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GENERATION OF COHERENT RADIATION BY A RELATIVISTIC ELECTRON BEAM IN AN ONDULATOR*

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A detailed study of the self-modulation of a relativistic electron beam in an ondulator in the single-pass regime is carried out. Beam-parameter conditions are obtained under which the radiative instability in question occurs. The possibility of constructing a source of coherent radiation based on this principle is discussed. The radiation specifications of such a source are analyzed. Control the mass of longitudinal motion with the help of an additional longitudinal magnetic field introduced in the ondulator is discussed. Numerical examples are given for sources of submillimeter and infrared-range radiation.

1. INTRODUCTION

In recent years there has been a move toward construction of sources of coherent radiation in which the electrons moving along a periodically curved trajectory are used. The so-called free-electron laser (FEL) belongs to this class of device. The relativistic electron beam in this laser passes through an ondulator (a periodic transverse magnetic field) located in an open optic resonator.

In this work we study a simpler situation in which the electron beam passes through an ondulator and unlike FEL, there is no resonator. We study the problem of radiative instability of the beam in an ondulator. With certain restrictions on the beam parameters, the harmonics of density whose wavelength at a given energy resonate with the ondulator period become unstable. Generally speaking, for the instability to be evident, some initial level of density (or of current) oscillations is required at the entrance of the ondulator. Statistical density fluctuations can play this role of initial excitation. For sufficient length of the ondulator, the resonant harmonics of density fluctuations become large enough during a pass that the modulated beam radiates from a definite section of the ondulator. Such a scheme may be used as an independent source of coherent radiation or as an amplifier.

The effects of relativistic-beam self-modulation in the single-pass regime were studied in Refs. 2-5. The case of an infinitely wide electron beam was analyzed in Refs. 2-4. The authors employed either the methods of plasma physics,[2] or those of high-frequency device theory.[3,4] The results of a comprehensive study of the effect of relativistic electron-beam self-modulation in an ondulator are presented in Ref. 5. It should be noted that the authors of that paper made use of the methods usually applicable in a study of coherent beam instabilities in storage rings. The existence of the important, from the practical point of view, case of the so-called "narrow" beam, when the parametric dependence of the basic quantities seems to be different from the case of a "wide" beam, was shown. The results of those papers show that the creation of high-power coherent sources in the infrared and submillimeter ranges of the wavelengths where they are do not now exist is a valuable application of the effect of beam self-modulation. Even at comparatively low beam currents, the amplitude of density modulation increases by a few times already in a time of the order of tens of ondulator periods in this range, which makes it possible to circumvent the use of resonator systems. The aim of the present paper is to draw conclusions basing upon the results of the paper of Ref. 5 and also to analyze a number of new problems. Most important is a study of the possibility of controlling the effective mass of longitudinal motion of the electrons in an ondulator with the help of an additional external longitudinal magnetic field. It is shown that this permits one to increase the frac-

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tion of the electron beam energy converted into radiation and also to shorten the ondulator length. Problems concerning the practical realization of a coherent source are also considered.

2. DESCRIPTION OF THE EFFECT OF ELECTRON-BEAM SELF-MODULATION IN AN ONDULATOR

Let us present a theory of the growth of longitudinal electron-beam modulation in an ondulator. Let all the electrons move with the same velocity $\mathbf{v}$. Let us send such a beam through the ondulator, whose field repeats periodically in a period λ_0: $\mathbf{H}(y)=\mathbf{H}(y+\lambda_0)$, where y is the coordinate along the axis of the ondulator coinciding with the direction of beam motion. Transverse to the axis, the field $H_\perp$ in the ondulator depends only on the longitudinal coordinate y. In particular, for a helical ondulator $H_\perp = H_x + iH_y = H_0 \exp(-i\kappa y)$, where $\kappa = 1/ƛ_0 = 2\pi/\lambda_0$ and H_0 is a constant. Upon deflection from the ondulator axis, the longitudinal magnetic field in the transverse direction (rot $H = 0$) is: $|H_{\shortparallel}{}^0| \sim \|H_\perp|$ $\sigma/ƛ_0$, where σ is the transverse beam dimension). This field results in the beam focusing in the transverse direction.

Besides the ondulator field introduce also an external longitudinal magnetic field, allowing additionally the control of the instability development. In such fields the forced velocity of electron motion may be written in the form

$$\mathbf{v}_s = v_y(\mathscr{E},y)\mathbf{e}_y + \mathbf{v}_\perp(\mathscr{E},y),$$

where the velocity components v_y and $\mathbf{v}_\perp$ are functions of the electron energy $\mathscr{E}$ and are periodic functions of y with period λ_0. Here we assume that $1 - v_y \ll 1$ and therefore we take $v_y = 1$ everywhere possible. In particular, in the field of the helical ondulator

$$v_\perp = v_x + iv_z = u \exp(-i\kappa y),$$

$$u = \mathrm{K}/(\gamma - \mathrm{K}_\parallel),\ v_y = \text{const},$$

where

$$\mathrm{K} = eƛ_0|H_0|/m,\ \mathrm{K}_\parallel = eH_\parallel ƛ_0/m$$

$$\equiv \gamma\omega_\parallel/\kappa,\ \gamma = (1 - v^2)^{-1/2}$$

Let us study the dynamics of beam modulation in the radiation field without the action of a Coulomb field taken into account. The transverse rotation of electrons is assumed to be given by the ondulator fields. In this case, radiation can result in changing the longitudinal motion only.

As the reaction of radiation on beam modulation is important only over lengths greatly exceeding the ondulator period, it is reasonable to carry out averaging of the equations of motion over times of the order of λ_0.

If one proceeds to the canonically conjugate variables $s = y - \int^t v_y(\mathscr{E}_0,y)dt$ and $P = \mathscr{E} - \mathscr{E}_0$, where $\mathscr{E} - \mathscr{E}_0$ stands for the energy deviation, and expand over small deviation P, one gets the Hamiltonian $\mathscr{H}(P, s, y)^v$ describing the relative motion of electrons with radiation. It is convenient to use the longitudinal coordinate as a time. The Hamiltonian can be derived with the help of the canonical transformation $[(\mathbf{P} - e\mathbf{A})^2 + m^2]^{1/2}$.

$$\mathscr{H} = P^2/2\mathscr{E}M - e\langle \mathbf{v}_\perp \hat{\mathbf{A}}\rangle,$$

where $\hat{\mathbf{A}}$ is the vector potential of the radiation field and $\mathscr{E}M$ the mass of the longitudinal motion, which relates the longitudinal velocity variation to energy change

$$M^{-1} = \mathscr{E}\left\langle \frac{ds/dy}{P} \right\rangle = \mathscr{E}\left\langle \frac{dv_y}{d\mathscr{E}} \right\rangle = \frac{1}{\gamma^2} - \gamma\left\langle \mathbf{v}_\perp \frac{\partial \mathbf{v}_\perp}{\partial \gamma} \right\rangle, \quad (2.1)$$

where the braces $\langle ... \rangle$ denote averaging over y. In particular, for a helical ondulator, the expression for M is

$$\frac{1}{M} = \frac{1}{\gamma^2} + \frac{\kappa|u|^2}{(\kappa - \omega_\parallel)}. \quad (2.2)$$

The vector potential of the radiation field $\hat{\mathbf{A}}(r,t)$ is connected with the periodic variation in the electron velocity in the ondulator by

$$\frac{\partial^2 \mathbf{A}}{\partial x^2} + \frac{\partial^2 \mathbf{A}}{\partial Z^2} + \frac{\partial^2 \mathbf{A}}{\partial y^2} - \frac{\partial^2 \mathbf{A}}{\partial t^2} = -4\pi e \mathbf{v}_\perp(\mathscr{E}_0,y)\rho(\mathbf{s}_\perp,s,y), \quad (2.3)$$

where the particle density is expressed in the variables $\mathbf{s}_\perp = \mathbf{r}_\perp - \int \mathbf{v}_\perp dy$ and s, coordinates characterizing the relative positions of the electrons in the transverse and longitudinal directions. The dependence of the density ρ on y thus characterizes a slow variation in the modulation amplitude, which takes place over wavelengths greatly exceeding the ondulator period.

Using Eq. (2.3), we obtain the Hamiltonian

$$\mathcal{H} = \frac{P^2}{2\mathcal{E}M} - e^2\left\langle \mathbf{v}_\perp \int_0^L dy' \int ds_\perp' \frac{\mathbf{v}_\perp(y')}{|r - r'|} \rho(\mathbf{s}_\perp', s + y' - y + V|\mathbf{r} - \mathbf{r}'|, y')\right\rangle, \quad (2.4)$$

where L is the length of the ondulator, $|\mathbf{r} - \mathbf{r}'| = [(y - y')^2 + (\mathbf{s}_\perp - \mathbf{s}_\perp')^2]^{1/2}$, $V = \langle v_y(\mathcal{E}_0, y)\rangle$. The part of the total integral $\sim\int_0^L dy'$ proportional to $\int_0^y dy'$ describes the "forward" action of radiation along the direction of beam motion, while the remaining part $\int_y^L dy'$ is the "backward" action. As can be seen from Eq. (2.4), the forward radiation resonates with harmonics of density that are modulated in the longitudinal direction at frequencies $K_n = n\,[\lambda\!\!\!^{-}_0(1 - V)]^{-1} \simeq 2n\gamma_{11}^2/\lambda\!\!\!^{-}_0$, where $n = \pm 1, \pm 2, \ldots, n/\lambda\!\!\!^{-}_0 = \kappa_n$ are the spectral frequencies of $v_\perp(y)$, and $\gamma_{11} = (1 - V^2)^{-1/2}$. The backward radiation resonates at lower frequencies $K_n = n[\lambda\!\!\!^{-}_0(1 + V)]^{-1}$. A larger increment at rate of instability is associated with the forward radiation and therefore the backward radiation is neglected below.

An equation that describes the variation of particle densities $\rho(s,y)$ ($\mathbf{s}_\perp$ = const) in the ondulator may be derived from the kinetic equation for the particle distribution $f(P,s,y)$, $\partial f/\partial y + (P/\mathcal{E}M)(\partial f/\partial s) - (\partial\mathcal{H}/\partial s)(\partial f/\partial P) = 0$. Hence

$$\frac{\partial^2\rho}{\partial y^2} = \frac{1}{\mathcal{E}M}\frac{\partial}{\partial s}\left[\frac{\partial\mathcal{H}}{\partial s}\rho + \frac{1}{\mathcal{E}M}\frac{\partial}{\partial s}\int P^2 f dP\right], \quad (2.5)$$

where $\rho = \int f dP$. The second term in the right-hand side of Eq. (2.5) is proportional to the square of the density modulation amplitude. Therefore, in the linear amplitude-modulated approximation this term is neglected and we get the simple result

$$\frac{\partial^2\rho}{\partial y^2} = \frac{1}{\mathcal{E}M}\frac{\partial^2\mathcal{H}}{\partial s^2}\rho_0, \quad (2.6)$$

where ρ_0 ($\mathbf{s}_\perp$, s) is the beam density at the entrance of the ondulator.

Let us expand $v_\perp$ in a Fourier series and express the density in the form where the strong dependence of ρ and s is obvious

$$v_\perp = \sum_n u_n \exp(i\kappa_n y),\ \rho = \rho_0 + \sum_{n=0} a_n(\mathbf{s}_\perp, s, y)\exp(-iK_n s).$$

After simple calculations with the help of Eqs. (2.6), (2.4), and $[|\mathbf{r} - \mathbf{r}'| \simeq (y - y') + |\mathbf{s}_\perp - \mathbf{s}_\perp'|^2/2\,(y - y')]$ one can derive equations for the modulation amplitudes a_n. For a narrow beam ($\sigma \to 0$) the equation has the form*

$$\frac{\partial^2 a_n}{\partial y^2} = \frac{\dot{N}r_e}{2\gamma M}|u_n|^2 K_n^2 \int_0^y \frac{a_n(\mathbf{s}_\perp, s + (1 - V)(y' - y), y')}{y - y' + iK_n\sigma^2}dy', \quad (2.7)$$

where $V\dot{N} = V\int \rho d\mathbf{s}_\perp$ is the beam current and $r_e = e^2/m$. For a wide beam, the equation for a_n is transformed into ($\sigma \to \infty$)

$$\frac{\partial^2 a_n}{\partial y^2} = \frac{\pi}{i}\frac{\rho_0(\mathbf{s}_\perp, s)}{\gamma M} r_e|u_n|^2 K_n \int_0^y a_n(\mathbf{s}_\perp, s + (1 - V)(y - y'), y')\,dy'. \quad (2.8)$$

The characteristic quantity σ^2 that distinguishes the narrow and wide beam is

$$\sigma_{cr}^2 = \frac{1}{K_n^2|u|}\sqrt{\frac{\gamma|M|}{\dot{N}r_e}}. \quad (2.9)$$

It is seen from Eq. (2.8) that for a wide beam the increment for a particle at the beam center is higher than that for the particles at a distance $|\mathbf{s}_\perp| \neq 0$ from the center.

All of the harmonics of modulation a_n increase independently at the initial stage, and we now omit the index n, assuming it to correspond to a maximum increment.

Let us solve Eqs. (2.7) and (2.8) for the case of a continuous beam. For the resonant harmonic of density ($K = 2\gamma_{\parallel}^2\kappa$), the amplitude a is inde-

* Eq. (2.7) is derived for a Gaussian distribution in the transverse dimension, $\rho_\perp = (2\pi\sigma^2)^{-1}\exp(-S_\perp^2/2\sigma^2)$. Due to the logarithmic dependence on the transverse beam dimensions, the increment is slightly sensitive to the specific form of the distribution $\rho_\perp(\mathbf{s}_\perp)$.

pendent of s. The initial conditions for a will be $(\partial a/\partial y)_{y=0} = 0$, $a_{y=0} = a_i(s_\perp)$. In the case of a wide beam, the solution of Eq. (2.8) is

$$a = \frac{a_i}{3}(\exp \Lambda_1 y + \exp \Lambda_2 y + \exp \Lambda_3 y),$$

where

$$\Lambda_1 = |\Lambda| \exp\left(-i\frac{\pi}{6}\right), \Lambda_2 = -|\Lambda| \exp\left(i\frac{\pi}{6}\right), \Lambda_3 = i|\Lambda| = i\left[\frac{\pi r_e \rho_0}{\gamma|M|}|u|^2 K\right]^{1/3}$$

Hence the amplitude of modulation grows exponentially. The characteristic length in which the amplitude becomes e times larger (at $|\Lambda|y \gg 1$), as one can see from Eq. (3.12), is

$$l = \frac{2}{\sqrt{3}|\Lambda|} = \frac{2}{\sqrt{3}}\sqrt{\frac{\gamma|M|}{\pi \dot{N} r_e \rho_\perp |u|^2 K}}, \qquad (2.10)$$
$$\left(\rho_\perp = \frac{\rho_0}{\dot{N}}, \int \rho_\perp ds_\perp = 1\right).$$

In the case when $M = \gamma_\parallel^2$, the expression (2.10) coincides with the corresponding formulas of Refs. 2-4. For a narrow beam, the corresponding expression for $l = Re\Lambda^{-1}$ satisfies the relation*

$$\Lambda^2 = \frac{\dot{N} r_e}{2\gamma M}|u|^2 K^2 \int_0^\infty \frac{\exp(-\Lambda y)}{y + iK\sigma^2}dy \simeq$$
$$\simeq \frac{\dot{N} r_e}{2\gamma M}|u|^2 K^2 [\ln (i\Lambda K\sigma^2)^{-1} - c + \ldots], \qquad (2.11)$$

where $c \simeq 0.58$ is the Euler constant. In particular, for a narrow beam and positive longitudinal mass, if $\ln (\sigma_{cr}^2/\sigma^2) \gg 1$, we get from Eq. (2.11)

$$l = \frac{ƛ}{|u|}\left[\frac{2\gamma|M|}{\dot{N} r_e \ln(\sigma_{cr}^2/\sigma^2)}\right]^{1/2} \qquad (2.12)$$

where $ƛ = ƛ_0(1 - V)$. In the case when $M = \gamma_\parallel^2$, formulas (2.11) and (2.12) coincide with the corresponding formulas of Ref. 5.

The characteristic width of the spectrum of density harmonics ΔK_c (the amplitude of the harmonic a at $K = 2\gamma_\parallel^2\kappa + \Delta K_c$ is e times less than the amplitude at $K = 2\gamma_\parallel^2\kappa$), which can be unstable for the narrow beam at $M > 0$, $\ln \sigma_{cr}^2/\sigma^2 \gg 1$ and the wide, correspondingly, is

$$\Delta K_c = \left[\frac{2 \ln \sigma_{cr}^2/\sigma^2}{\ln \rho_0/\tilde{\rho}_i}\right]^{1/2} \frac{ƛ_0}{l}|K|, \Delta K_c = 2[3/\ln \rho_0/\tilde{\rho}_i]^{1/2}\frac{ƛ_0}{l}|K|.$$

These results can also be obtained from qualitative considerations which give another representation of the process of modulation development. Let us evaluate the radiation fields acting on the particles in the beam. Consider a continuous electron beam completely modulated in the longitudinal direction and passing through the ondulator of length y_0. The beam in the ondulator is a set of periodic radiators that oscillate in such a way that the radiation fields add coherently at zero angle in the direction of beam motion. It follows from simple geometric considerations that the angle at which such coherence still occurs (the angle of diffraction) is of the order of $(ƛ/y_0)^{1/2}$ for a small area and $ƛ/\sigma$ for a large area. The characteristic quantity distinguishing these limiting cases is the area of the radiation flux cross section in the ondulator in the narrow beam, i.e., of the order of $ƛ y_0$.

To estimate the radiation field $\hat{E}$, energy balance can be used, which enables one to estimate also the coherent radiation power W of the completely modulated beam. On the one hand, the power W is determined by the flux of radiating energy $W = = \frac{1}{4\pi}\int \hat{E}^2 d\hat{s} \sim \hat{E}^2\hat{S}$, where $\hat{S}$ is the characteristic radiation flux cross section in the ondulator. On the other hand, the field inside the beam $\hat{E}$ does work on the electrons that must be equal to W per unit time: $W \approx e\dot{N}\int_0^{y_0} \hat{E}v_\perp dy$. In resonance, when the period of beam modulation resonates with the ondulator period ($\lambda = \lambda_0(1 - V)$), the quantity $\langle \hat{E}v_\perp\rangle$ is maximum ($\langle \hat{E}v_\perp\rangle \approx |\hat{E}u|$) and $W \approx e\dot{N}|\hat{E}u|y_0$. Thus we obtain $W \approx \hat{E}^2\hat{S} \approx e\dot{N}\hat{E}uy_0$ and we can now estimate the field value and radiation power. For a narrow

* Formulas (2.10) and (2.11) hold for a bunch length much longer than $l/\gamma_\parallel^2$.

beam ($\sigma^2 \ll \lambdabar y_0$) the transverse radiation flux cross section is approximately $\lambdabar y_0$ and*

$$\hat{E}_{\max} \approx e\dot{N}|u|/\lambdabar,\ W \approx e^2\dot{N}^2|u|^2 y_0/\lambdabar \quad (2.13)$$

For a wide beam ($\hat{S} \approx \sigma^2$), we have

$$\hat{E}_{\max} \approx e\dot{N}uy_0/\sigma^2,\ W \approx e^2\dot{N}^2|u|^2 y_0{}^2/\sigma^2. \quad (2.14)$$

Taking into account the reaction of the radiation field on the beam leads to instability of the resonant harmonics of density because of the dissipative part of radiation field. In order to increase the amplitude of initial modulation $\tilde{\rho}$ by a few times, the particles must shift in the longitudinal direction under the influence of radiation field $\hat{E} \approx \hat{E}_{\max}\tilde{\rho}/\rho_0$ by an amount of the order of $\Delta s \approx \lambdabar\tilde{\rho}/\rho_0$. In the resonance region we have

$$\Delta S \approx \Delta V \cdot l \approx \frac{\Delta\mathscr{E}}{M\mathscr{E}}\, l$$

$$\approx \frac{l}{M\mathscr{E}}\, e|\hat{E}_{y_0=l}||u| \approx \lambdabar\tilde{\rho}/\rho_0. \quad (2.15)$$

It is easy from this relation to derive expressions for the characteristic growth length that agree with Eqs. (2.10) and (2.12) up to numerical factors.

Thus, having traversed the length $L = l \cdot \ln(\rho_0/\tilde{\rho}_i)$ ($\tilde{\rho}_i/\rho_0$ is the initial degree of beam modulation) in the ondulator, the beam's modulation amplitude becomes as large as possible. The characteristic length on which the modulation is close to the maximum is determined by the spread of longitudinal velocities that appears under the radiation. As seen from relations (2.15), the characretistic length in which the radiation power is maximum, is of the order of the growth length l. It is not difficult to estimate the total coherent radiation power W. Using formulas (2.13) and (2.14), we get*

$$W \approx \gamma m\dot{N}(|M|\lambdabar/l). \quad (2.16)$$

It is seen that the fraction of the beam energy $|M|\lambdabar/l$ converting into radiation can also be varied by M.

3. LIMITS OF APPLICABILITY OF THE RESULTS

The limitations due to the finite number of the particles in the beam are most obvious; in a coherent volume whose size is the order of a wavelength in the longitudinal direction and the order of a diffraction length $(\lambdabar l)^{1/2}$ in the transverse direction, the number of particles should be very large. Correspondingly, for the narrow and wide beams the following conditions constrain the lower limit of the current.

$$N\lambdabar \gg 1,\ \dot{N}\lambdabar^2 l/\sigma^2 \gg 1. \quad (3.1)$$

We have taken into account that in the initial section of the ondulator the distribution of longitudinal velocities is a δ-function, which is true when particles shifts from the spreads of longitudinal velocities can be neglected. Hence the energy and angular spreads of the particles should not exceed ($\Delta Vl \ll \lambdabar$)

$$\Delta\mathscr{E}/\mathscr{E} \ll |M|\lambdabar|l,\ (\Delta\theta)^2 = (\Delta v_\perp)^2 \ll \lambdabar/l. \quad (3.2)$$

Let us find the limitation on feasible gradient of the longitudinal velocities $|\partial V/\partial \mathbf{r}_\perp|$ in the trans-

* Strictly speaking, this method allows estimation of the dissipative fraction of a radiation field whose phase coincides with the phase density of modulation. There is also the nondissipative part of the radiation field for a narrow beam, whose work over the beam is on the average equal to zero. This part of the field leads to the energy distribution inside the beam. It can be interpreted as a retarding action of the radiation fields of particles placed behind the test particle at distances much longer than a wavelength (unlike the dissipative part of the field whose influence is most strong at distances of the order of a wavelength). As the radiation field falls with distance as r^{-1}, for a narrow beam the nondissipative part of the field with respect to the amplitude is approximately $\ln(\lambdabar y_0/\sigma^2)$ times higher as compared with the dissipative one ($\sigma^2/\lambdabar$ is the shortest distance at which the fields can add coherently). For a narrow beam, when the ondulator is helical, an exact calculation yields

$$\hat{E} = \frac{e\dot{N}u}{2\lambdabar}\left[\frac{\pi}{2}\cos Ks + \left(\ln\frac{y}{K\sigma^2}\right)\sin Ks\right]\exp(-i\kappa y),$$

$$W = \frac{\pi}{4}\gamma_\parallel^2|u|^2 e^2\dot{N}^2\kappa y_0,$$

and for a wide beam, with a Gaussian electron distribution in cross section, we have

$$\hat{E} = \frac{e\dot{N}uy}{2\sigma^2}(\cos Ks)\exp\left(-\frac{r_\perp{}^2}{2\sigma^2} - i\kappa y\right),\ W = \frac{e^2\dot{N}^2}{16\sigma^2}|u|^2 y_0{}^2.$$

* To derive formula (2.16) in the case of a narrow beam, it is supposed that $\ln(\sigma^2/\sigma_{cr}^2) \approx 1$.

verse beam cross section at the entrance of the ondulator. The gradient can be negligible if the relative change in velocity over a coherence length $[(\lambda l)^{1/2}]$ in the transverse direction is small. For the narrow and wide beams respectively, we get

$$\sigma|\partial V/\partial \mathbf{r}_\perp| \ll \lambda/l, \ |\partial V/\partial \mathbf{r}_\perp| \sqrt{\lambda l} \ll \lambda/l. \quad (3.3)$$

When the last condition is fulfilled, the variation in velocity V in the transverse direction may be regarded as an adiabatic one.

Besides the collective radiation fields considered above, the Coulomb field acts on the beam particles. Let us define the range of parameters when the Coulomb field can be neglected. To calculate the Coulomb field E_c, it is convenient to proceed to the particle reference system. In this system the beam density is modulated with period $\gamma_\parallel\lambda$. In the case when $\sigma^2 \ll \gamma_\parallel^2\lambda^2$ the periodic part of the Coulomb field inside the beam is equal to

$$E_c = \frac{2e\dot{N}}{\gamma_\parallel^2\lambda} \ln\left(\frac{2\gamma_\parallel\lambda}{\sigma}\right) \sin Ks \quad (3.4)$$

If the radial dimension is large ($\sigma^2 \gg \gamma_{\parallel}{}^2\lambda^2$), the Coulomb field on the beam axis will be determined by the charge density.

$$E_c = \frac{2e\dot{N}\lambda}{\sigma^2} \sin Ks. \quad (3.5)$$

The radiation fields will play the main role in the dynamics of density modulation if the periodic part of the Coulomb-field projection on the particle velocities is much less than the projection of radiation field: $|\mathbf{E}_c\cdot\mathbf{v}| \approx E_c \ll |\hat{\mathbf{E}}\cdot\mathbf{v}| \approx |\hat{E}u|$. Hence, with the help of formulas (2.13), (2.14), and (3.4) and (3.5) we get as the conditions that must be satisfied.

$$|u|^2 \gg \min\left(\frac{\dot{N}r_e}{\gamma|M|}, \left|\frac{\dot{N}r_e\lambda^2}{\gamma M\sigma^2}\right|^{1/2}\right), \ \sigma^2 \gg \frac{\lambda^2}{|u|^2}. \quad (3.6)$$

It follows from condition (3.6) that $l \gg \lambda_0$ in the region of application for our study. Existence of a range of parameters in which the main interaction between the particles is carried out through the radiation fields is due to the fact that the Coulomb interaction becomes sensitive to the geometry of a beam larger than $\gamma_\parallel\lambda$ in size, while the radiation fields begin to change when to the beam is of diffraction size, $(\lambda l)^{1/2} \approx \gamma_\parallel\lambda\,(l/\lambda_0)^{1/2}$. With increase of the beam cross section, the Coulomb field decreases by l/λ_0 times before the radiation field starts to decrease.

The results obtained are valid under the assumption of relatively small spreads both in energy and longitudinal mass of the particles in the beam. The spread in energies, and consequently in mass is maximum at the ondulator section where the beam modulation is close to complete.* Here

$$\frac{\delta\gamma}{\gamma} \approx |M|\frac{\lambda}{l} \ll 1, \ \left|\frac{\delta M}{M}\right| \approx \left|\frac{\delta\gamma}{M}\frac{\partial M}{\partial\gamma}\right| \ll 1 \quad (3.7)$$

Hence the upper limit of the longitudinal motion mass M is determined by the conditions (3.7). The lower limit is due to the fact that near the resonance $|\omega_{11} - \kappa| \ll \kappa$ only in the region in which M can be decreased; the growth length l must be longer than $\lambda_0/|1 - \frac{\omega_{11}}{\kappa}| \approx \lambda_0/|Mu|^2$.

Thus the limits to variation in M for the helical ondulator are determined by the conditions

$$\frac{\lambda}{l|u|^2} \ll \frac{M}{\gamma_\parallel^2} \ll \left(\frac{l}{\lambda_0}, \gamma|u|\sqrt{\frac{l}{\lambda_0}}\right). \quad (3.8)$$

The constraints on the value of M can be obtained by substituting l from formulas (2.10), or (2.12).

It is possible from Eq. (3.6) to define maximum permissible current ($M \approx \gamma_\parallel^2$ at a current close to the maximum)

$$e\dot{N}_{\max} \approx e\frac{\gamma\sigma^2}{r_e\lambda_0{}^2}\gamma_\parallel^6|u|^4. \quad (3.9)$$

The maximum possible radiation power achieved at a current $e\dot{N} \approx 10^4\gamma^3\sigma^2/\lambda_0{}^2A$, $\gamma|u| \approx 1$, $M \approx \gamma_{11}^2$ can also be estimated.

$$W_{\max} \approx \gamma m\dot{N} \approx 10^{10}\gamma^4\sigma^2/\lambda_0{}^2\,(Wt).$$

We have listed all the principal important restrictions on the parameters of the problem. In addition, it should be noted that the fulfilment of

* The condition for neglecting the effect of the ondulator field on the transverse motion of electrons ($|H_\perp| \gg |\hat{E}|(1 - V)$) follows from Eqs. (3.6) and (3.7). Furthermore, the smallness of the quantity, $|\delta u/u| \ll 1$, also follows from these equations,

the condition (3.6) excludes any direct influence of the Coulomb interaction on the dynamics of developing the instability but this condition is insufficient to neglect entirely the influence of Coulomb fields. For instance, Coulomb interaction of the beam particles can enlarge the beam cross section adiabatically, with respect to the growth length, thereby increasing this length under certain conditions. It is important to emphasize that, in principle, such an adiabatic interaction may be removed and, therefore, probably does not impose any additional limitations. Several methods can be suggested to get rid of this influence. In particular, the Coulomb repulsion can be removed by ion compensation of the beam charge or by magnetization of the electrons in the ondulator by a longitudinal magnetic field.

There is a critical current above which the Coulomb expansion (at $\omega_{\parallel} = 0$) decreases the power of coherent radiation*

$$N_c \approx \frac{\gamma}{r_e} \frac{\gamma_{\parallel}^4 |u|^2}{|M| \ln^4(\rho_0/\bar{\rho}_i)} \qquad \sigma_i \lesssim \frac{\lambda}{|u|} \sqrt[2]{\frac{|M|}{\gamma_{\parallel}^2}} \ln \frac{\rho_0}{\bar{\rho}_i},$$

$$N_c \approx \frac{\gamma}{r_e} \frac{\sigma_i^2}{\lambda^2} \frac{\gamma_{\parallel}^4 |u|^4}{|M|^2 \ln^6(\rho_0/\bar{\rho}_i)} \qquad \sigma_i \gtrsim \frac{\lambda}{|u|} \sqrt[2]{\frac{M}{\gamma_{\parallel}^2}} \ln \frac{\rho_0}{\bar{\rho}_i}, \tag{3.10}$$

where σ_i is the transverse beam dimension at the ondulator entrance.

If the beam current exceeds the values determined by formula (3.10), it is sufficient to introduce a longitudinal field for the narrow and wide beams, correspondingly**

$$|\omega_{\parallel}| \gg \left(\frac{\dot{N} r_e}{\gamma}\right)^{3/4} \left(\frac{\gamma_{\parallel}{}^4 |u|^2}{|M|}\right)^{1/4} \cdot \frac{1}{\lambda_0},$$

$$|\omega_{\parallel}| \gg \frac{1}{\gamma_{\parallel}\sigma_i} \left(\frac{\dot{N} r_e}{\gamma}\right)^{1/2}$$

Adiabatic expansion of the beam can be associated with the initial angular spread of electron velocities. This is possible when many characteristic lengths of growth ($L/l = \ln \rho_0/\bar{\rho}_i \gg 1$) are required for developing the instability. This effect of expansion is also readily avoided by introducing a longitudinal field. It is worthwhile to note that, in practice, the deliberate introduction of a longitudinal field $\omega_{\parallel} l \approx 1$ removes the effects of beam expansion connected with the Coulomb repulsion and the angular spread mentioned above.

4. CONTROL OF THE MASS OF LONGITUDINAL MOTION

The effects of a series of factors that limit the power of the output radiation can be successfully compensated by means of a longitudinal field. When $H_{\parallel}$ is large enough, the longitudinal field ceases to be only a neutralizer of damaging factors and starts, together with the transverse field of the ondulator H_0, to play a determining role in the dynamics of instability. Qualitatively new is the fact that the value of the longitudinal mass M and the transverse velocity amplitude $|u|$ become two independent parameters. For example, we can always choose a value of H_0 such that $|u|$ is constant when $H_{\parallel}$ varies, and hence, the wavelength of the resonant harmonic of density is constant as well, while, according to Eq. (2.1), we can control the quantity and sign of the longitudinal mass M. Note that it is possible, if desired, to change M (adiabatically) in some sections of the ondulator only. So, for example, in the initial section of the ondulator, where the difference in particle energy inside the beam is only determined by the initial spread, we can shorten the growth length l by decreasing M and, therefore, make the whole ondulator shorter. Of course, this is possible if the initial energy spread of the beam is small because, according to Eq. (3.2), the decrease in M leads to a more rigid requirement for $\Delta\mathscr{E}/\mathscr{E}$. Otherwise, in the final section of the ondulator, where the beam modulation is close to complete, it is desirable to increase M because, according to Eq. (2.16), in this case the output power of coherent radiation W increases. Let us find how large W can become when M increases, within the limitations of (3.6) and (3.7). Let us first consider a practically interesting case when $\gamma|u| \lesssim 1$ (the ondulator is helical). We get from Eqs. (3.6) and (3.8) that if the beam area σ^2

* Besides the increase of the beam cross section, the angular spread between the particle velocity and beam axis increases as well, but the condition for a maximum possible angle θ_c in the practical region where $\sigma_i \lesssim \lambda_0|\gamma_{\parallel}^2 u^2|^{-1} \ln^2(\rho_0/\bar{\rho}_i)$ is always fulfilled if Eq. (3.10) is satisfied.

** If the own focusing by ondulator fields is strong enough, the introduction of the longitudinal field may appear not necessary.

$< \sigma_0^2 = (\lambda\!\!\!^{-2}/|u|^2)(\gamma_\parallel\gamma^2|u|^2/\dot{N}r_e)^{2/3}$, then the maximum possible value of M is $M_{\max} \approx \gamma_\parallel^2(\gamma^3|u|^2/\dot{N}r_e)^{1/3}$. In the limiting case, the fraction of the beam energy converted into radiation is

$$\left(\frac{|M|\lambda\!\!\!^{-}}{l}\right)_{\max} \approx \left(\frac{\gamma^5|u|^2}{\gamma_\parallel^2\dot{N}r_e}\right)^{1/6}\frac{\lambda\!\!\!^{-}_0}{l_0},$$

where l_0 is the growth length at $M = \gamma_\parallel^2$.

In the case when $\sigma^2 > \sigma_0^2$,

$$M_{\max} \approx \gamma_\parallel^2\left(\frac{\gamma^x|u|^4\sigma^2}{\dot{N}r_e\lambda\!\!\!^{-}_0{}^2}\right)^{1/5}, \quad \left|\frac{M\lambda\!\!\!^{-}}{l}\right|_{\max} \approx \left(\frac{\gamma^x|u|^4\sigma^2}{\dot{N}r_e\lambda\!\!\!^{-}_0{}^2}\right)^{2/15}\frac{\lambda\!\!\!^{-}_0}{l_0}.$$

Thus, at currents much lower than the maximum permissible, the gain in power can be significant.

5. ON THE LEVEL OF BEAM MODULATION AT THE ENTRANCE OF AN ONDULATOR

To reveal the effect of self-modulation, a knowledge of the initial level of the harmonics of beam density is necessary. In a realistic situation, if the initial conditions are not prepared in a special manner, there exists a continuous spectrum of fluctuations of density harmonics that arise from the fact that there is a finite number of particles in the beam. Hence all the harmonics in a band of width $(\Delta K/K) \approx \lambda\!\!\!^{-}_0/l$ will become unstable and grow by a few times in the length l. After passage of a length $L \approx l \cdot \ln(\rho_0/\tilde{\rho}_i)$ all the harmonics achieve a size of the order of ρ_0.

The spectrum width $\Delta K \sim K\lambda\!\!\!^{-}_0/l$ corresponds to a correlation length of the order of $l/2\gamma_\parallel^2$. Hence the values of harmonics of density fluctuations for the narrow and wide beams respectively are

$$\tilde{\rho}_i/\rho_0 \approx (\dot{N}l/2\gamma_\parallel^2)^{-1/2}, \quad \tilde{\rho}_i/\rho_0 \approx (\dot{N}l/2\gamma_\parallel^2)^{-1/2}(\sigma^2/\lambda\!\!\!^{-}l)^{1/2}.$$

[It is taken into account for a wide beam that the correlation length in the transverse direction is of the order of $(\lambda\!\!\!^{-}l)^{1/2}$.] For example, in the case of a narrow beam at $\gamma|u| \approx 1$, $\gamma^2 \approx 10$, $\lambda\!\!\!^{-}_0 \approx 1$ cm, $e\dot{N} \approx 1\,A$, we get $\tilde{\rho}_i/\rho_0 \approx 10^{-5}$.

Any source of coherent radiation with wavelength λ can be used for the preparation of the initial modulation of a beam with the same period. For this purpose, in the initial section of the ondulator the beam is affected by an electromagnetic wave propagating in the direction of particle motion. In this case, the beam is modulated with period λ. It is worthwhile to consider the interaction between the particles and external wave only in the length of the initial section of the ondulator which is not longer than the characteristic growth length l. One can estimate a maximum value of the initial modulation of the particles in the beam which can be obtained through the use of external coherent radiation of power W_{ext}. Supposing that the external radiation is focused in an optimal way,* we have $\tilde{\rho}_i/\rho_0 \approx (W_{\text{ext}}/W)^{1/2}$ in the resonance region ($\lambda \approx \lambda_0/2\gamma_\parallel^2$), where W is the coherent radiation power emitted from the ondulator length l upon the complete beam modulation (i.e., the output power of the source under study).

The spectrum of initial conditions determines the width of the output radiation spectrum. If the beam modulation is developed from the spectrum of density fluctuations, the degree of monochromaticity is equal to $\Delta\omega/\omega \approx \lambda\!\!\!^{-}_0/l$. In the case when the initial state $\tilde{\rho}_i$ is prepared with an external monochromatic radiator, the output radiation is also monochromatic (for a continuous beam). Note that as a modulating radiator, the source can be used based on the principle under consideration where the necessary spectrum width is cut off with a monochromator. There also exists the possibility of creating a back-coupling source where a small fraction of the output radiation after monochromatization is supplied again to the ondulator entrance. In such a scheme the whole length of the ondulator is shortened as well.

The angular divergence of output radiation is also connected to the initial conditions for beam modulation.** If the beam modulation is developed from the spectrum of density fluctuations,

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* In the case of a small beam cross section, the external radiation is focused up to the diffraction limit ($\hat{S} \approx \lambda\!\!\!^{-}l$) and in the case of a large beam cross section up to the beam dimensions ($\hat{S} \approx \sigma^2$)

** The angular divergence of radiation for a wide beam can be connected with the widening of the beam because of Coulomb repulsion: $\hat{\theta}_c \approx \theta_c \approx \dot{N}r_e\angle/(\gamma\gamma_\parallel^2\sigma)$. In the case $\theta_c \gg \hat{\theta}$, the angular divergence of radiation can be decreased to the diffraction limit $\hat{\theta}$ by a longitudinal field whose strength satisfies the relation $|\omega_\parallel^2| \gtrsim \dot{N}r_e/(\gamma\gamma_\parallel^2\sigma\theta)$.

then the angle of radiation divergence θ, taking into account that the correlation length in the transverse direction does not exceed the quantity $(\lambda\!\!\!^{-} l)^{1/2}$, is approximately equal to $\hat{\theta} \approx (\lambda\!\!\!^{-}/l)^{1/2}$ for both narrow and wide beams.

For a wide beam, the angle of divergence can be decreased if the initial modulation is prepared with constant phase over the transverse beam cross section. In this case, the angle of radiation divergence is determined by the diffraction angle of a coherent radiator with dimension σ: $\hat{\theta} \approx \lambda\!\!\!^{-}/\sigma \ll (\lambda\!\!\!^{-}/l)^{1/2}$.

This initial condition can be prepared if, as a source which gives the initial modulation, radiation is used that is generated by a narrow beam widened to dimensions of the order of σ. This can be done in the back-coupling scheme, too. Such a scheme allows one not only to improve the monochromaticity, but also to decrease the angular divergence of radiation for a wide beam.

For some applications the polarization properties of the output radiation can be important. The type of polarization is determined by the structure of the ondulator field in the radiating section. An additional possibility to control the polarization arises when a strong magnetic field is introduced into a linear ondulator. At small field ($|\omega_{\|}| \ll \kappa$) the radiation has linear polarization. With increase of the longitudinal field the polarization becomes elliptic and in the limiting case circular. Variation of the longitudinal field direction makes it possible to change the sign of the radiation helicity.

6. NUMERICAL EXAMPLES

It seems quite promising to apply the above-described principle of beam self-modulation for creation of a source that operates in the submillimeter range, where high-power sources of coherent radiation are now lacking. Let us consider the following example. Parameters of the beam are: $\gamma = 5.5$, beam current 100 A, emittance $\sigma \cdot \Delta\theta = 5 \cdot 10^{-3}$cm. Parameters of the helical ondulator are: period $\lambda_0 = 2$ cm, $H_0 = 3$ kG. Then the period of free oscillations in transverse plane is equal $\lambda_b = 25$ cm, the radiation wavelength is 0.045 cm. The growth length is calculated from the formula for the narrow beam and is equal to $l = 14$ cm. The beam of electrons radiates a power of order of $5 \cdot 10^6$ W in ondulator. In the case when modulation develops from the fluctuation spectrum, $\Delta\omega/\omega \approx 10^{-2}$, $\hat{\theta} \approx 2 \cdot 10^{-2}$. Introduction of a 28 kG longitudinal field will enables one to increase the limiting output power of the source by a factor of 3.

To produce a beam of electrons with these parameters, one can use electrostatic acceleration. At present, this type of source is being developed for the problem of electron cooling of antiproton beams in storage rings.[6] Electrostatic acceleration is very convenient in that it allows one to recuperate the energy of the used electron beam in the simplest way, thereby increasing the source efficiency up to the order of unity.

This method of producing the coherent radiation enables one to use pulsed electron sources as well. An increase of duty factor (with the same power of the electron source) decreases the characteristic growth length and increases the fraction of the beam energy converting into radiation. For example, the following parameters* are chosen: $\gamma = 20$, the period of the helical ondulator $\lambda_0 = 6$ cm, $e\dot{N} = 3 \cdot 10^4$ A, emittance $\sigma\Delta\theta = 5 \cdot 10^{-3}$ cm, $H_0 = 1$ kG, $\lambda = 10^{-2}$ cm, $\lambda_b = 350$ cm. The growth length is calculated from the formula for the wide beam and is equal to $l = 22$ cm.

The beam will radiate at the angle $\hat{\theta} \approx 5 \cdot 10^{-2}$ and the degree of monochromatization is $\Delta\omega/\omega \approx 2 \cdot 10^{-2}$ if the modulation occurs from the spectrum of fluctuations and at the angle $\hat{\theta} \approx 3 \cdot 10^{-3}$ when the modulation occurs from one harmonic of density. Radiation power will be $\approx 10^{10}$W.

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* These parameters of the beam may be attained in existing high-current accelerators with pulsed current duration of about 10^{-7} sec (see, e.g., Ref. 7). Note that in our case the local energy spread in the beam is of importance and it can be less than the total spread.

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