conditions ranging from 10% to 100% of all boundary conditions at the surface nodes. These tests serve to determine the amount of error in angle distributions we might expect from a random, and possibly poor spatial uniformity of measurements. We define the angle error $e^{\angle}$ as

$$e^{\angle}=|\arccos(\boldsymbol{f}\cdot\boldsymbol{f}^{\mathrm{int}})|, \quad (11)$$

where $\boldsymbol{f}$ and $\boldsymbol{f}^{\mathrm{int}}$ are the exact and interpolated fiber orientation.

Figure 6 illustrates representative fiber interpolation errors from 10%, top left, to 90%, bottom right, of all possible boundary conditions used. Since we have chosen the random subsets of the boundary conditions randomly for each ratio, we expect the angle error to be more or less randomly distributed across the mesh. As the subset of assigned boundary condition grows from 10% to 90%, the angle error decreases.

Figure 7 displays the average angle error for varying random boundary condition ratios from 10% to 100%. As a rule of thumb, when using randomly distributed boundary conditions, to obtain an angle error of 10% or less on average, we have to prescribe at least 70% of the surfaces nodes as boundary conditions.

### 3.4 Sensitivity analysis for uniformly distributed boundary conditions

Last, since real fiber data extracted from histology or diffusion tensor imaging are typically acquired in a spatially uniform manner, we performed a mesh refinement test. We generated a coarse patient-specific mesh of 4,563 nodes (Kotikanyadanam et al. 2010) and refined it recursively twice using edge bi-section subdivision. For each of the three meshes, we performed a Poisson interpolation to generate fiber orientations and assigned boundary conditions only to the original 4,563 nodes. We then calculated the angle error (11) between the two subdivided meshes averaged over all nodes. Table 3.4 summarizes the generating parameters and mesh data. Figure 8 illustrates the results of the mesh refinement study. With spatially uniform subsets of the boundary conditions, in contrast to the previous problem, we observe that the error is less than 10% for only 6.25% of the surface nodes prescribed as boundary conditions.

Figure 9 illustrates the fiber angle error in first subdivision mesh compared to second subdivision mesh. When using uniformly distributed boundary conditions, most fibers display an error of 20% or less. This indicates that our algorithm works best when interpolating uniformly distributed features across the heart.

## 4. Discussion

We believe that the results from the previous section demonstrate the advantages of a geometrical approach, compared to a mathematical approach, when generating fiber orientation maps for computational human heart models. The proposed method is able to reproduce fiber orientations for given fiber angles on an idealized geometry (Nielsen et al. 1991). As the analytical fiber distribution elegantly enforces a linear interpolation across the thickness of the ellipsoidal ventricle (Göktepe & Kuhl 2010), a Poisson interpolation with uniform properties on the same geometry generates identical results. However, by using our geometrical approach, we can avoid singularities in the septum and at the apex inherent to the mathematical approach by directly controlling the fiber orientation when $\boldsymbol{n}$ and $\boldsymbol{z}$ are perfectly aligned.

Our geometrical approach allows us to easily transfer the elegance of the mathematical approach to arbitrary patient-specific geometries (Kotikanyadanam et al. 2010). For finite element models of anisotropic electrical conduction (Chen et al. 2012; Dal et al. 2012) or anisotropic cardiac mechanics (Göktepe et al. 2011; Wong et al. 2012), our geometrical approach is convenient since its results can immediately feed into a finite element analysis. The Poisson interpolation is easy, fast, and robust at various mesh granularities.

Here we have chosen linear tetrahedral elements since they are closely related to concepts of discrete exterior calculus used in computer graphics to interpolate surface features on arbitrary geometries (Fisher et al. 2007). In addition, tetrahedral elements are widely used and easy to understand. However, the proposed concept is easily expandable to hexahedral elements. In fact, it is even compatible with cubic Hermitian elements (Nielsen et al. 1991), which already contain the necessary information regarding surface normals, sheet direction, and tangent direction.

Another advantage of solving the Poisson interpolation with finite element solvers is that the finite element method ensures the best approximating solution if the governing differential equation is linear. This has some interesting implications. For example, it means that the method presented can also properly account for non-constant value boundary conditions, which may occur if more detailed fiber orientation measurements are available. Given experimental measurements, our algorithm returns the best approximating solution that matches the fiber orientation for different mesh refinement levels. To further fine-tune control of the fiber orientation profile, the finite element method allows us to easily integrate additional constraints.

Previously, least squares methods have been proposed to generate fiber orientation distributions (Costa et al. 1996). However, there are fundamental differences between the finite element method and the least squares methods. The least squares approach will try to find the solution that most closely resembles the measured fiber orientation (Toussaint et al. 2010). As such, it strongly depends on the character of the interpolating shape functions and on the underlying discretization scheme. The finite element method, however, enforces user-defined fiber orientations at the specific nodes. It satisfies the Dirichlet boundary conditions in a strong sense, and determines the best approximating fiber orientation at all other nodes. However, least squares methods and this finite element method are not exclusive approaches and can be used in tandem when necessary. For example least squares methods can be used to smoothen out the distribution on surfaces, which our finite element method can then interpolate through the thickness.

Perhaps the most exciting and most useful application of our feature-based Poisson interpolation compared to analytical and least squares approaches is the ability to fill in gaps when there is lack of experimental data. Since it is known that the heart has an organized fiber orientation and it may be significantly easier to measure the surfaces of tissue rather than the inner layers, our method can provide a reasonable approximation using a combination of observable results and a more algorithmically-methodical approach. Even if we only measure a relatively sparse, yet spatially uniform, set of fiber directions, either histologically (Ennis et al. 2008) or from diffusion tensor imaging (Toussaint et al. 2010a), fiber diffusion can fill in the gaps and create a reasonable and smooth solution as shown in Figure 8. In addition, we can easily integrate experimental data at no-nodal points, e.g., from magnetic resonance imaging (Böl et al. 2011), using constraint boundary conditions.

The proposed method provides a variety of natural extensions and generalizations that may be of interest to the biomechanics community. For example, we could easily apply the underlying concept to other muscular tissues such as skeletal muscle (Böl & Reese 2008; Lu et al. 2011), or to collagenous structures in other biological tissues. If we assume that the transitioning nature between two surfaces in a body is not linear, we could varying the diffusion tensor at varying points within the body to match the description accurately. While the heart’s transmural fiber orientation transition is almost linear (Ennis et al. 2008), by varying the transmural or in-plane conductivity, we could potentially approximate nonlinear transmural and regional fiber variations. Likewise, the fiber orientation distribution can be more carefully adjusted and tuned by possibly using inequality constraints, which might be more physiological. Another possible extension would be using noisy source terms with the finite element model to account for biological variability. Along a similar vein, fiber interpolation can be used as a tool to generate smoothly varying fiber fields with random defects to represent diseased tissue conditions (Schmitt et al. 2009), similar to the bottom row of Figure 7. We could generate and control the amount of disorganization within the fiber structure, and virtually investigate the impact of different clinical therapies on electrical and mechanical function in cases where the fiber orientation is globally irregular or locally distorted.

## 5. Conclusion

We have presented a novel Poisson interpolation-based approach to create fiber orientation maps in real patient-specific heart geometries. The algorithm makes creative use of existing and common linear finite element element programs for diffusion or thermal problems. Adopted from computer graphics, the underlying concept is based on diffusing a defined subset of fiber orientations throughout the domain of interest to create smoothly interpolating fields. In the finite element setting, this subset is simply treated as Dirichlet boundary condition and enforced in a strong sense. Here we have shown the application of the Poisson interpolation scheme to create fiber orientation maps across the heart, but the general concept can easily be applied to other arbitrarily shaped biological tissues with known organized fiber orientations. We have shown that the algorithm does generally work for both arbitrarily and uniformly distributed input data. However, our interpolation errors were significantly lower for uniform distributions. In summary, the proposed method is computationally efficient, robust, and easy to solve. It is immediately transformative to generate smoothly interpolated fields in experimental and clinical settings, in which data sets are sparse but uniformly distributed.

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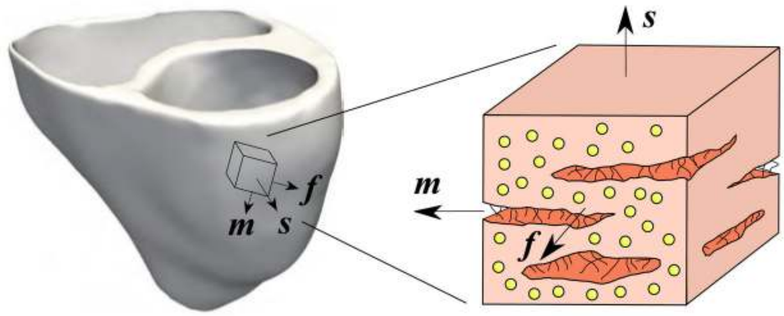


**Figure 1.**

Characteristic microstructure in the human heart. Fiber directions $\boldsymbol{f}$ vary transmurally from approximately −70° in the epicardium, the outer wall, to +80° in the endocardium, the inner wall. Fiber vectors $\boldsymbol{f}$ and sheet vectors $\boldsymbol{m}$ locally span the sheet plane, with the sheet plane normal $\boldsymbol{s}$ pointing outwards, approximately orthogonal to the epicardial wall.

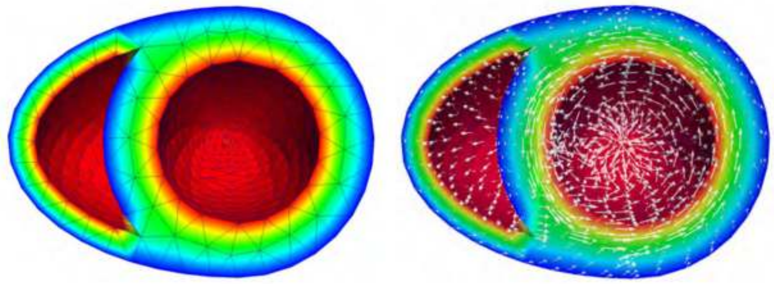

**Figure 2.**
(Left) Poisson interpolation results of fiber orientation angle with respect to circumferential fiber. Epicardial and right ventricular septal surfaces are assigned as −70°, while the endocardial surfaces are assigned as 80° as boundary conditions. (Right) Resulting interpolated fiber directions throughout the heart.

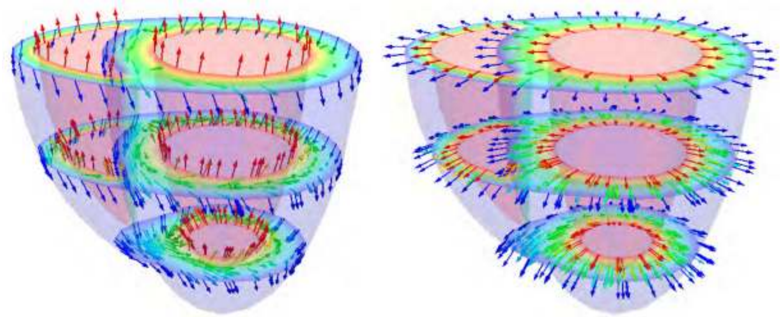

**Figure 3.**
(Left) Fiber directions at various slices in the heart are shown. (Right) The normal directions at the corresponding slices are shown. The colors at each point of the cross-sectional slices correspond to the fiber angle orientation with blue representing −70° and red representing +80°.

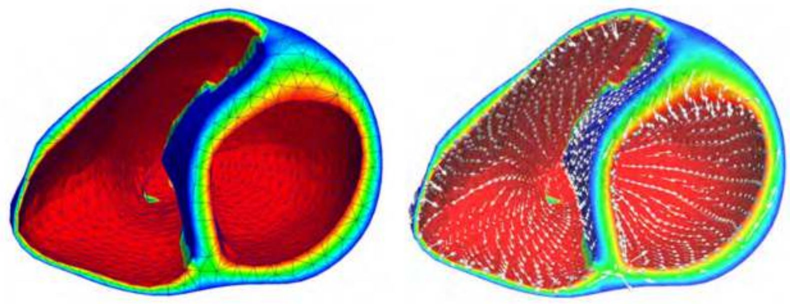

**Figure 4.**

(Left) Poisson interpolation results of fiber orientation angle with respect to circumferential fiber. Epicardial and right ventricular septal surfaces are assigned as −70°, while the endocardial surfaces are assigned as 80° as boundary conditions. (Right) Resulting interpolated fiber directions throughout the heart.

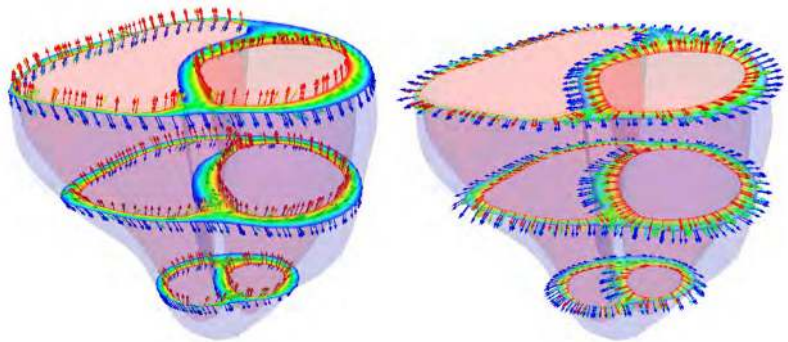

**Figure 5.**
(Left) Fiber directions at various slices in the heart are shown. (Right) The normal directions at the corresponding slices are shown.

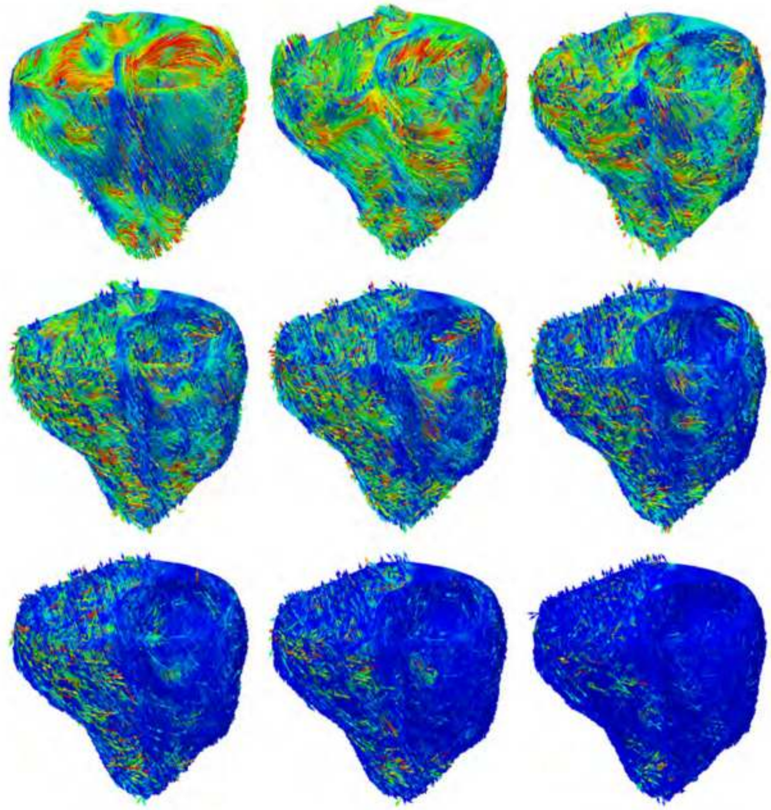

**Figure 6.**
Representative figures of angle fiber error given random subsets of boundary conditions for the fiber interpolation algorithm. Fibers highlighted in blue have no error (0°) whereas red fibers are closer to perpendicular (90°). Increasing sizes of random subsets are chosen and are shown starting from 10% (top left), 20% (top center), 30% (top right), 40% (left), 50% (center), 60% (right), 70% (bottom left), 80% (bottom center), 90% (bottom right).

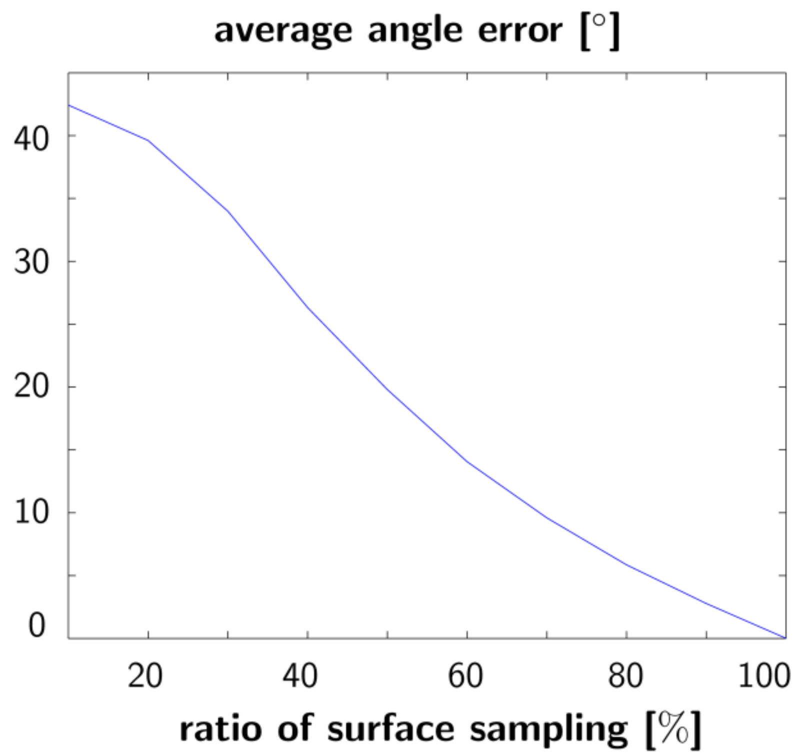


**Figure 7.**
Average angle error when 10% to 100% of the surface nodes used as boundary conditions to generate fiber orientation distribution.

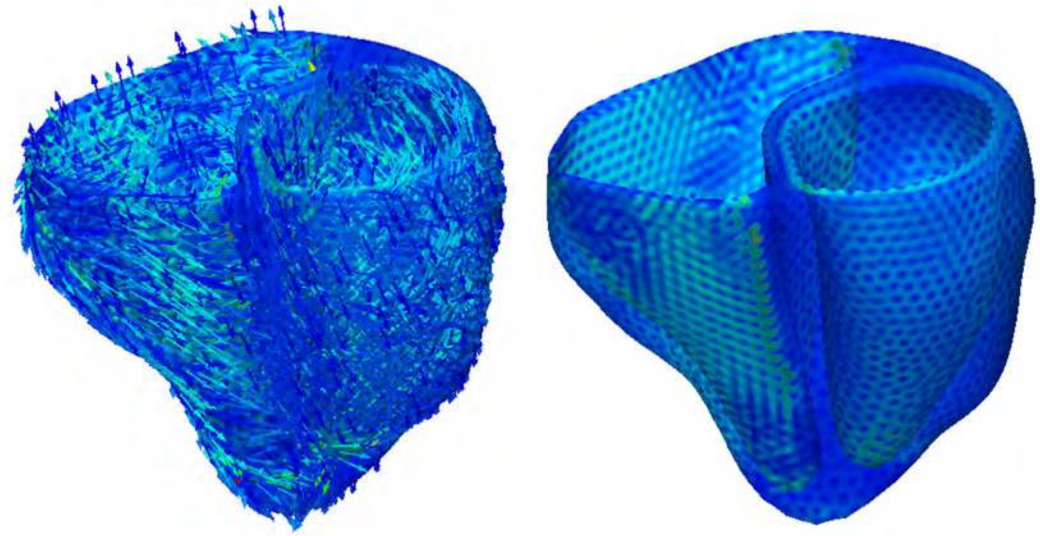

**Figure 8.**

(left) Fiber direction color coded by angle error. (right) Fiber angle error over the heart without fibers. Small hexagonal patterns are formed, because the nodes at the coarsest mesh are used as the boundary condition set, and therefore the new nodes on the surfaces are not part of this boundary condition set.

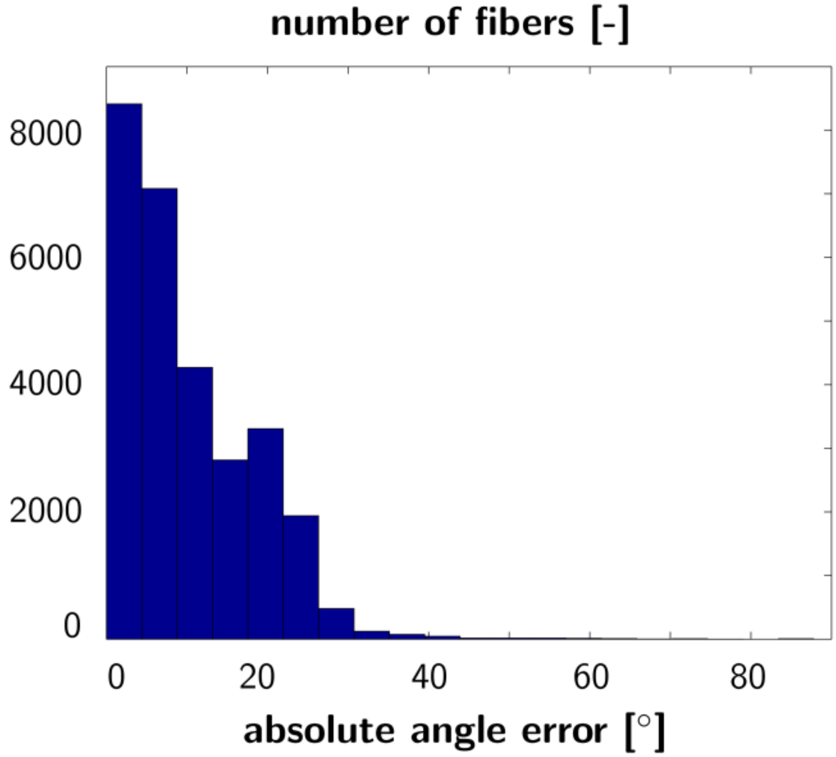


**Figure 9.**
Histogram of fiber angle error in first subdivision mesh compared to second subdivision mesh. The histogram is organized into bins of 5 ° error.

**Table 1**

Algorithm for creating smoothly varying fiber and sheet orientations ***f*** and ***s*** within a human heart, for a given mesh and prescribed fiber angles $\alpha$ on the surfaces of the heart.

| | |
|---|---|
| Identify epicardial and endocardial surfaces $\partial\mathcal{B}_{\phi+}$ and $\partial\mathcal{B}_{\phi-}$ | |
| For all nodes on $\partial\mathcal{B}_{\phi+}$ and $\partial\mathcal{B}_{\phi-}$ | |
| Assign local cardiac coordinate system $\{\boldsymbol{n}, \boldsymbol{c}, \boldsymbol{z}\}$ | (3) |
| Assign sheet plane normal ***s*** | (6) |
| Assign fiber angles $\alpha$ with respect to horizontal circumferential direction ***c*** | (7) |
| Calculate projection ***p*** of the fiber direction | (8) |
| Calculate fiber direction ***f*** from rotation of projection into sheet plane | (9) |
| Solve Poisson equation for each component of ***f*** and ***s*** interpreted as feature $\theta$ using ***f*** and ***s*** on $\partial\mathcal{B}_{\phi+}$ and $\partial\mathcal{B}_{\phi-}$ as Dirichlet boundary conditions | (1) (2) |
| Collect and normalize the three components of ***f*** and ***s*** at each node | |
| [*Optional*] Correct sheet normals such that ***f*** and $\tilde{\boldsymbol{s}}$ form an orthonormal basis | (10) |
| [*Optional*] Interpolate ***f*** and ***s*** to integration points or element centroids | |

**Table 2**

Table of parameters for random and subdivision examples.

| **Surfaces** | **Fiber angle [°]** |
|---|---|
| Epicardium, right septal wall, $\partial \mathcal{B}_{\phi^+}$ | −70 |
| Left endocardium, right free wall, $\partial \mathcal{B}_{\phi^-}$ | +80 |

| **Subdivision** | **# of surface nodes** | **# of total nodes** |
|---|---|---|
| Initial mesh | 3,839 | 4,563 |
| 1st Subdivision | 15,350 | 28,639 |
| 2nd Subdivision | 61,394 | 198,041 |
| Average angle error | 9.965° | |