Generation Capacity Expansion in Imperfectly Competitive Restructured Electricity Markets

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Abstract

Investments in generation capacity in restructured electricity systems remain a relatively unexplored subject in the modeling community. We consider three models that differ by their underlying economic assumptions and the degree to which they depart from the old capacity expansion representations. The first model assumes a perfect, competitive equilibrium. It is computationally very similar to the old capacity expansion models even if its economic interpretation is different. The second model (open-loop Cournot game) extends the Cournot model that is sometimes used for modeling operations in restructured electricity systems to include investments in new generation capacities. This model can be interpreted as describing investments in an oligopolistic market where capacity is simultaneously built and sold on long-term contracts when there is no spot market (Power Purchase Agreements). The third model (closed-loop Cournot game) separates the investment and sales decision. It describes a situation where investments are decided in a first stage and sales occur in a second stage, both taking place in oligopolistic markets. The second stage is a spot market. This makes the problem a two-stage game and corresponds to investments in merchant plants where the first stage equilibrium problem is solved subject to equilibrium constraints. Because two-stage models are relatively unusual in discussions of

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electricity, we characterize the properties of this game and compare them with those of the open-loop game. We show that despite some important differences, the open and closed-loop games share many properties. One of the important results is that the solution of the closed-loop game, when it exists, falls between the solution of the open-loop game and the competitive equilibrium.

Keywords: Electric utilities; Existence and characterization of equilibria; Non cooperative games; Programming; Oligopolistic Models

1 Introduction: Investments in power generation

Capacity expansion models have a long tradition in both the power sector and the operations research literature. They were one of the first applications of linear programming in the fifties when the industry was organized as regulated monopolies (Massé and Gibrat (1957)).

While capacity expansion models have evolved into quite complex tools in operations research terms, their economics remained relatively simple. Generation plants differ by their investment and operation costs. Capacity expansion models select the mix of plants that minimizes the total cost of satisfying a time-varying demand with randomness over a typical horizon of say twenty years. This formulation assumes away some important phenomena. Having capacity investments with long lives imply risks. Except for the prudence reviews that developed in the US (see Joskow (1998) for a discussion of the evolution of the US power sector), these risks were generally passed on to the consumer. This allowed the industry and the modeler to assume away most risk factors including the uncertainty of future demand and fuel costs. Even though our goal is to look at capacity investments in competitive situations, we retain this simplification in this paper. It is indeed common practice to increase the discount factor with respect to the risk free interest rate in order to account for uncertainty. We assume such a procedure and consider annual values of equipment capacity costs using an exogenous discount rate. All capacity costs are therefore expressed in terms of their annualized value in the rest of the paper. Extensions of these standard capacity expansion models can account for long-term uncertainties in demand and fuel cost. Stochastic programming models (Louveaux and Smeers (1988), Janssens de Bisthoven et al. (1988)) as well as the literature on investments under uncertainty (Leland

(1972), Drèze and Gabszewicz (1967), Smith (1969), Drèze and Sheshinski (1976)) are relevant to this problem.

A capacity expansion model designed for a regulated monopoly converts directly into one applicable to a perfectly competitive market. One first introduces a demand model that accounts for the dependence of demand on prices. Assuming that the associated inverted demand function is the gradient of a utility function ($p(q) = \nabla_q U(q)$, the minimum cost capacity expansion problem can be readily extended into a nonlinear model where the objective maximizes the net present value of producers and consumers surplus over a given horizon. With additional computational complexity, the method also extends to demand models that do not derive from a utility function (see Wu and Fuller (1996)). An alternative model where profits are maximized subject to a regulatory constraint has been used to understand the inefficiencies associated with rate-of-return regulation (Murphy and Soyster, (1983)).

Perfect competition is a strong assumption when it comes to restructured electricity markets. A more suitable hypothesis is to presume that the power market is oligopolistic, or in other words that the number of suppliers is sufficiently small that each can influence prices. While the regulated monopoly model can be extended readily to deal with a perfectly competitive market, representing imperfect competition is much more complex. This paper is about investment in oligopolistic restructured electricity systems.

Market power is an actively researched area in the literature on restructured electricity systems. Several models exist that look at the operations of a market with oligopolistic players when capacities are given (see for instance the POWER (www.ucei.org) or the HEPG (www.ksg.harvard.edu/hepg) web sites). An extensive stream of less formalized literature also treats the subject.

In contrast, very little is available when it comes to investment. Chuang, Wu, and Varaiya (2001) formulate a single-period Cournot model and solve examples of equilibria. Except for this paper and to the best of our knowledge, neither qualitative nor qualitative results from market models dealing with both investments and operations in an oligopolistic electricity market exist at this time. But economic theory provides us with at least two frameworks, both equally interesting for looking at the issue. We briefly review some of these notions as they constitute the conceptual backbone of this paper.

We begin with strategic investments, which are investments that are made to modify a rival's actions. They are best interpreted in a two-stage decision context where investment decisions are made first while operations (generation, trading and sales) are decided in the second stage. Second-stage uncertainties, when they are present, influence first-stage decisions. In the first model of this type, Spence (1977) considers the case where an incumbent builds capacity in the first stage while a potential entrant invests in the second-stage. Both operate in the market in the second stage. The potential entrant incurs a fixed cost to enter the market, which the incumbent has already paid. The entrant optimizes its decision assuming that the incumbent will utilize all its capacity in the second stage, given the entrant effectively enters the market. The potential entrant decides to enter the market only if it can make a positive profit after paying for the fixed cost. The incumbent selects its capacity and operating levels in order to maximize its profit subject to the condition that it wants to bar the entrant from the market. Once the potential entrant decides to not enter, the incumbent operates below its capacity to maximize its profits.

Dixit (1980) retains most elements of Spence's model but allows the incumbent to add capacity in the second stage. The entrant invests and operates in the second stage if it can make a positive profit after covering the fixed cost. Another difference from Spence is the second-stage market is Cournot. The problem of the incumbent is thus an optimisation subject to equilibrium constraints (the Cournot conditions of the second stage), that is to say an MPEC (Mathematical Program subject to Equilibrium Constraints) (Luo et al. (1996)). MPEC problems are more general than the bilevel mathematical program that subsumes Spence's model.

In contrast to both Spence and Dixit, Gabsewicz and Poddar (1997) assume that the two firms may simultaneously enter the market. They do so by choosing their capacities in the first stage, and they cannot revise this choice in the second stage. Only operational decisions are made in second-stage, and as in Dixit's model, the second-stage market equilibrium is Cournot. Gabsewicz and Poddar also modify the description of the technology by dropping the fixed cost to enter the market. Uncertainty is a key element of the Gabsewicz and Poddar (1997) model. They assume that the demand function is revealed in the second stage only and that the achieved equilibrium is contingent on this demand information. This implies that investments must be decided before knowing the intensity of the demand. Firms invest so as to reach a Nash equilibrium in the first stage knowing the expected outcome of their decision in the second stage. This nesting of two equilibrium problems (a subgame-perfect equilibrium) leads to an equilibrium problem subject to equilibrium constraints. To make things more interesting, the uncertain character of the demand in the second stage makes this problem a stochastic equilibrium problem subject to equilibrium constraints.

These problems are unusual but not totally absent from the literature. In a different but related context, Allaz (1991) and Allaz and Vila (1993) study the forward commodity markets with market power through an equilibrium model subject to equilibrium constraints. The first paper offers a multistage deterministic model while the latter is two stage but stochastic. Both models look at the incentive for producers of some commodity to trade in the forward market (first stage) before going into the spot market (second stage). In the deterministic model, when duopolists sell into a forward market, the resulting equilibrium is closer to the competitive equilibrium. Furthermore, both players enter the forward market despite the profit reductions because the decision to enter has the form of a prisoners dilemma game. The stochastic equilibrium model allows one to explore the mix of strategic and hedging incentives for trading in the forward market. While Allaz (1991) and Allaz and Vila (1993) models do not immediately apply to our investment problem, the former can be adapted to fit a realistic power market by considering two forward electricity markets namely peak and off peak. Generators would then first trade peak and off peak forwards before generating in peak and off peak on the spot market. This extension is left for further research.

These two-stage models provide a realistic framework for looking at investments in the restructured power sector. However, they are stylized analytic models that do not involve the computable tools necessary for solving realistic models. It is our objective in this paper to move a few steps from the economic concepts towards computable models of capacity expansion in restructured electricity systems. We first note that the introduction of an uncertain demand function by Gabsewicz and Poddar (1997) is suitable for modeling demand in the power sector. As indicated above, electricity demand is both time varying and uncertain. The time varying character of electricity demand is often represented by a load duration curve. This curve is best characterized by its inverse which gives the amount of time t at least m megawatts are demanded. When the time interval is scaled to 1, this inverse can be interpreted as an operating characteristic curve, which is one minus the corresponding probability distribution of a certain demand level. This probabilistic interpretation allows one to incorporate the overall uncertainty driving demand. The time varying and uncertain demand can thus be represented as a set of demand states each occuring with a given probability. In conclusion some representation of uncertainty, such as found in Gabszewicz and Poddar (1997) is necessary for dealing with investments in electricity.

Both the oligopolistic investment problem and the issue of entry deterrence are relevant to model the restructured power industry. The oligopolistic problem is more directly applicable to the US situation where investor-owned utilities in restructured systems have largely divested their power plants. In contrast, entry deterrence appears directly applicable to the European market where this divestiture process has not taken place to such a large extent and a dominant player remains in place in most instances. The Spence and Dixit models as well as Schmalensee's (1981) and Bulow, Geanakoplos, and Klemperer's (1985) variants involve fixed costs or economies of scale. We disregard both phenomena. Restructuring in electricity is indeed rooted in the idea that phenomena of natural monopoly have largely disappeared from generation (see Stoft 2002, Chapter 2-2). We therefore assume that economies of scale and fixed entry costs are negligible. In contrast with the above mentioned models that assume a single technology, the representation of the electricity sector requires several types of plants in order to economically satisfy the time varying demand. We thus concentrate on the oligopolistic investment problems with no explicit entering fixed cost but with the players using different technologies having different cost characteristics. Also, we do not use the asymmetry of an incumbent versus a new entrant. In short, this paper concentrates on problems of the Gabszewicz and Poddar (1997)'s type, that is, problems of oligopolistic competition in investment. But we adapt their assumption to the realities of the power sector and suppose that agents invest in different technologies. This diversity of technologies and agents has consequences. While Gabsewicz and Poddar rely on the symmetry of their problem (both agents use the same technologies) in order to prove the existence of equilibrium, the asymmetry of our agents may invalidate this existence.

Summing up, we assume a restructured power system where firms select their capacity and compete in the day-to-day power market. In order to simplify the problem while retaining the key aspects of the power sector, we assume only two types of capacity namely peak units and baseload units (Stoft, chapter 2-2). Peak units have lower investment and higher operating costs than baseload units. This technological diversity is a major departure from the cited economic literature and is a consequence of electricity not being storable. The time-varying nature of demand is combined with a representation of the uncertainty of the demand as a load duration curve which is discretized into a finite set of demand scenarios. Price responsiveness is included by supposing a price responsive demand in each scenario. One can formulate each player's optimization problem as a two-stage stochastic program with multiple demand curves. Since the operating costs are invariant in our model, the order in which plants are dispatched does not change across multiple demand scenarios and the scenarios collapse into a single scenario. This is very much akin to the Gabsewicz and Poddar (1997) framework.

The paper presents three models, a perfect-competition model, an open-loop Cournot model and a closed-loop Cournot model (in mathematical programming terms, and equilibrium subject to equilibrium constraints). See Fudenberg and Tirole (1986) for the notions of open and closed loop equilibrium. We use perfect competition as a benchmark. The open-loop Cournot model is a relatively simple representation of imperfect competition. Its strategic variables are investment and operations; the two players select these variables at the same time. This is a Cournot model because the players choose quantities and then set the price. Even though the model is mathematically simple, it has a realistic interpretation, namely plants are built and their output is sold under long-term contract at the same time. This model does not assume any spot market; it corresponds to an industry structure organized around Power Purchase Agreements (Hunt and Shuttleworth (1996)). The closed-loop Cournot model is structured around the same strategic variables. The main difference is that these variables are not decided at the same time. Capacity decisions are made in the first period and operating decisions in the second period. The closed-loop Cournot model can be seen as an industry structure organized around a spot market (Hunt and Shuttleworth (1996)). Generators compete by building plants with no guarantee that they will be able to sell their output or any guarantee on price. The game is then truly a two-stage game where competition takes place in two steps. The generators play against each other when making investments, knowing how they will play against each other when operating their plants. This feature makes the closed-loop game a first period equilibrium subject to equilibrium constraints in the second period. Note however that our framework does not include forward contracting (e.g. one year), a problem that has received a consider-able attention in the literature (e.g. Green (1999), Newbery (1998), Wolak (1999)). The inclusion of this forward market could be done in different ways and is a subject of research in itself. One possibility is to assume that forward contracts are concluded at the same time the plants are built (see Newbery (1999), section 5). This keeps the problem two stage. It becomes multistage if there are several rounds of forward contracting after building time). The analysis of forward contracting is left to further research.

The paper is organized as follows. The next section (Background) introduces the description of the power sector adopted in the paper, that is the representation of the demand and technologies and some market assumptions. The perfect-competition case used as the reference is discussed in Section 3 together with the equilibrium conditions and standard properties. The open-loop Cournot model is presented in Section 4 together with equilibrium conditions and some properties of the solution. A sensitivity analysis of the short-term equilibrium is also presented in this section. These sensitivity results are first applied in Section 5 to discuss short and long-term reaction functions and to derive some of their properties. The closed-loop Cournot model is introduced in Section 6. Its analysis constitutes the core of the rest of the paper. After a definition of the problem and a statement of its equilibrium conditions, the solution of the closed and open-loop Cournot models are compared to establish their similarities and differences. These properties allow one to derive results comparing the investments in both models. Finally, Section 7 deals with the difficult issue of existence and uniqueness of a solution of the closed-loop Cournot model. Conclusions close the paper. In order to facilitate the exposition, all proofs are given in the appendix.

2 Background

We consider a simple electricity system where all demand and supply is concentrated at a single node. The paper, therefore, neglects network congestion issues. This simplification is mandatory at this stage of the research. First a model that includes the network and accounts for congestion would be untractable at this stage. Second the main forces driving investments in restructured markets are still so unknown that it seems better not to cloud the issue with the impact of congestion which itself still generates a lot of discussion. This being said, it is an often heard argument that conestion should influence the location of new plants. But this issue has never been studied in a formal model. We approximate the load duration curve with a step function (Figure 1).

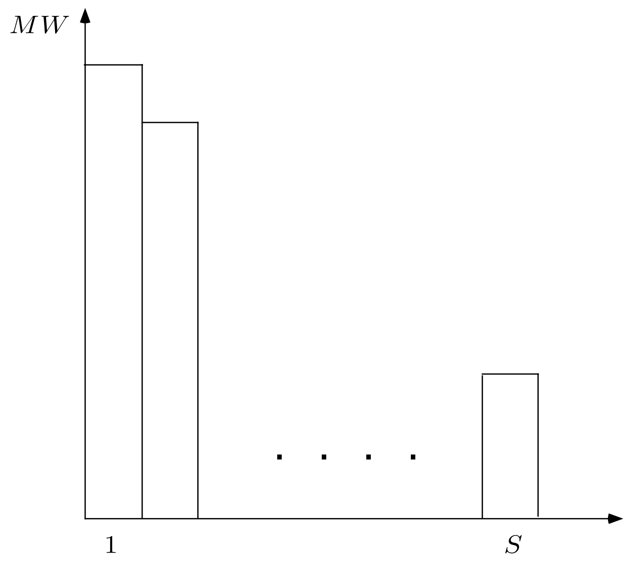

Figure 1: Yearly demand decomposition

In order to simplify notation, we assume that these segments are one unit wide. We index these segments by $s = 1, \cdots, S$, where $s = 1$ is the peak segment and $s = S$ is the base segment.

The different models considered in this paper deal only with two types of generation equipment, each characterized by their annual (per kW) investment (K) and operations (per kWh) (ν) cost. We use p to denote a peaker (e.g. Gas Turbine)and b to denote a base-load plant (e.g. Combined Cycle Gas Turbine). A two technology power system is described in Stoft (2002) (chapter 2-2-2) together with typical cost figures. By assumption, peakers are cheaper for peak demand, $K_p + \nu_p < K_b + \nu_b$, and base plants are cheaper for base demand, $K_b + S\nu_b < K_p + S\nu_p$.

These assumptions are illustrated in Figure 2.

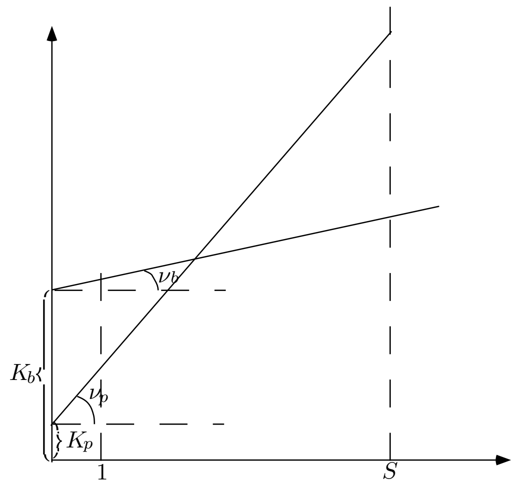

Figure 2: Peakers and base plants

Generation capacities are built and operated by two generators denoted $i = p, b$. In order to simplify the structure of the model, we assume that generator p builds and operates only peak plants while generator b builds and operates only base plants. Essentially, we are looking at the case where companies develop expertise and specialize. We have developed results for the case where they do not specialize. However, we have left them out because of length considerations. The main result when players do not specialize is that the players have identical capacity levels for each type at equilibrium. Specialization creates an asymmetry just as incumbency has for the models in the cited papers. Investment variables are denoted $x_i, i = p, b$ for investments by generators p and b respectively and are continuous. Operations variables are denoted $y_i^s, i = p, b; s = 1, \cdots, S$ for the production of generator i in time segment s. Needless to say we have $x_i \geq y_i^s \geq 0 \;\; i = p, b; s = 1, \cdots S$.

Finally, demand in each time segment s of the second stage is given by an affine inverted demand curve: $p^s = \alpha^s - \beta q^s \quad s = 1, \cdots, S$ and $\beta > 0$. We use this demand model for two reasons. First, it is a good approximation to a nonlinear demand curve in the immediate neighborhood of the equilibrium. Second, it makes the mathematics of the proofs simpler and more understandable. We use the same slope for all steps to simplify the notation and some of the resulting formulas. What is critical to the character of our results is that the demand curves do not cross. Demand is higher in the peak segment and decreases as one moves towards the base segments. This is expressed as $\alpha^1 > \alpha^2 > \ldots > \alpha^S$. The inverted demand curves for the different time segments are depicted in Figure 3.

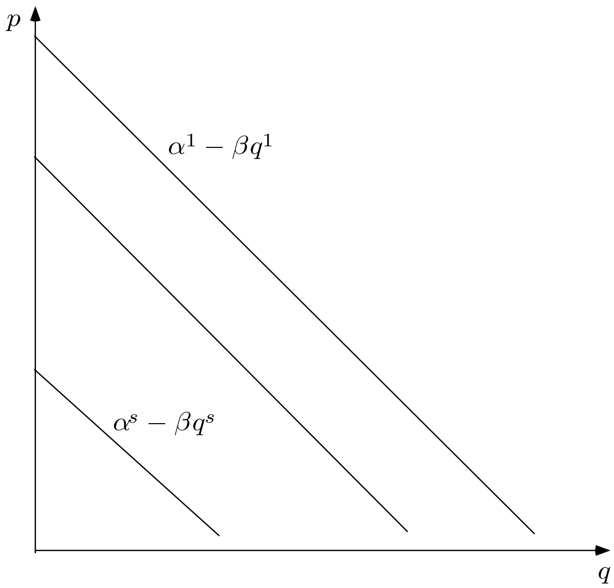

Figure 3: Inverted demand curves

3 The perfect-competition case: equilibrium conditions

Consider first the case where both generators compete with given capacities x without exerting market power. That is, they generate until marginal cost equals price. Each of the generators has the following optimization problem when it takes the prices p^s as given.

$$\begin{array}{l} \max_{x_i, y_i^s} \sum_s [p^s - \nu_i y_i^s] - K_i x_i \\ \text{s.t. } 0 \leq y_i^s \leq x_i^s. \end{array} \tag{1}$$

Let ω_i^s be the dual on the nonnegativity of y_i^s and λ_i^s the dual on the capacity upper bound. Note that outside the context of the equilibrium, because the prices are fixed, the solutions are $x_i = 0$ when $\sum_{s=1}^{\tilde{S}} p_s < K_i + \tilde{S}\nu_i, \forall\, \tilde{S}; x_i = \infty$ when

$\sum_{s=1}^{\tilde{S}} p_s > K_i + \tilde{S}\nu_i$ for some $\tilde{S}$ and $x_i = [0, \infty)$ when $\sum_{s=1}^{\tilde{S}} p_s = K_i + \tilde{S}\nu_i$ for all $\tilde{S}$.

The vector of generation levels $y_i^s, s = 1, \cdots, S,\ i = p, b$ at equilibrium satisfies the following short-term (operations) equilibrium conditions where the producer does not see the demand response to price. (See Sherali et al. (1982) for the derivation of these conditions in the fixed demand case).

$$\begin{aligned}
&-\alpha^s + \beta y_i^s + \beta y_{-i}^s + \nu_i + \lambda_i^s = \omega_i^s \geq 0, \qquad y_i^s \geq 0, \omega_i^s y_i^s = 0 \\
&x_i - y_i^s \geq 0, \quad \lambda_i^s \geq 0, \quad (x_i - y_i^s)\lambda_i^s = 0 \\
&i = p, b; s = 1, \cdots, S
\end{aligned} \tag{2}$$

Equilibrium capacity levels x satisfy the following long-term (investment) equilibrium conditions.

$$\begin{aligned}
&K_i - \textstyle\sum_{s=1}^{S} \lambda_i^s \geq 0, \quad x_i \geq 0, \quad x_i(K_i - \textstyle\sum_{s=1}^{S} \lambda_i^s) = 0 \\
&i = p, b
\end{aligned} \tag{3}$$

The following properties follow directly from these equilibrium conditions.

Production Efficiency:

Peak plants are built and operated only for the time segments $s = 1, \cdots S_p$ for which they are most cost effective, that is, such that $K_p + S_p\nu_p < K_b + S_p\nu_b$ and $K_p + (S_p + 1)\nu_p > K_b + (S_p + 1)\nu_b$. In contrast base plants are built and operated in all time segments $s = 1, \cdots, S$.

Pricing Efficiency:

Prices are equal to long run marginal costs which are themselves equal to short-run marginal costs plus scarcity rents in all time segments. This is seen by noting that

$$p^s = \alpha^s - \beta(y_p^s + y_b^s) \quad = \begin{cases} \nu_p + \lambda_p^s & \text{when } x_p = y_p^s \\ \nu_p & \text{when } x_p > y_p^s > 0 \\ \nu_b + \lambda_b^s & \text{for } s = 1, \cdots, S \end{cases}$$

Investment Criterion:

The criterion is to invest when the capital cost equals the sum of margins on operation costs in all time segments. This can be restated as $K_i = \sum_{s=1}^{S} \max(p^s - \nu_i; 0),\ i = p, b$. This expression has the flavor of a call option in the sense that the value of the plant is equal to the payoff of a strip of call options of strike prize ν_i in all time segments. It is useful to note here that the price of electricity p^s is endogenous to the process (the price depend on the investment). Models of this type are discussed in Dixit and Pindyck (1994). They provide the natural economic context for looking at this investment criterion.

4 The open-loop Cournot model

4.1 Equilibrium conditions

We now take up the first imperfect-competition model, referred to as the open-loop Cournot model. In this model, each generator selects its capacity x_i and generation plan y_i^s, taking the generation levels y_{-i}^s of the other player as given. In short, generator i, $i = p, b$, solves the continuous quadratic programming problem,

$$\begin{aligned} &\min_{x_i, y_i^s} \sum_s \left[-\alpha^s + \beta(y_i^s + y_{-i}^s)\right] y_i^s + \nu_i \sum_{s=1}^S y_i^s + K_i x_i \\ &\text{s.t. } x_i - y_i^s \geq 0, \quad y_i^s \geq 0, \quad s = 1, \cdots, S \end{aligned} \tag{4}$$

The solution to this problem satisfies the following short-term equilibrium conditions, which are the Karush, Kuhn, Tucker (KKT) conditions for each player. (The equilibrium condition is that the KKT conditions for both players are satisfied simultaneously.)

$$\begin{aligned} &-\alpha^s + 2\beta y_i^s + \beta y_{-i}^s + \nu_i + \lambda_i^s = \omega_i^s \geq 0, \quad y_i^s \geq 0, \quad \omega_i^s y_i^s = 0 \\ &x_i - y_i^s \geq 0, \quad \lambda_i^s \geq 0, \quad (x_i - y_i^s)\lambda_i^s = 0 \\ &i = p, b; \qquad s = 1, \cdots, S \end{aligned} \tag{5}$$

The solution also satisfies the following long-term equilibrium conditions

$$K_i - \sum_{s=1}^S \lambda_i^s \geq 0, \quad x_i \geq 0, \quad x_i(K_i - \sum_{s=1}^S \lambda_i^s) = 0 \qquad i = p, b \tag{6}$$

In contrast with the perfect-competition model, generation is not necessarily least cost in the open-loop model. Peak plants may indeed operate in time segments $s > S_p$ where they are not most cost efficient (Reminder: $K_p + S_p\nu_p < K_b + S_p\nu_b$.) Also, prices are greater than marginal cost in all time segments where generation is positive

$$\begin{aligned} p^s = \alpha^s - \beta(y_p^s + y_b^s) > \alpha^s - 2\beta y_p^s - \beta y_b^s \quad &= \nu_p + \lambda_p^s \qquad \text{when } x_i = y_i^s, \\ &= \nu_p \qquad\qquad \text{when } x_i > y_p^s > 0 \\ p^s = \alpha^s - \beta(y_p^s + y_b^s) > \alpha^s - \beta y_p^s - 2\beta y_b^s \quad &= \nu_b + \lambda_b^s \qquad \text{when } y_b^s > 0 \end{aligned}$$

Finally, players invest until capital cost equals the sum of the marginal gross margins made on the different time segments

$$K_i = \sum_{s=1}^S \max(p^s - \beta x_i - \nu_i; 0) \quad i = p, b.$$

The relationship between this criterion and its possible interpretation in term of option should again be noted here. One first note that the value of the plant can still be expressed as the value of a strip of call options. But the payoff of these options is quite unusual to the extent that it is not equal to the maximum between the spread between electricity price and fuel costs and zero. If one were to refer to the usual criterion of a real option, one would find $K_i < \sum_{s=1}^{S} \max(p^s - \nu_i; 0)$, $i = p, b$, which implies that the value of the plant is smaller than a strip of call options on the spark spread. The seemingly exaggerated prices sometimes offered on the market for existing plants may thus be interpreted as the resulting from a blind use of the real option formula in oligopolistic markets.

4.2 Solution existence and uniqueness

Existence and uniqueness of Cournot solutions are generally easy to analyze. The present model is no exception. In order to proceed towards this analysis, we first consider a variational inequality reformulation of the problem (See Harker and Pang (1990) for a survey of the theory of variational inequalities). Define for $s = 1, \cdots, S$

$$y^s = \begin{pmatrix} y_p^s \\ y_b^s \end{pmatrix}, \qquad G_i^s(y^s) \equiv -\alpha^s + 2\beta y_i^s + \beta y_{-i}^s + \nu_i, \qquad i = p, b \tag{7}$$

(Note: $-G_i^s(y^s)$ is equal to the marginal revenue minus the short-run marginal cost of generator i.)

Also define

$$y = (y^1, \cdots, y^S); G^{sT}(y^s) = (G_p^s(y^s), G_b^s(y^s)); G^T(y) = (G^{1T}(y^1), \cdots, G^{ST}(y^S)) \tag{8}$$

$$x = (x_p, x_b), \quad K^T = (K_p, K_b), \quad F(x, y)^T = (K^T, G^T(y)) \tag{9}$$

Let Z be the set of feasible (x, y). By definition, the solution to the variational equality $VI(Z, F)$ is a point $z^* = \begin{pmatrix} x^* \\ y^* \end{pmatrix}$ belonging to Z satisfying

$$F^T(z^*)(z - z^*) \geq 0 \tag{10}$$

for all $z \in Z$. The mapping $F(z)$ is monotone if for all $(x, y) \in Z$, that is, $[F(z^1) - F(z^2)](z^1 - z^2) \geq 0$. It is strictly monotone when this inequality is strict whenever $z^1 \neq z^2$. The following lemma provides the basic technical result for analyzing the open-loop Cournot model.

Lemma 1 *$G(y)$ is strictly monotone, $F(x, y)$ is monotone.*

It is now possible to restate the Open-Loop Cournot Competition Problem as

Seek $(x^*, y^*) : x_i^* - y_i^{s*} \geq 0, y_i^{s*} \geq 0,\ i = p, b;\ s = 1, \cdots, S$ satisfying

$$F(x^*, y^*)^T(x - x^*, y - y^*) \geq 0 \tag{11}$$

for all $(x, y) : x_i - y_i^s \geq 0,\ y_i^s \geq 0,\ i = p, b;\ s = 1, \cdots, S.$

The properties of this model are summarized in the following theorem which also invokes the notion of dynamic consistency introduced in Newbery (1984). By definition the multiperiod solution of a game is dynamically consistent when the optimal actions for future periods as part of the period 1 solution remain optimal in the subsequent periods, given the first period solution. Note that dynamic consistency is a weaker concept than subgame perfection (see Haurie et al (1999) for a discusion of these two concepts).)

Theorem 1 *There always exists an open-loop Cournot equilibrium. It is unique. This equilibrium is dynamically consistent. The base player always invests a positive amount at equilibrium. The peak player does not necessarily do so, except if the equilibrium demand in some segment* $s \leq S_p$ *is larger than the equilibrium demand in segment* S_{p+1}.

4.3 Sensitivity analysis

This section presents a set of results that pave the way towards the analysis of the closed-loop equilibrium problem. These results show how the solution of the short-term equilibrium problem varies with the generation capacities and the demand parameters. Using Theorem 1 and the variants introduced below, we can state the following definitions and lemma.

Definition 1 *Let* $y_i^s(x)$, $i = p, b$; $s = 1, \cdots, S$ *be the solution of the short-term equilibrium condition (4) for a given* x*, the vector of capacity for both players.*

The $y_i^s(x)$ satisfy the following properties.

Lemma 2 $y_i^s(x)$ *is well defined (*y_i^s *is unique for given* x*). Each* $y_i^s(x)$ *is left and right differentiable with respect to* x_j, $j = i, -i$.

The following model is similar in nature (but particular to the power sector) to the one introduced by Dixit (1980) for studying strategic investments. It models the second-stage equilibrium after the entrance of player $-i$ with a capacity x_{-i}.

Definition 2 *An Open-Loop Cournot Competition problem contingent on capacity* x_{-i} *is an Open-Loop Cournot Competition problem where the capacity*

x_{-i} *is fixed. Denote by* $(x_i(x_{-i}), ((y_i^s(x_{-i}), y_{-i}^s(x_{-i})),\ s = 1, \cdots, S)$ *the solution to an Open-Loop Cournot Competitive problem contingent on capacity* x_{-i} *as a function of this capacity.*

The solution to this problem satisfies the following properties.

Lemma 3 $(x_i(x_{-i}), y_i(x_{-i}), y_{-i}(x_{-i}))$ *are well defined (they exist and are unique for every* x_{-i}*). Each of these functions is left and right differentiable.*

Consider now the solution of the short-run equilibrium in time segment s as a function of the demand level in that time segment. The solutions satisfy the following properties.

Lemma 4 *Define* $y_i^s(\alpha^s)$*,* $\lambda_i^s(\alpha^s)$ *and* $\omega_i^s(\alpha^s)$ *to be the solutions of the short-run equilibrium conditions (4) as functions of* α^s*.* $y_i^s(\alpha^s)$ *and* $\lambda_i^s(\alpha^s)$ *are monotonically non-decreasing in* α^s*.* $\omega_i^s(\alpha^s)$ *is monotonically decreasing in* α^s *when nonzero.*

This result is intuitive. It says that the generation level of each agent increases with the willingness to pay (α^s) for electricity. It also states that the marginal value of capacity in some time segment increases with the willingness to pay for electricity in that time segment. Because willingness to pay for electricity in the different time segments decreases with the index of these time segments, this lemma implies the following corollary.

Corollary 1 *If* $y_i^s = x_i$*, then* $y_i^{s'} = x_i$ *for* $s' < s$*.*

The peak generator has a higher operating cost than the base generator. It is thus reasonable to expect that, barring the case where base generation is limited by available capacity, peaker generation will be lower than base generation in any given time segment. This is stated in Lemma 5.

Lemma 5 *$y_p^s < y_b^s$ in any time segment s such that the baseload capacity is not binding ($y_b^s < x_b$).*

The next equally intuitive sensitivity result, stated formally in Lemma 6, says that the marginal value of plant capacity decreases as its total capacity increases.

Lemma 6 *Given x_{-i}, the λ are non-increasing with x_i and strictly decreasing with x_i as long as the λ are positive.*

Lemma 2 implies that one can state the solution of the short-run equilibrium conditions as a function of first-stage investments. This will be used for characterizing the closed-loop Cournot equilibrium. Lemma 3 implies that one can study the solution of a game where an incumbent chooses capacity x_i, taking into account the future investment and operations of an entrant. This is used for studying strategic investments (Dixit (1980) and Poddar's (1997) extension). The rest of this paper concentrates on the closed-loop equilibrium à la Gabszewicz and Poddar (1997). In order to proceed further, we use Corollary 1 to introduce the following definition.

Definition 3

$$s_i(x) = \max\{s \mid y_i^s = x_i\}, \qquad i = p, b \tag{12}$$

For a given vector x of generation capacities, it is thus possible to partition the set of the different time segments into the following different classes.

$$
\begin{array}{llll}
\text{(a)} & -\alpha^s + 2\beta x_i + \beta x_{-i} + \nu_i + \lambda_i^s = 0 & 0 < y_i^s = x_i & \lambda_i^s \geq 0 \\
 & & & i = p, b \\
\text{(b)} & -\alpha^s + 2\beta y_i + \beta y_{-i} + \nu_i = 0 & 0 < y_i^s < x_i & \lambda_i^s = 0 \\
 & & & i = p, b \\
\text{(c)} & -\alpha^s + 2\beta x_i + \beta y_{-i} + \nu_i + \lambda_i^s = 0 & 0 < y_i^s = x_i & \lambda_i^s \geq 0 \\
 & -\alpha^s + \beta x_i + 2\beta y_{-i} + \nu_{-i} = 0 & 0 < y_{-i}^s < x_{-i} & \lambda_{-i}^s = 0 \\
\text{(d)} & -\alpha^s \qquad + \beta y_{-i}^s + \nu_i = \omega_i^s & y_i^s = 0 & \omega_i^s \geq 0 \\
 & -\alpha^s \qquad + 2\beta y_{-i}^s + \nu_{-i} = 0 & 0 < y_{-i}^s < x_{-i} & \lambda_{-i}^s = 0 \\
\text{(e)} & -\alpha^s \qquad + \beta x_{-i} + \nu_i = \omega_i^s & y_i^s = 0 & \omega_i^s \geq 0 \\
 & -\alpha^s \qquad + 2\beta x_{-i} + \nu_{-i} + \lambda_{-i}^s = 0 & 0 < y_{-i}^s = x_{-i} & \lambda_{-i}^s \geq 0
\end{array} \qquad (13)
$$

Define $B_i y_j(x) = \dfrac{\partial y_j}{\partial x_i}$ when the derivative exists. The derivative of the second-stage equilibrium variables with respect to the first-stage capacities can be characterized as follows.

Lemma 7 *The derivative of y_j with respect to x_i when it exists can be stated as follows for the above cases*

$$
\begin{array}{lllll}
\text{(a)} & B_i y_i(x) = 1 & i = p, b & B_i y_{-i}(x) = 0 & i = p, b \\
\text{(b)} & B_i y_j(x) = 0 & i = p, b; j = p, b & & \\
\text{(c)} & B_i y_i(x) = 1 & B_{-i} y_{-i}(x) = 0 & B_{-i} y_i(x) = 0 & B_i y_{-i}(x) = -\frac{1}{2} \\
\text{(d)} & B_i y_j(x) = 0 & i = p, b; j = p, b & & \\
\text{(e)} & B_{-i} y_{-i}(x) = 1 & B_i y_i(x) = 0 & B_i y_{-i}(x) = 0 & B_{-i} y_i(x) = 0
\end{array} \qquad (14)
$$

5 Reaction curves

Reaction curves play a significant role in the study of oligopolistic equilibria in economics. In order to conduct our analysis, consider the following definition of the short-term reaction curve.

Definition 4 *The short-run reaction curve of player i with respect to the action y^s_{-i} of player $-i$ in time segment s, for given capacities x is the solution of the system*

$$-\alpha^s + 2\beta y^s_i + \beta y^s_{-i} + \nu_i + \lambda^s_i = \omega^s_i;\ y^s_i \geq 0,\ \omega^s_i y^s_i = 0$$
$$x_i - y^s_i \geq 0,\ \lambda^s_i \geq 0,\ (x_i - y^s_i)\lambda^s_i = 0$$

It is denoted $y^s_i(y^s_{-i}; x)$.

This reaction curve satisfies the following property.

Lemma 8 *$y^s_i(y^s_{-i}; x)$ exists and is well defined. It is piecewise affine with*

$$\begin{aligned} \frac{dy^s_i}{dy^s_{-i}} &= 0 && \textit{when}\ \ y^s_i = x_i\ \textit{or}\ 0 \\ &= -\frac{1}{2} && \textit{whenever}\ \ 0 < y^s_i < x_i \end{aligned}$$

The long-term reaction curves $x_i(x_{-i}), (y^s_i(x_{-i}), y^s_{-i}(x_{-i}), s = 1, \cdots, S)$ have been introduced in Definition 2. They can be used to study strategic investments as in Dixit. They are characterized by the following propositions.

Proposition 1 *$x_i(x_{-i})$ is piecewise affine and continuous. Each affine segment has a slope strictly between 0 and -1.*

Proposition 2 *$y^s_i(x_{-i})$ and $y^s_{-i}(x_{-i})$ are piecewise affine and continuous. Each affine segment of $y^s_i(x_{-i})$ has a slope between 0 and -1. Each affine segment of $y^s_{-i}(x_{-i})$ has a slope of 0 or $+1$.*

6 The closed-loop Cournot model

6.1 Definition and equilibrium conditions

To define generator i's problem in the closed-loop Cournot model, consider first the solution $y_i^s(x_i, x_{-i})$ of the short-run equilibrium conditions (4).

For given x, seek $y_i^s(x_i, x_{-i})$ that satisfies

$$
\begin{aligned}
&-\alpha^s + 2\beta y_i^s + \beta y_{-i}^s + \nu_i + \lambda_i^s = \omega_i^s \geq 0, \quad y_i^s \geq 0, \quad \omega_i^s y_i^s = 0 \\
&x_i - y_i^s \geq 0, \quad \lambda_i^s \geq 0, \quad \lambda_i^s(x_i - y_i^s) = 0 \\
&i = p, b; \quad s = 1, \cdots, S.
\end{aligned}
$$

The long run problem of generator i is then

$$
\min_{x_i \geq 0} K_i x_i + \sum_{s=1}^{S} \left[-\alpha^s + \beta(y_i^s(x_i, x_{-i}) + y_{-i}^s(x_i, x_{-i})) + \nu_i\right] y_i^s(x_i, x_{-i}) \tag{15}
$$

By definition (x_p^*, x_b^*) is a subgame-perfect equilibrium (Selten (1975)) or a closed-loop Cournot equilibrium (Fudenberg and Tirole (1986), Haurie et al. (1999)) if x_i solves generator i's long run problem for given x_{-i}^*, $i = p, b$.

In order to characterize the solution to this problem, consider a point x such that the $y_i^s(x)$ are differentiable. A closed-loop equilibrium at such a point would satisfy the condition

$$
\begin{aligned}
K_i + \textstyle\sum_s \left[-\alpha_s + 2\beta y_i^s(x) + \beta y_{-i}^s(x) + \nu_i\right] B_i y_i^s(x) \\
+ \textstyle\sum_s \beta y_i^s(x) B_i y_{-i}^s(x) = \xi_i \geq 0, \quad \xi_i x_i = 0
\end{aligned} \tag{16}
$$

We temporarily disregard points of non-differentiability and characterize an equilibrium point where all $y_i^s(x)$ are differentiable. This will be done by investigating the relationship between the solution of the closed and open-loop problems.

6.2 Closed-loop versus open-loop Cournot model

The following lemma is intuitively reasonable: if one player does not generate in some time segments in the Cournot equilibrium, it must be the one with higher short-term operating costs, that is the peak player.

Lemma 9 *Suppose the closed-loop Cournot equilibrium problem has a solution with time segments of type e. Then the peak player is the one operating at zero level in these time segments.*

This result allows one to derive a first characterization of the relation between the closed and open-loop problems. It gives a sufficient condition for the two equilibria to be identical. Essentially, this happens when neither player has load segments where the operating decisions change in response to the other player's capacity decision.

Theorem 2 *When all segments s in the closed-loop Cournot equilibrium problem are of type e with $\omega_i^s > 0$ or a, b or d, the equilibrium is the same as the solution of the open-loop Cournot problem.*

The following can be seen as a restatement of this result in terms of the investment criterion. Specifically the equality between the K_i and $\sum_i \lambda_i^s$ will play an important role in relating the open and closed-loop equilibria.

Corollary 2 *If for every load segment $x_i = y_i^s$ implies $x_{-i} = y_{-i}^s$, then $K_i = \sum_s \lambda_i^s$.*

This corollary states that in this case the solution of the optimization subject to the equilibrium constraints is the same as in the pure optimization in the open-loop game. The above results indicate that differences between the open and closed-loop equilibria require the solution to have time segments of type c or e. A first characterization of a solution with time segments of type c is given by the following lemma, which states that if the solution has multiple segments of type c in the equilibrium, then the same player is below capacity in all of these segments.

Lemma 10 *In case c,* $0 < y_i^s = x_i$ *and* $0 < y_{-i}^s < x_{-i}$ *for some segment* s *implies that there is no segment* s' *for which* $0 < y_{-i}^{s'} = x_i$ *and* $0 < y_i^{s'} < x_{-i}$*.*

This lemma allows us to introduce the following theorem which establishes a major divergence between the solution of the open and closed-loop Cournot equilibria. That is, the solution of the player's optimization subject to equilibrium constraints is different from the optimization in the open-loop game and the KKT conditions are violated.

Theorem 3 *Consider the case in which segments fall into the five cases* a, b, c, d *and* e *with* $\omega_i > 0$*. Then the solution to the closed-loop Cournot equilibrium problem is different from the solution of the open-loop Cournot equilibrium problem. Moreover one of the two following pairs of relations holds*

$$K_i > \sum_{s=1}^{S} \lambda_i^s, \quad K_{-i} = \sum_{s=1}^{S} \lambda_i^s \tag{17}$$

for $i = p$ *or* b *depending on whether* $y_i^s = x_i$ *and* $y_{-i}^s < x_i$ *in the segments of type* c*.*

The interpretation of the theorem is as follows. The investment cost of some plant, at the closed-loop equilibrium, may be higher than the sum of its short-term marginal values in the different time segments as would be implied by the KKT conditions. The difference between the two characterizes the value for the player of being able to manipulate the short-term market by its first-stage investments. This value is not captured in the standard optimization duals because the second-stage problem is an equilibrium solution.

These relations can again be interpreted in terms of a real option. First, note that the relation $K_{-i} = \sum_{s=1}^{S} \lambda_i^s$ implies $K_{-i} \leq \sum_{s=1}^{S} \max(p^s - \nu_i; 0)$. As already argued in the discussion of the open loop problem, the interpretation is that the value of a merchant plant may be smaller than the strip of call options on spark spread between electricity and fuel cost. As will be shown in Corollary 3, this happens for gas-fired units. In contrast the relation $K_i > \sum_{s=1} \lambda_i^s$ leaves us with a complete indeterminacy as far as the comparison between K_i and $\sum_{s=1}^{S} \max(p^s - \nu_i; 0)$ is concerned. It would seem that the usual interpretation of real options breaks down here.

The following lemma establishes a relatively intuitive property that is common to the solution of the open and closed-loop equilibria. It states that the solution of the short-run equilibrium first takes advantage of the existing capacity with low operating costs. As expected, this holds both in the open and closed-loop equilibria.

Lemma 11 *Assume an equilibrium of the open or closed-loop Cournot equilibrium problem. At such an equilibrium, if the peak player is at capacity in some time segment s, then the base player is also at capacity in that time segment.*

The following corollary takes advantage of Lemma 11 to refine the result expressed by Theorem 3. It says that if closed and open-loop equilibria differ, the base player manipulates the short-run market through investment. Accordingly, the per-unit investment cost in the base plant is higher than the sum of the marginal values of this plant in the different time segments.

Corollary 3 *If there exists a closed-loop equilibrium with time segments of type c, then*

$$K_b > \sum_{s=1}^{S} \lambda_b^s \text{ and } K_p = \sum_{s=1}^{S} \lambda_p^s \tag{18}$$

The next results complete the comparison between the closed and open-loop equilibria. Cournot equilibria are known to reduce quantities put on the market. Theorem 4 says that this effect is less pronounced with the closed-loop game.

Theorem 4 *Suppose there exists a closed-loop equilibrium. Then the total capacity in the closed-loop equilibrium is at least as large as the total capacity in the open-loop equilibrium and is larger when there are segments of type c or e.*

The explanation of the above phenomenon can be found in the capability of the base player to manipulate the short-term market by its investment. Specifically the base load player has a stronger incentive to invest than in the open-loop model.

Theorem 5 *Suppose there exists a closed-loop equilibrium. Then the base capacity in the closed-loop equilibrium is at least as great as the base capacity in the open-loop equilibrium.*

The overall outcome of this added investment is a reduction of prices compared to those prevailing in the open-loop equilibrium.

Theorem 6 *Suppose there exists a closed-loop equilibrium. Then the total production in the closed-loop equilibrium is larger than in the open-loop equilibrium for each time segment. Hence prices are lower in each time segment of the closed-loop equilibrium.*

The following result is a priori surprising: even though the solutions of the open and closed-loop equilibria may be different (and are also different from the perfect-competition equilibrium), the marginal value of the peak capacity in all time segments is the same in all these equilibria. Although the results may look strange, the underlying reason is simple: the investment criterion $K_p = \sum \lambda_p^s$ is the same in the three equilibria and it can easily be shown that this implies the equality of the λ_p^s.

Lemma 12 *Let m, c and o respectively indicate the competitive, closed-loop and open-loop equilibria. Then $\lambda_p^{sm} = \lambda_p^{sc} = \lambda_p^{so} = \lambda_p^s$ $\forall$ s if one invests in the peak plant in the three equilibria. One has $\lambda_p^{sm} \geq \lambda_p^{sc}, \forall$ s if $x_p^c = 0$ at equilibrium.*

The following theorem concludes the comparison among the different equilibria.

Theorem 7 *The total capacity and production in the closed-loop equilibrium falls between the open-loop equilibrium and the competitive equilibrium.*

7 Existence and uniqueness of the solution of the closed-loop Cournot model

The existence and uniqueness of the solution of the open-loop Cournot model was rather straightforward to establish. In contrast the analysis of these same questions is much more involved for the closed-loop model. We first introduce some notation. For a given $x = (x_p, x_b)$ use the monotonicity properties of

$\lambda(\alpha)$ stated in Lemma 4 and Lemma 5 to define

$$\begin{aligned} S_1 &= \{1, \cdots, s_1\} = \{s \mid \lambda_p^s > 0\} \\ S_2 &= \{s_1+1, \cdots, s_1+s_2\} = \{s \mid \lambda_p^s = 0, \lambda_b^s > 0, y_p^s > 0\} \\ S_3 &= \{s_1+s_2+1, \cdots, s_1+s_2+s_3\} = \{s \mid y_p^s = 0, \lambda_b^s > 0\} \\ S_4 &= \{s_1+s_2+s_3+1, \cdots, S\} = \{s \mid \lambda_p^s = 0, \lambda_b^s = 0\}. \end{aligned} \tag{19}$$

Note that this definition is local to this section and these s_i are not the same as those defined in Section 4.3 "sensitivity analysis". We also write the S_i and s_i as $S_i(x)$ and $s_i(x)$ if dependence of these elements on x is to be emphasized. Finally, we write

$$\Sigma = \{S_1, S_2, S_3\} \quad \text{and} \quad \sigma = \{s_1, s_2, s_3\}.$$

As a preliminary goal, we want to study the first-stage reaction of player b (investment x_b) as a function of the first-stage action of player p (investment x_p).

Rewriting the objective function of player b using the above sets for a given x_p, we can state that player b minimizes

$$\begin{aligned} OC_b(x_b \mid x_p) &= K_b x_b + \textstyle\sum_{s \in S_1(x)} (-\alpha^s + \beta x_p + \beta x_b + \nu_b) x_b \\ &+ \textstyle\sum_{s \in S_2(x)} (-\alpha^s + \beta y_p^s(x) + \beta x_b + \nu_b) x_b \\ &+ \textstyle\sum_{s \in S_3(x)} (-\alpha^s + \beta x_b + \nu_b) x_b \\ &+ \textstyle\sum_{s \in S_4(x)} (-\alpha^s + \beta y_b^s + \nu_b) y_b \end{aligned} \tag{20}$$

Define the derivative of $OC_b(x_b \mid x_p)$ with respect to x_b where it exists as

$$\begin{aligned} MC_b(x_b \mid x_p) &= K_b + \textstyle\sum_{s \in S_1(x)} (-\alpha^s + \beta x_p + 2\beta x_b + \nu_b) \\ &+ \textstyle\sum_{s \in S_2(x)} (-\alpha^s + \beta y_p^s(x) + 2\beta x_b + \nu_b) \\ &+ \textstyle\sum_{s \in S_2(x)} y_b^s B_b y_p^s(x) + \textstyle\sum_{s \in S_3(x)} (-\alpha^s + 2\beta x_b + \nu_b) \end{aligned} \tag{21}$$

Finally, we denote the expressions (20) and (21) as $OC_b(x_b \mid x_p; \Sigma), OC_b(x_b \mid x_p; \sigma), MC_b(x_b \mid x_p; \Sigma), MC_b(x_b \mid x_p; \sigma)$ when we want to express them as functions of Σ (or σ) that are defined exogenously, independent of x. Specifically it is useful to refer to $OC_b(x_b \mid x_p; S_1)$, $OC_b(x_b \mid x_p; s_1)$, $MC_b(x_b \mid x_p; S_1)$, $MC_b(x_b \mid x_p; s_1)$ where only the first element S_1 (or s_1) is defined exogenously, independent of x, but the other S (or s) depend on x.

The following lemma states that the objective function of player b is not convex. However it has partial convexity properties.

Lemma 13 *$OC_b(x_b \mid x_p)$ is a piecewise convex function of x_b for given x_p. Separation between convexity intervals occur at points $b_{s_1}(x_p) = \dfrac{\alpha^{s_1} - \nu_p - 2\beta x_p}{\beta}$, $s_1 = 1, \cdots, S$.*

The $b_{s_i}(x_p)$ identify levels of x_b where the marginal value of peak plants becomes zero. The lemma states that $OC_b(x_b \mid x_p)$ is convex in x_b as long as the sets of time segments with zero and nonzero marginal values of peak plants do not change.

It is easy to see that the function $OC_b(x_b \mid x_p)$ is piecewise quadratic. Separation between quadratic pieces occur when some of the $S_i(x)$ change. These changes may also create non-differentiable points in the function $OC_b(x_b \mid x_p)$. Non-convexities can occur only at these points. Consider the points (x_p, x_b) where this non-differentiability of $OC_b(x_b \mid x_p)$ can occur. $MC_2(x_b \mid x_b; \Sigma)$ is still defined if one specifies the values of s_1, s_2, s_3. Using these definitions and the proof of Lemma 13, one can state the following corollary.

Corollary 4 *Let x_p, x_b be a point where $OC_b(x_b \mid x_p)$ is not differentiable.*

Define $\sigma = (s_1, s_2, s_3) = \lim_{\substack{\varepsilon>0 \\ \varepsilon \to 0}} \sigma(x_p, x_b - \varepsilon)$. *Then*

$$\begin{aligned} MC_b(x_b \mid x_p; s_1 - 1, s_2 + 1, s_3) &< MC_b(x_b \mid x_p; s_1, s_2, s_3\} \\ MC_b(x_b \mid x_p; s_1, s_2 - 1, s_3 + 1) &= MC_b(x_b \mid x_p; s_1, s_2, s_3\} \\ MC_b(x_b \mid x_p; s_1, s_2, s_3 - 1) &= MC_b(x_b \mid x_p; s_1, s_2, s_3\}. \end{aligned}$$

Consider now, for x_p given, the evolution of $MC_b(x_b \mid x_p)$ as x_b increases. Elements of $S_1(x)$ can move into $S_3(x)$ and similarly elements of $S_2(x)$ and $S_3(x)$ can move into $S_3(x)$ and $S_4(x)$ respectively. Using Corollary 4, we obtain a graph of $MC_b(x_b \mid x_p)$ as depicted on Figure 4.

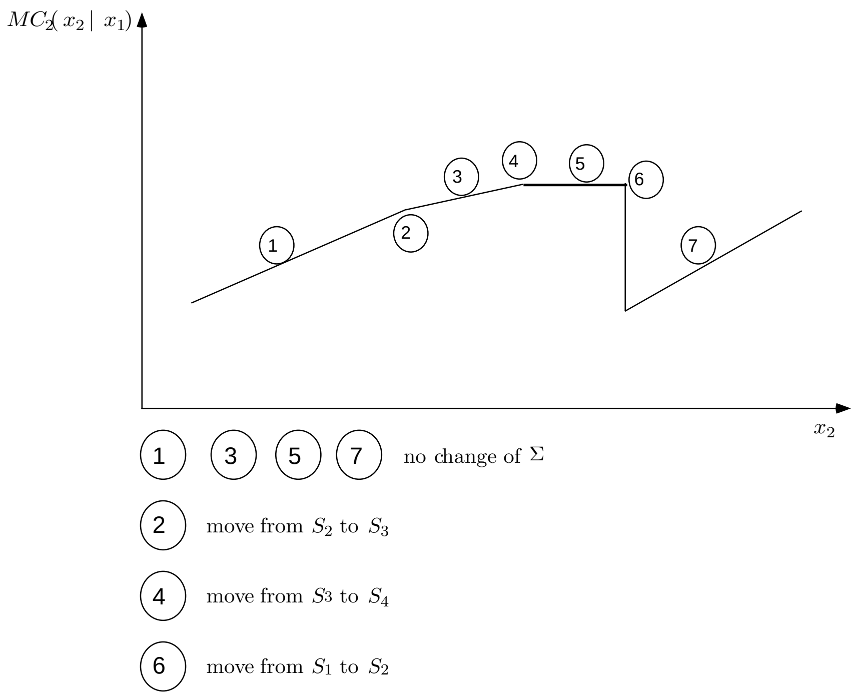

Figure 4: Pattern of $MC_b(x_b \mid x_p)$

7.1 Discussion

It appears from the preceeding discussion that the constraint $x_b \leq b_{s_1}(x_p)$ determines the set $S_1 = \{1, \cdots, s_1\}$ and defines the region of convexity of the function $OC_b(x_b \mid x_p)$. Even though, $\min_{x_b} OC_b(x_b \mid x_p)$ is not a convex problem, it is piecewise convex and $\min_{x_b \leq b_{s_1}(x_p)} OC_b(x_b \mid x_p) = \min OC_b(x_b \mid x_p; s_1)$ is a convex problem. The problem $\min_{x_b} OC_b(x_b \mid x_p; s_1)$ has an economic interpretation: it represents the behavior of player b when this latter optimizes its capacity in the domain of x_b that makes $\lambda_p^s = 0$ (the marginal value of the plant of player p is zero) in all market segments $s > s_1$. Comparing the expressions of the objective functions, one can easily see that $OC_b(x_b \mid x_p; s_1) \geq OC_b(x_b \mid x_p; s_1 - 1)$. It is also easy to see that $x_b \leq b_{s_1}(x_p)$ is contained in $x_b \leq b_{s_1-1}(x_p)$. Reassembling these different remarks, one can conclude that $\min_{x_b} OC_b(x_b \mid x_b) = \min_{s_1} \min_{x_b} OC_b(x_b \mid x_p; s_1)$.

While the objective function of player b is piecewise convex, it is useful to note as stated in the following lemma, that its optimum never lies at the boundary between two zones of convexity.

Lemma 14 *The reaction of base player b (investment x_b) to the action of player p (investment x_p) can never be on a boundary $x_b = b_{s_1}(x_p)$ for some s_1.*

The above discussion leads to a first characterization of the reaction function of player b to the investments of player p.

Proposition 3 *The solution to $\min_{x_b} OC_b(x_b \mid x_p; s_p)(= \min_{x_b \leq b_{s_1}(x_p)} OC_b(x_b \mid x_p))$ exists and is unique. Let $x_b(x_p; s_1)$ be this solution. Then $x_b(x_p; s_1)$ is piecewise affine and continuous, with the slope of each affine segment strictly between 0 and -1.*

Proposition 3 suggests, but does not prove, that the overall reaction function of player b has the form depicted in Figure 5 where a particular piecewise affine function $x_b(x_p; s_1)$ constitutes the reaction function in an interval strictly between two lines $b_s(x_p)$ and $b_{s-1}(x_p)$ (because of Lemma 14). In order to further elaborate on this intuition, consider the first segment of this reaction function.

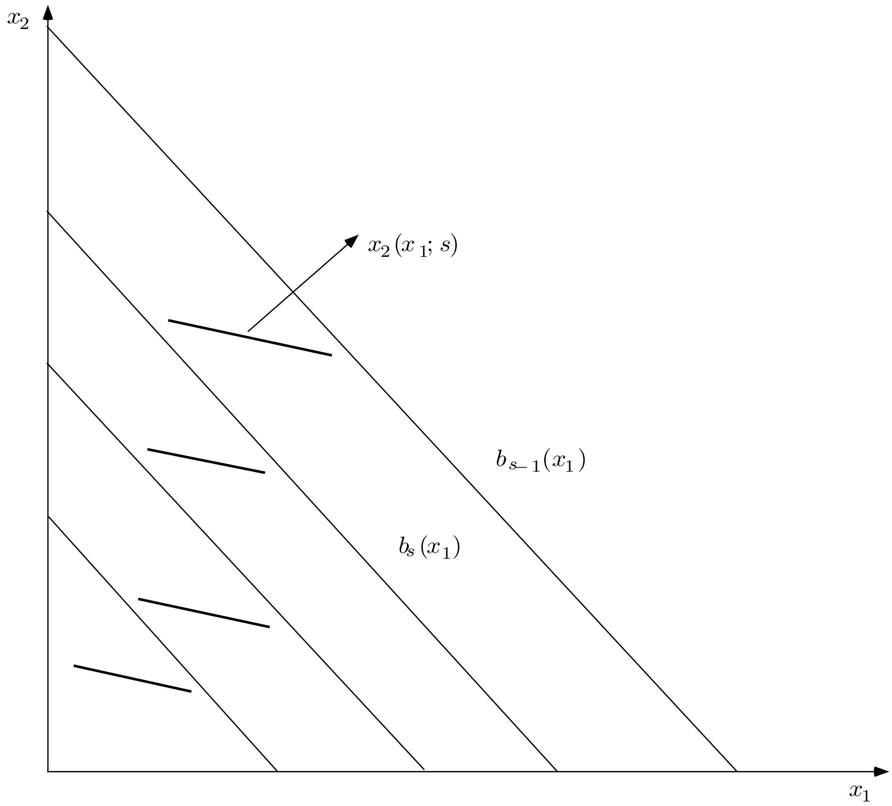

Figure 5: Interpretation

7.2 Construction of the first segment of the reaction function

Consider the initial condition where $x_p = 0$ (no peak capacity). Player b reacts to this situation by solving $\min_{x_b} OC_b(x_b \mid 0)$ which is a convex problem. This solution defines the set of time segments $s = 1, \cdots, s_1^0$ where the marginal value of an investment in the peaker is positive. In order to define s_2 and s_3 and avoid the degeneracy $y_p^s = x_p = 0$ with $\lambda_p^s > 0$, simply take a perturbation $\varepsilon > 0$ of the zero capacity of equipment p. Let s_2^0, s_3^0 be obtained accordingly.

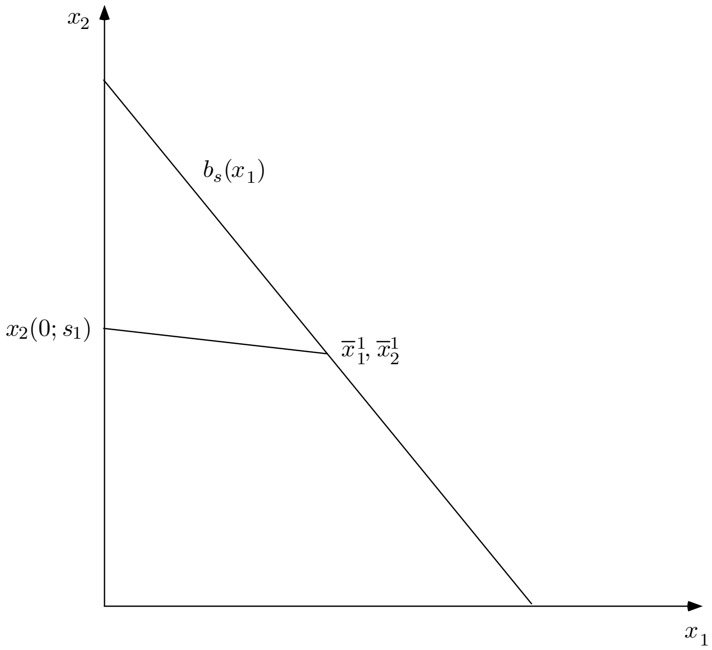

Figure 6: Construction of the reaction curve when x_p departs from 0

Let $x_b(0; s_1^0)$ (or $x_b(0; s_1^0, s_2^0, s_3^0)$) be this solution. By construction $x_b(0; s_1^0)$ $\leq b_{s_1^0}(0)$. Consider now the function $x_b(x_p; s_1^0)$ that is where s_1^0 is kept fixed but the s_2 and s_3 are functions of the point x. By Proposition 3 $x_b(x_p; s_1^0)$ is continuous piecewise affine with slope between 0 and -1 for each affine segment. It thus has an intersection with $x_b = b_{s_1^0}(x_p)$ because $b_{s_1^0}(x_p)$ has a slope -2. Let $\overline{x}_p^1, \overline{x}_b^1$ be this intersection. We know that $\overline{x}_p^1$, $\overline{x}_b^1$ cannot be on the reaction function. Indeed,

Lemma 15 *There exists a point* x_p^1 *strictly between* 0 *and* $\overline{x}_p^1$ *where* $x_b(x_p; s_1^0)$ *ceases to be the optimal response when* $x_p > x_p^1$*. From that point on and on some interval the optimal response is a function* $x_b(x_p; s_1^1)$ *with* $s_1^1 < s_1^0$*. Moreover one has* $x_b(x_p^1; s_1^1) > x_b(x_p^1, s_1^0)$*.*

This leads one to extend the reaction function as depicted on Figure 7.

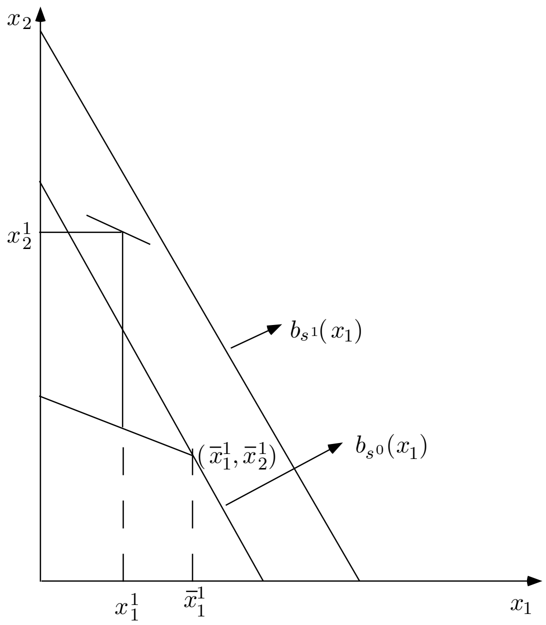

Figure 7: First and second segments of the reaction functions

$x_b(x_p; s_1^0)$ is thus the reaction function until some point x_p^1 where s_1^0 decreases and $x_b(x_p)$ jumps by a positive amount. Let x_p^1, x_b^1(where $x_b^1 = x_b(x_p^1; s_1^1)$) be the point after the jump; $k = 1$ denotes the first jump. This construction can be generalized. Indeed

Lemma 16 *Let (x_p^k, x_b^k) and $x_b(x_p; s_1^k)$ be the point and the reaction function obtained after jump k. One has*

$$x_b(x_p^k; s_1^k) < b_{s_1^k}(x_p^k).$$

If $s_1^k > 1$, $x_b(x_p; s_1^k)$ defines the reaction function until a point x_p^{k+1}, x_b^{k+1}. At that new point, the optimal response is a function $x_b(x_p^{k+1}; s_1^{k+1})$ with $s_1^{k+1} < s_1^k$. Moreover, one has

$$x_b(x_p^{k+1}; s_1^{k+1}) > x_b(x_p^k; s_1^k).$$

This construction can be summarized in the following theorem.

Theorem 8 *The capacity reaction function of the baseload player in the closed-loop game is piecewise continuous with upward jumps. In each interval of continuity, it is monotonically decreasing with slope between 0 and -1.*

Up to this point, we have said nothing about the reaction function of player p. Since this player sees player b at capacity whenever it is, its reaction function is continuous and monotonically decreasing with slope between 0 and -1. Combining the properties of the reaction functions, if they intersect, they intersect at only one point. Summing up we obtain the following existence and uniqueness result.

Theorem 9 *The closed-loop game does not necessarily have a pure strategy equilibrium. If it has, the equilibrium cannot occur when $\lambda_p^s = 0$ and $x_p = y_p^s$. If there is an equilibrium, it is unique.*

The discontinuities have a flavor of strategic substitute and complement effects as discussed in Bulow, Geanakoplis, and Klemperer (1985). The downward sloping affine segments reflect substitute effects. They are driven by the linear demand curves as in Dixit's model. The upward jumps look like extreme cases of complement effects where an increase of capacity of one player (here the peak) induces a simultaneous increase of the other player. But the effect is more difficult to relate to Bulow et al.'s analysis. It indeed results from a re-optimization of the generation of the peak player at certain levels of peak and base capacity. This rearrangement is rooted in the discrete decomposition of the demand curve into different time segments, something that is not directly interpretable in Bulow et al.'s framework.

8 Conclusion

This paper analyzes three capacity expansion models related to the restructured electricity industry. The first model assumes a perfectly competitive market. It is an idealized situation which is useful only for reference. The second model, referred to as the open-loop Cournot model, corresponds to a market where commitments are simultaneously made on investment and sales contracts. It represents an organization based on Power Purchase Agreements. This model has the standard Cournot properties and it is also easy to handle numerically. Finally, the third model represents an industry organized around merchant plants. It is an equilibrium problem subject to equilibrium constraints. In general this type of model is extremely difficult to handle numerically. Indeed, the problem is a true two-stage equilibrium problem that exhibits non-convexities in the first stage. This non-convexity is not surprising. Two-stage equilibrium models are extensions of bilevel and MPEC problems that are well known to be nonconvex. Because of this nonconvexity, some form of enumeration is required not only to find a pure-strategy equilibrium but also to identify whether it exists.

In order to explore the different games, the models elaborated in this paper have been simplified to the case of two agents, each specializing in a particular type of plant, namely peak and base plants. This simplified context facilitates the analysis. Specifically, it makes it relatively easy to identify whether there exists an equilibrium, and to characterize it when it exists. The simplification also makes it possible to structure the set of possible second-stage equilibria, using sensitivity analysis. It is the exploitation of those results that allows us to derive results on an a priori badly behaved problem. This characterization can also help reduce the enumeration required to handle the nonconvexity of the problem in case one tries to solve it numerically. We expect that some of this analysis can be carried through to more general models. In principle, the search through second-stage equilibria needs to be done by enumerating all complementarity sets of the second-stage problem. This may be an impossible task for a general problem with several agents controlling several technologies or when agents are spatially distributed on a grid. One longer-term objective of the paper is to show that this enumeration can be reduced by sensitivity analysis. Also, we expect that economic intuition could help develop this sensitivity analysis and characterize the nature of the relevant nonconvexities. One next step of the research will include exploring which sensitivity properties can be retained in a more general context in order to reduce the enumeration.

Sequential games pervade all electricity restructuring experiences even though the literature remains relatively underdeveloped. Most of the attention in the area so far has concentrated on the contract market (e.g. Green

(1999), Newbery (1998), Wolak (1999), Bessembinder and Lemmon (1999)) or multisettlement systems (Kamat and Oren (2002)). The subject that retained most attention is the extent to which forward markets reduce market power and the incentive of players to engage into these contracts. This problem finds its academic origin in Allaz (1992) and Allaz and Vila (1993). It has been recently highlighted by the contrast between the Californian debacle (where these contracts were forbidden) and the good performance of the British reform (where they were allowed). We look at a somewhat complementary problem as we do not consider the forward/spot markets but compare two situations that differ by the existence of a spot market. Our Cournot model assumes long term contracts but no spot market while the closed loop models disregard long term contracts to concentrate on the spot market. Models such as these and extensions thereof could shed some light on the new situation that the experiences create. This would be particularly relevant to the extent that the impact of the structure of the industry on the incentive to invest and its ultimate consequences on electricity prices remain largely unexplored. An important qualitative feature of the results of this paper is that the model with a spot market has lower prices and higher quantities than the one without the spot market. This result holds even when there is no financial contracts as was the case in the initial stage of restructuring in California. The model also shows the potential for no equilibrium in the model with a spot market. Needless to say, these conclusions could be tempered by the value of long-term contracts in managing risk. Furthermore, a market with a mix of spot and contract prices could potentially lower prices even further than the closed-loop solution, along the lines of the work by Allaz and Vila (1993).

The model also leads to investment criteria. These can be interpreted in terms of real options. But the interesting feature is that these interpretations are not the standard ones. In short a blind application of standard calls on spark spread could be totally erroneous for valuing plants in an oligopolistic market. Thi s would certainly justify further research. How to characterize the solution with continuously growing demand and existing capacity is another topic for future research. Lastly, the development of multistage models could also be warranted. For instance, the second-stage game misses another feature that has appeared in California, using planned maintenance games to raise the price. In short, we believe that two-stage, and multistage game models could help explore many realistic situations that appear in electricity restructuring.

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Appendix

Proof of Lemma 1

To prove that $G^s(y^s)$ is strictly monotone, note that

$$\begin{aligned}
& [G^s(y^{s,1}) - G^s(y^{s,2})]^T(y^{s,1} - y^{s,2}) \\
= \quad & \beta(y_p^{s,1} - y_p^{s,2}, y_b^{s,1} - y_b^{s,2}) \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} y_p^{s,1} - y_p^{s,2} \\ y_b^{s,1} - y_b^{s,2} \end{pmatrix} > 0
\end{aligned}$$

whenever $y^{s,1} \neq y^{s,2}$ (recall that $\beta > 0$). The strict monotonicity of $G(y)$ follows from the strict monotonicity of each $G^s(y^s)$. To show the monotonicity of $F(x, y)$ note that

$$\begin{aligned}
& [F(x^1, y^1) - F(x^2, y^2)]^T \begin{pmatrix} x^1 - x^2 \\ y^1 - y^2 \end{pmatrix} \\
= \quad & (0, 0, G(y^1) - G(y^2))^T \begin{pmatrix} x_p^1 - x_p^2 \\ x_b^1 - x_b^2 \\ y^1 - y^2 \end{pmatrix} = [G(y^1) - G(y^2)]^T(y^1 - y^2).
\end{aligned}$$

This expression is ≥ 0, strictly positive when $y^1 \neq y^2$ and zero otherwise.

Proof of Theorem 1

(i) There always exists an open-loop Cournot equilibrium. Because the demand function is affine and demand is constrained to remain non negative, one has $0 \leq y_i^s \leq \frac{\alpha^s}{\beta}$ for all i and s. There is thus no loss of generality in bounding x_i by $\frac{\alpha^1}{\beta}$ (the largest of the $\frac{\alpha^s}{\beta}$).

The open-loop Cournot equilibrium problem can then be reformulated as a problem in a non empty compactness set. This together with the continuity of the mapping insures the existence of a solution (Harker and Pang (1990)).

(ii) The solution is unique in y because of the strict monotonicity of $G(y)$. It is also unique in x because $x_i = y_i^1$, $i = p, b$ since each generator minimizes its cost.

(iii) The solution is dynamically consistent.

This comes trivially from the statement of the equilibrium conditions (condition (5) is a subset of conditions (5) and (6)).

(iv) The base player always invests a positive amount in the open-loop Cournot equilibrium. The peak player does not necessarily do so, except if the equilibrium peak demand is greater than the other equilibrium demand.

Suppose $x_b = 0$ and assume $x_p > 0$ to avoid the trivial case of no demand.

- The short-run equilibrium conditions imply for $s = 1, \cdots, S$

$$\begin{aligned} -\alpha^s + 2\beta x_p + \nu_p + \lambda_p^s &= 0 \\ -\alpha^s + \beta x_p + \nu_b + \lambda_b^s &\geq 0 \quad \text{or } \alpha^s - \beta x_p \leq \nu_b + \lambda_b^s \end{aligned}$$

- The long run equilibrium conditions, together with the above relations, imply

$$\begin{aligned} & K_p = \textstyle\sum_s \lambda_p^s = \sum_s \alpha^s - 2\beta S x_p - S\nu_p < \sum_s \alpha^s - \beta S x_p - S\nu_p \\ \text{or} \quad & K_p + S\nu_p < \textstyle\sum_s \alpha^S - \beta S x_p \leq S\nu_b + \sum \lambda_b^S \leq K_b + S\nu_b \end{aligned}$$

which contradicts relation (1).

- To show that x_p can be 0 at equilibrium, we use the following example. Let $K_b = 10, \nu_b = 0, K_p = 5.5, \nu_p = 4$, $\alpha^p = 10$, $\alpha^b = 8$, $\beta = 1$. Relation (1) is clearly satisfied. The short-run equilibrium conditions of the base player can be written as

$$\begin{aligned} -\alpha^1 + 2\beta + \nu_b + \lambda_b^1 = 0, \\ -\alpha^2 + 2\beta x_b + \nu_b + \lambda_b^2 = 0 \end{aligned}$$

which are satisfied for $x_b = b, \lambda_b^1 = 6, \lambda_b^2 = 4$.

The short-run equilibrium condition of the peak player can be written as

$$-\alpha^1 + \beta x_b + \nu_p + \lambda_p^1 \geq 0,$$
$$-\alpha^b + \beta x_b + \nu_p + \lambda_p^2 \geq 0$$

which are satisfied for $x_b = b, \lambda_p^1 = 0.5, \lambda_1^b = 0$.

Last we assume that $x_p = 0$ and $q^1 = \alpha^1 - \beta(y_p^1 + y_b^1) > q^s = \alpha^s - \beta(y_p^s + y_b^s), s = 2, \cdots, S$.

The short-run equilibrium conditions for player p and b respectively imply $-\alpha^1 + \beta x_b + \nu_p + \lambda_p^1 \geq 0$ and $-\alpha^1 + 2\beta x_b + \nu_b + \lambda_b^1 = 0$. Using $K_b = \lambda_b^1$ (recall that $\lambda_b^s = 0$ for $s \geq 2$ because $q^1 > q^s$, $s = 2, \cdots, S$). One obtains $K_p \geq \lambda_p^1 \geq \alpha^1 - \beta x_b - \nu_p > \alpha^1 - 2\beta x_b - \nu_p = \alpha^1 - 2\beta x_b - \nu_b + (\nu_b - \nu_p) = \nu_b + \lambda_b^1 - \nu_p = \nu_b - \nu_p + K_b$ or $K_p + \nu_p > K_b + \nu_b$ which contradicts (1).

Proofs of Lemma 2 and Lemma 3

Proof of Lemma 2

The proof follows from the strict monotonicity of $G(y)$ as shown in the proof of Lemma 1. The $y(x)$ are unique for all x, because with x given, the second-stage problem decomposes into a set of standard, single-stage Cournot problems. The $y(x)$ are also continuous in x and hence left and right differentiable with respect to x_p and x_b.

Proof of Lemma 3

The proof follows from the strict monotonicity of $G(y)$ and the monotonicity of $F(x, y)$ as shown in the proof of Lemma 1. Because all these expressions are unique, they are also continuous in x and hence left and right differentiable with respect to x_{-i}.

Proof of Lemma 4

Consider successively the four cases

(i) $0 < y_i^s < x_i, y_{-i} = 0, \omega_{-i}^s > 0$

(ii) $y_i^s < x_i$ and $\omega_i^s = 0$, $i = p, b$

(iii) $y_i^s < x_i$ and $y_{-i}^s = x_{-i}$

(iv) $y_i^s = x_i$, $i = p, b$

Case (i)

From (4), $-\alpha^s + 2\beta y_i^s + \nu_i = 0$ or $y_i^s = \dfrac{\alpha^s - \nu_i}{2\beta}$. Similarly $-\alpha^s + \beta y_i^s + \nu_{-i} = \omega_{-i}^s$ and $\omega_{-i}^s = -\alpha^s + \beta \frac{\alpha^s - \nu_i}{2\beta} + \nu_{-i}$ or $\omega_{-i}^s = -\frac{\alpha^s}{2} - \frac{\nu_i}{2} + \nu_{-i}$.

Case (ii)

Suppose $y_i^s < x_i$, $i = p, b$. $\lambda_i^s = 0$ and hence satisfies the lemma. With $\lambda_i^s = \omega_i^s = 0$, (5) is two equations with two unknowns having a solution

$$y_i^s = \frac{1}{3\beta}\left[\alpha^s - (2\nu_i - \nu_{-i})\right], \quad i = p, b.$$

This proves the lemma for case (ii).

Case (iii)

$$-\alpha^s + 2\beta y_i^s + \beta x_{-i} + \nu_i = 0$$

implies

$$y_i^s = \frac{1}{2\beta}\left(\alpha^s - \beta x_{-i} - \nu_i\right).$$

Inserting this expression in $-\alpha^s + \beta y_i^s + 2\beta x_{-i} + \nu_{-i} + \lambda_{-i}^s = 0$ one gets

$$-\alpha^s + \frac{1}{2}(\alpha^s - \beta x_{-i} - \nu_i) + 2\beta x_{-i} + \nu_{-i} + \lambda_{-i}^s = 0$$

which gives

$$\lambda_{-i}^s = \frac{1}{2}\left[\alpha^s - 3\beta x_{-i} + (\nu_i - 2\nu_{-i})\right]$$

which again proves the lemma.

Case (iv)

$$\lambda_i^s = \alpha^s - 2\beta x_i - \beta x_{-i} - \nu_i$$

which proves the lemma.

The above shows that the $y_i^s(\alpha^s)$ are monotone with α^s in each of the (ii), (iii) and (iv) cases. Because the y_i^s are unique for each α, $y_i^s(\alpha^s)$ is continuous in α and hence the lemma is satisfied. Because we do not have any uniqueness result for the λ, one needs to be somewhat more careful for proving their global monotonicity. Consider a change from (ii) to (iii). The result is trivial since the λ can only move from 0 to a nonnegative value.

Consider now a change from (iii) to (iv). The result is trivial for the λ that goes from zero to a nonnegative value. It is easy to see from the equilibrium conditions (5) that the other λ is continuous with y and hence that the result also holds.

Proof of Lemma 5

Relaxing the constraint $y_p^s < x_p$, consider case (ii) of the proof of Lemma 4. One has $y_p^s = \frac{1}{3\beta}\left[\alpha^s - (2\nu_p - \nu_b)\right]$ and $y_b^s = \frac{1}{3\beta}\left[\alpha^s - (2\nu_b - \nu_p)\right]$. The result follows from the fact that $\nu_p > \nu_b$ implies $2\nu_p - \nu_b > 2\nu_b - \nu_p$. Since $y_p^s = \min\{x_p, \frac{1}{3\beta}[\alpha^s - (2\nu_p - \nu_b)]\}$, the lemma holds.

Proof of Lemma 6

Consider again the different cases of the proof of Lemma 4.

(i) and (ii) $y_i^s < x_i \quad i = p, b$ implies that the $\lambda_i^s = 0$. The lemma is thus satisfied in this case.

(iii) $y_i^s < x_i$ and $y_{-i}^s = x_i$. The relation (proven in Lemma 4)

$$\lambda_{-i}^s = \frac{1}{2}\left[\alpha^s - 3\beta x_{-i} + (\nu_i - 2\nu_{-i})\right]$$

shows that λ_{-i}^s strictly decreasing with x_{-i} and invariant with x_i. The lemma is thus satisfied in this case.

(iv) $y_i^s = x_i \quad i = p, b$ implies

$$\lambda_i^s = \alpha^s - 2\beta x_i - \beta x_{-i} - \nu_i$$

which again proves the lemma.

The result can then be obtained globally by using the same reasoning as in Lemma 4.

Proof of Lemma 7 and Lemma 8

Proof of Lemma 7

The result immediately follows from the expressions used to define segments (a) to (e).

Proof of Lemma 8

The result immediately follows from short-term equilibrium relation (5).

Proof of Proposition 1

Let x_{-i} be given. x_i satisfies

$$-\alpha^s + 2\beta x_i + \beta y_{-i}^s + \nu_i + \lambda_i^s = 0, s \leq s_i(x) \text{ (denoted } s_i \text{ in the following).}$$

Adding up all these relations and noting that $K_i = \sum_s \lambda_i^s = \sum_{s \leq s_i} \lambda_i^s$

$$-\sum_{s=1}^{s_i} \alpha^s + s_i 2\beta x_i + \beta \sum_{s=1}^{s_i} y_{-i}^s + s_i \nu_i + K_i = 0$$

or at a point x_{-i} where the derivative is defined

$$2s_i \frac{dx_i}{dx_{-i}} + \sum_{s=1}^{s_i} \frac{dy_{-i}^s}{dx_{-i}} = 0.$$

Consider the following cases

(i) $s_{-i} \geq s_i$ and hence $\frac{dy_{-i}^s}{dx_{-i}} = 1$ for $s = 1, \cdots, s_i$ which implies $\frac{dx_i}{dx_{-i}} = -\frac{1}{2}$

(ii) $s_{-i} < s_i$ and hence $\frac{dy_{-i}^s}{dx_{-i}} = 1$ for $s = 1, \cdots, s_{-i}$. For $s = s_{-i} + 1, \cdots, s_i$ one has

$$-\alpha^s + \beta x_i + 2\beta y_{-i}^s + \nu_{-i} = 0$$

and hence

$$\frac{dy_{-i}^s}{dx_i} = -\frac{1}{2}.$$

Combining these relations we write

$$2s_i \frac{dx_i}{dx_{-i}} + s_{-i} + (s_i - s_{-i}) \frac{dy_{-i}^s}{dx_i} \frac{dx_i}{dx_{-i}} = 0.$$

This gives

$$\frac{dx_i}{dx_{-i}} \left(2s_i - \frac{1}{2}(s_i - s_{-i}) \right) + s_{-i} = 0$$

or

$$\frac{dx_i}{dx_{-i}} = -\frac{s_{-i}}{\frac{3}{2}s_i + \frac{1}{2}s_{-i}} = -\frac{2s_{-i}}{3s_i + s_{-i}} \geq \frac{-2s_i}{3s_i + s_{-i}} > -1.$$

Proof of Proposition 2

Let x_{-i} be given. One has

(i) $\quad -\alpha^s + \beta y_i^s + 2\beta y_{-i}^s + \nu_{-i} = 0 \quad \text{if} \quad y_i^s < x_i,\ i = p, b \quad \text{and hence}$ $\dfrac{dy_{-i}^s}{dx_{-i}} = 0$

(ii) $\quad -\alpha^s + \beta x_i + 2\beta y_{-i}^s + \nu_{-i} = 0 \quad \text{if} \quad y_i^s = x_i$ and $y_{-i}^s < x_{-i}$ and $\lambda_{-i} = 0$. This implies $\dfrac{dy_{-i}^s}{dx_{-i}} = -\dfrac{1}{2}\dfrac{dx_i}{dx_{-i}}$ which is strictly between 0 and 1 by Proposition 1

(iii) $\quad \dfrac{dy_{-i}^s}{dx_{-i}} = 1 \quad \text{if} \quad y_{-i}^s = x_{-i}$ and $\lambda_{-i} > 0$

Let x_{-i} be given. One has

(i) $-\alpha^s + 2\beta y_i^s + \beta y_{-i}^s + \nu_{-i} = 0$ if $y_i^s < x_i$ and $y_{-i}^s \le x_{-i}$ and $\lambda_{-i} = 0$ and hence $\dfrac{dy_i^s}{dx_{-i}} = 0$

(ii) $-\alpha^s + 2\beta y_i^s + \beta x_{-i} + \nu_{-i} = 0$ if $y_i^s \le x_i$ and $\lambda_i^s = 0$ and $y_{-i}^s = x_{-i}$ and $\lambda_i^s > 0$ and hence $\dfrac{dy_i^s}{dx_{-i}} = -\dfrac{1}{2}$

(iii) $\dfrac{dy_i^s}{dx_{-i}} = \dfrac{dx_i}{dx_{-i}}$ if $y_i^s = x_i$ and $\lambda_i^s > 0$ and hence $\dfrac{dy_i^s}{dx_{-i}}$ is strictly between 0 and -1 by Proposition 1.

Proof of Lemma 9

Let s be a segment of type e. Let $y_i^s = 0$ and $y_{-i}^s = x_{-i}$. Suppose $x_i > 0$, one gets

$$\alpha^s - \beta x_{-i} - \nu_i = -\omega_i^s \ge 0$$

$$\alpha^s - 2\beta x_{-i} - \nu_{-i} - \lambda_{-i}^s = 0 \quad \text{or} \quad \alpha^s - 2\beta x_{-i} - \nu_{-i} \ge 0$$

Combining the two relations, one gets

$$\nu_i \ge \alpha^s - \beta x_{-i} > \alpha^s - 2\beta x_{-i} > \nu_{-i}$$

which shows that i is generator 1.

Proof of Theorem 2

Consider a point of differentiability and the associated equilibrium conditions

$$\begin{aligned} K_i &+ \textstyle\sum_s \left[-\alpha^s + 2\beta y_i^s(x) + \beta y_{-i}^s(x) + \nu_i\right] B_i y_i^s(x) \\ &+ \textstyle\sum_s \beta y_i^s(x) B_i y_{-i}^s(x) = 0 \qquad i = p, b \end{aligned}$$

The $B_i y_{-i}^s(x)$ are zero when s belongs to a, b or d or e when $\omega_i^s > 0$ (Lemma 7). The equilibrium condition becomes

$$K_i + \sum_s \left[-\alpha^s + 2\beta y_i^s(x) + \beta y_{-i}^s(x) + \nu_i\right] B_i y_i^s(x) = 0.$$

This expression can be rewritten (using Lemma 7) as

$$K_i + \sum_{s \in a \cup e} \left[-\alpha^s + 2\beta y_i^s(x) + \beta y_{-i}^s(x) + \nu_i\right] = K_i - \sum_s \lambda_i^s = 0$$

which shows that the solution is also a solution to the Open-Loop Cournot problem.

Proof of Lemma 10

Suppose $0 < y_i^s = x_i$ and $0 < y_i^{s'} < x_i$ with $s' < s$. One then has $y_i^{s'} < x_i = y_i^s$ with $\alpha^{s'} > \alpha^s$ which contradicts Lemma 4. Alternatively $0 < y_i^s < x_i$ and $0 < y_{-i}^s = x_{-i}$ with $s' > s$ shows the same contradiction.

Proof of Theorem 3

Using the reasoning of the proof of Theorem 2, one can restate the equilibrium condition as

$$K_i + \sum_{s \in a \cup c \cup e} (-\lambda_i^s) + \sum_{s \in c} \beta y_i^s(x) B_i y_{-i}^s(x) = 0, i = p, b$$

Suppose $y_i^s = x_i$ and $y_{-i}^s < x_{-i}$ for $i \in c$, then $B_i y_{-i}^s(x) = -\frac{1}{2}$. We obtain

$$\begin{array}{lll} & K_i - \sum_s \lambda_i^s - \frac{1}{2}\sum_{s\in c}\beta y_i^s(x) = 0 & \text{and} \quad K_{-i} - \sum_s \lambda_{-i}^s = 0 \\ \text{or} & K_i = \Sigma_s \lambda_i^s + \frac{1}{2}\sum_{s\in c}\beta y_i^s(x) > \sum_s \lambda_i^s & \text{and} \quad K_{-i} = \sum_s \lambda_{-i}^s \end{array}$$

Proof of Lemma 11

Note that the result holds trivially if both players are at capacity throughout all time segments. Suppose not and let s_i be defined as in relation (10). Suppose $s_b < s_p$. One has

$$\begin{array}{ll} & \lambda_p^s = \lambda_b^s = 0 \qquad s > s_p \\ & \lambda_p^s \geq \lambda_b^s = 0 \qquad s_p \geq s > s_b \\ & \lambda_p^{s_b} - \lambda_p^{s_b+1} = \alpha^{s_b} - \alpha^{s_b+1} - \beta(x_b - y_b) \\ \text{because} & -\alpha^{s_b} + 2\beta x_p + \beta x_b + \nu_p + \lambda_p^{s_b} = 0 \ \text{ and} \\ & -\alpha^{s_b+1} + 2\beta x_p + \beta y_b + \nu_p + \lambda_p^{s_b+1} = 0 \end{array}$$

$$\begin{array}{ll} & \lambda_b^{s_b} - \lambda_b^{s_b+1} = \alpha^{s_b} - \alpha^{s_b+1} - 2\beta(x_b - y_b) \\ \text{because} & -\alpha^{s_b} + \beta x_p + 2\beta x_b + \nu_b + \lambda_b^{s_b} = 0 \text{ and } -\alpha^{s_b+1} + \beta x_p + 2\beta y_b \\ & +\nu_b = 0 \end{array}$$

and hence

$$\begin{array}{l} \lambda_p^{s_b} - \lambda_p^{s_b+1} > \lambda_b^{s_b} - \lambda_b^{s_b+1}, \text{ and } \lambda_p^s - \lambda_p^{s+1} = \alpha^s - \alpha^{s+1}, \\ \lambda_b^s - \lambda_b^{s+1} = \alpha^s - \alpha^{s+1} \qquad s_b > s \end{array}$$

and hence

$$\begin{array}{l} \lambda_p^{s_b} - \lambda_p^{s_b+1} > \lambda_b^{s_b} - \lambda_b^{s_b+1}, \text{ and } \lambda_p^s - \lambda_p^{s+1} = \alpha^s - \alpha^{s+1}, \\ \lambda_b^s - \lambda_b^{s+1} = \alpha^s - \alpha^{s+1} \qquad s_b > s. \end{array}$$

Combining these relations we get

$$\sum_s \lambda_p^s > \sum_s \lambda_b^s.$$

Suppose we are dealing with an Open-Loop Cournot equilibrium. Then

$$K_p = \sum_s \lambda_p^s > \sum_s \lambda_b^s = K_b$$

which is a contradiction. Suppose we are dealing with a Closed-Loop Cournot equilibrium and player p is at capacity in segment c. Then $s_p > s_b$ and by Theorem 3

$$K_p > \sum_s \lambda_p^s > \sum_s \lambda_b^s = K_b$$

which is a contradiction.

Proof of Theorem 4

Let o and c indicate the closed and open-loop Cournot equilibrium respectively. Suppose $x_p^o + x_b^o > x_p^c + x_b^c$. We prove the contradiction in two parts.

Part 1: We show that $x_p^o > x_p^c$ and $x_p^o + x_b^o > x_p^c + x_b^c$ implies $\sum_s \lambda_p^{sc} > K_p$ which contradicts the above corollary.

Part 2: We show that $x_b^o > x_b^c$ and $x_p^o + x_b^o > x_p^c + x_b^c$ implies $\sum_s \lambda_b^{sc} > K_b$ which again contradicts the above corollary.

Part 1

Suppose $x_p^o > x_p^c$, $x_p^o + x_b^o > x_p^c + x_b^c$. We know that $K_p = \sum_s \lambda_p^{sc}$ (long-term equilibrium condition (6) of the open-loop problem) and will show that $\lambda_p^{sc} \geq \lambda_p^{s0}$ for all s with some inequalities holding strictly. Let $K_p = \sum_{s'} \lambda_p^{s'o} + \sum_{s''} \lambda_p^{s''o}$ with $\lambda_p^{s'o} > 0$ and $\lambda_p^{s''o} = 0$

$$\lambda_p^{s'o} > 0 \text{ implies } y_p^{s'o} = x_p^0 > x_p^c \geq y_p^{s'c} \tag{A.1}$$

By Lemma 11, $\lambda_p^{s'o} > 0$ implies $\lambda_b^{s'o} > 0$ and hence $y_b^{s'o} = x_b^o$. Adding up $y_p^{s'o}$ and $y_b^{s'o}$ one gets

$$y_p^{s'o} + y_b^{s'o} = x_p^o + x_b^o > x_p^c + x_b^c \geq y_p^{s'c} + y_b^{s'c} \tag{A.2}$$

Adding (A.1) and (A.2) one gets

$$2y_p^{s'o} + y_b^{s'o} > 2y_p^{s'c} + y_b^{s'c}$$

and hence

$$\lambda_p^{s'c} > \lambda_p^{s'o}.$$

Therefore,

$$\sum_{s'} \lambda_p^{s'c} + \sum_{s''} \lambda_p^{s''c} > \sum_{s'} \lambda_p^{s'o} + \sum_{s''} \lambda_p^{s''o} = K_p$$

which is the desired contradiction.

Part 2

Suppose $x_p^o < x_p^c$ and $x_b^o > x_b^c$ and $x_p^o + x_b^o > x_p^c + x_b^c$. Let $K_b = \sum_{s'} \lambda_b^{s'o} + \sum_{s''} \lambda_b^{s''o}$ with $\lambda_b^{s''o} = 0$ and $\lambda_b^{s'o} > 0$.

$$\lambda_b^{s'o} > 0 \text{ implies } y_b^{s'o} = x_b^o > x_b^c \geq y_b^{s'c} \tag{A.3}$$

Suppose $\lambda_p^{s'o} > 0$, then

$$y_p^{s'o} = 0 \text{ and } y_p^{s'o} + y_b^{s'o} = x_p^o + x_b^o > x_p^c + x_b^c \geq y_p^{s'c} + y_b^{s'c} \tag{A.4}$$

Adding (A.3) and (A.4) one gets

$$y_p^{s'o} + 2y_b^{s'o} > y_p^{s'c} + 2y_b^{s'c}$$

and hence

$$\lambda_b^{s'c} > \lambda_b^{s'o}.$$

Suppose $\lambda_p^{s'o} = 0$, the short-term equilibrium conditions of player p in the open and closed-loop games are

$$-\alpha^s + 2\beta y_p^{s'o} + \beta x_b^o + \nu_p = 0$$

and

$$-\alpha^s + 2\beta y_p^{s'c} + \beta y_p^{s'c} + \nu_p + \lambda_p^{s'c} = 0$$

which gives

$$2\beta y_p^{s'o} + \beta y_b^{s'o} = \alpha^s - \nu_p \geq \alpha^s - \nu_p - \lambda_p^{s'c} = 2\beta y_p^{s'c} + \beta y_b^{s'c} \tag{A.5}$$

Adding (A.1) multiplied by 3β to (A.5) and simplifying one gets

$$y_p^{s'o} + 2y_b^{s'o} > y_p^{s'c} + 2y_b^{s'c}$$

and hence

$$\lambda_b^{s'c} > \lambda_b^{s'o}.$$

We then get

$$\sum_{s'} \lambda_b^{s'c} + \sum_{s''} \lambda_b^{s''c} > \sum_{s'} \lambda_b^{s'o} + \sum_{s''} \lambda_b^{s''o} = K_b$$

which is the desired contradiction.

Proof of Theorem 5

Suppose $x_p^o < x_p^c$. This relation together with $x_p^o + x_b^o < x_p^c + x_b^c$ shown in Theorem 4 implies $2x_p^o + x_b^o < 2x_p^c + x_b^c$. Using the corollary of Lemma 11, we write

$$K_p = \sum_{s'} \lambda_p^{s'c} + \sum_{s''} \lambda_p^{s''c} \text{ with } \lambda_p^{s'c} > 0 \text{ and } \lambda_p^{s''c} = 0.$$

Because $\lambda_p^{s'c} > 0$ implies $\lambda_b^{s'c} > 0$, by Lemma 11 we have

$$2y_p^{s'o} + y_b^{s'o} \leq 2x_p^o + x_b^o < 2x_p^c + x_b^c = 2y_p^{s'c} + y_b^{s'c}$$

which proves that $\lambda_p^{s'o}$ must be greater than $\lambda_p^{s'c}$. This implies

$$K_p = \sum_{s'} \lambda_p^{s'c} + \sum_{s''} \lambda_p^{s''c} < \sum_{s}^{\prime} \lambda_p^{s'o} + \sum_{s''} \lambda^{s''} = K_p$$

and hence a contradiction.

Proof of Theorem 6

Using Theorems 4 and 5, we know that $x_p^o + x_b^o < x_p^c + x_b^c$ and $x_b^o \leq x_b^c$. Suppose first that $\lambda_p^{sc} > 0$, then $\lambda_b^{sc} > 0$ and $y_p^{so} + y_b^{so} \leq x_p^o + x_b^o < x_p^c + x_b^c = y_p^{sc} + y_b^{sc}$ and the result is proven for these load segments.

Suppose now that $\lambda_p^{sc} = 0$ and $\lambda_b^{sc} > 0$, the two following relations hold at (y_p^{sc}, x_b^c)

$$-\alpha^s + 2\beta y_p^s + \beta x_b + \nu_p = 0 \tag{A.6}$$

$$-\alpha^s + \beta y_p^s + 2\beta x_b + \nu_b + \lambda_b^s = 0 \tag{A.7}$$

Consider a decrease of x_b from the value x_b^c towards x_b^o. Using (A.6) one sees that $y_p^s + x_b$ decreases $\left(\frac{d}{dx_b}(y_p^s + x_b) = \frac{1}{2}\right)$ as well as $y_p^s + 2x_b$ $\left(\frac{d}{dx_b}(y_p^s + 2x_b)\right.$

$= \frac{3}{2}\Big)$. Relation (A.7) will thus continue to hold with an increased λ_b^s for any decrease of x_b. Relation (A.6) will also continue to hold until x_b hits x_b^o or y_p^s hits x_p^o. In the first case, (y_p^s, x_b^o) satisfying (A.6) is the open-loop second-stage equilibrium in time segment s; it satisfies $y_p^s + x_b^o < y_p^{sc} + x_b^c$ which proves the result. In the second case we continue decreasing the value of x_b until it hits x_b^o while keeping y_p bounded at its upper limit x_p^o. This will result in a further decrease of $y_p^s + x_b$ until the point (x_p^o, x_b^o). This point is the open-loop second-stage equilibrium in time segment; it satisfies $x_p^o + x_b^o < y_p^{sc} + x_b^c$ which proves the result.

Proof of Lemma 12

Suppose first $x_p^c > 0$ and $x_p^o > 0$. Recall that the base player is at capacity when the peak player is at capacity (Lemma 11 and a similar and trivial result for the perfect-competition equilibrium). The equilibrium conditions (2) and (5) imply

$$\begin{aligned}
&-\alpha^s + \beta x_p^m + \beta x_b^m + \nu_p + \lambda_p^{sm} = 0 && s \in \{s \mid \lambda_p^{sm} > 0\} \\
&-\alpha^s + 2\beta x_p^o + \beta x_b^o + \nu_p + \lambda_p^{so} = 0 && s \in \{s \mid \lambda_p^{so} > 0\} \\
&-\alpha^s + 2\beta x_p^c + \beta x_b^c + \nu_p + \lambda_p^{sc} = 0 && s \in \{s \mid \lambda_p^{sc} > 0\}
\end{aligned}$$

or

$$\begin{aligned}
&\lambda_p^{sm} = \lambda_p^{(s-1)m} + (\alpha^s - \alpha^{s-1}) && s \in \{s \mid \lambda_p^{sm} > 0\} \\
&\lambda_p^s = 0 && s \notin \{s \mid \lambda_p^{sm} > 0\}
\end{aligned}$$

with similar relations for the open-loop and closed-loop equilibria. This is a set of simultaneous equations with one degree of freedom. Since

$$K_1 = \sum_s \lambda_p^{sm} = \sum_s \lambda_p^{sc} = \sum_s \lambda_p^{so},$$

we have three identical sets of simultaneous equations with the same solutions and the result holds.

Suppose now $x_p^c = 0$. (We know that $x_p^m > 0$ by our assumption on the cost parameters.) The equilibrium condition (5) becomes

$$-\alpha^s + \beta x_b^c + \nu_p + \lambda_p^{sc} = \omega_p^{sc} \qquad s \in \{s \mid \lambda_p^{sc} > 0\}$$

Select the minimal λ_p^{sc}, that is those for which $\omega_p^{so} = 0$. One has as before

$$\begin{aligned} \lambda_p^{sc} &= \lambda_p^{(s-1)c} + (\alpha^s - \alpha^{s-1}) \qquad && s \in \{s \mid \lambda_p^{sc} > 0\} \\ \lambda_p^{sc} &= 0 && s \notin \{s \mid \lambda_p^{sc} > 0\} \end{aligned}$$

The same reasoning as before implies $\lambda_p^{sm} \geq \lambda_p^{sc}$.

Proof of Theorem 7

Suppose first $x_p^c > 0$. (By assumption $x_p^m > 0$). Then by Lemma 12

$$K_p = \sum_{s \in S_p} (\alpha^s - \beta x_p^m - \beta x_b^m - \nu_p) = \sum_{s \in S_p} (\alpha^s - 2\beta x_p^c - \beta x_b^c - \nu_p)$$

where $S_p = \{s \mid \lambda_p^s > 0\}$. Thus

$$0 = |S_p|\{\beta x_p^c + \beta[(x_p^c + x_b^c) - (x_p^m + x_b^m)]\}$$

or

$$0 = x_p^c + (x_p^c + x_b^c) - (x_p^m + x_b^m)$$

and hence since $x_p^c > 0$,

$$x_p^m + x_b^m > x_p^c + x_b^c.$$

Suppose now $x_p^c = 0$. Then by Lemma 12

$$K_p = \sum_{s \in S_p} (\alpha^s - \beta x_p^m - \beta x_b^m - \nu_p) \geq \sum_{s \in S_p} (\alpha^s - 2\beta x_p^c - \beta x_b^c - \nu_p)$$

where $S_p = \{s \mid \lambda_p^{sm} > 0\}$, or

$$0 \geq |S_p|\{\beta[(x_p^c + x_b^c) - (x_p^m + x_b^m)]\} \qquad \text{(since } x_p^c = 0\text{)}$$

or again

$$x_p^m + x_b^m \geq x_p^c + x_b^c.$$

By Theorem 4 we know that $x_p^c + x_b^c \geq x_p^o + x_b^o$ and the capacity result holds.

By Theorem 6 we see that the production in each time segment in the closed-loop game is greater than the production in the open-loop game. To see that the competitive equilibrium has higher production than the closed-loop equilibrium, we need only consider the cases where the capacity of player 1 is not binding in the competitive case. First assume $y_p^{sc} > 0$. Then in equilibrium

$$\begin{aligned} & -\alpha^s + \beta y_p^{sm} + \beta y_b^{sm} + \nu_p = \omega_p^{sm} \geq 0 \\ & -\alpha^s + 2\beta y_p^{sc} + \beta y_b^{sc} + \nu_p + \lambda_p^{sc} = 0 \\ \text{or} \quad & y_p^{sm} + y_b^{sm} \geq 2y_p^{sc} + y_b^{sc} > y_p^{sc} + y_b^{sc}. \end{aligned}$$

For $y_p^{sc} = 0$, note that y_b^{sm} must also be zero (the marginal revenues are the same in both cases at 0 and below marginal cost) and we need only consider the case where $y_b^{sm} < x_b^m$, which leads to

$$\begin{aligned} & -\alpha^s + \beta y_b^{sm} + \nu_b = 0 \\ & -\alpha^s + 2\beta y_b^{sc} + \nu_b + \lambda_b^{sc} = 0 \\ \text{or} \quad & y_b^{sm} \geq 2y_b^{sc} + \lambda_b^{sc} > y_b^{sc}. \end{aligned}$$

Thus, the production levels are highest in competitive markets.

Proof of Lemma 13

Consider the derivative $MC_b(x_b \mid x_p)$ given by

$$\begin{aligned} K_b \quad & + \quad \textstyle\sum_{s \in S_1(x)} (-\alpha^s + \beta x_p + 2\beta x_b + \nu_b) \\ & + \quad \textstyle\sum_{s \in S_b(x)} (-\alpha^s + \beta y_p^s(x) + 2\beta x_b + \nu_b) \\ & - \quad \frac{1}{2} \textstyle\sum_{s \in S_2(x)} \beta x_b + \sum_{s \in S_3(x)} (-\alpha^s + 2\beta x_b + \nu_b). \end{aligned}$$

Recalling from the equilibrium condition (5) that

$$2\beta y_p^s(x) = \alpha^s - \beta x_b - \nu_p, s \in S_b(x)$$

we get after grouping terms

$$\begin{aligned} MC_b(x_b \mid x_p) &= K_b + \textstyle\sum_{s \in S_1(x) \cup S_2(x) \cup S_3(x)} (-\alpha^s + \nu_b) \\ &+ \textstyle\sum_{s \in S_1(x)} \beta x_p + \sum_{s \in S_2(x)} \left(\dfrac{\alpha^s - \nu_p}{2} \right) \\ &+ 2(|S_1(x)| + |S_3(x)| \beta x_b + |S_2(x)|) \beta x_b \end{aligned}$$

Note the following

(i) the expression is increasing with x_b as long as the $S_i(x)$ do not change.

(ii) the expression is constant when an element $\overline{s}$ goes from $S_2(x)$ into $S_3(x)$. To see this, take the equilibrium condition of player 1 when $y_p^{\overline{s}}$ becomes zero

$$-\alpha^{\overline{s}} + \beta x_b + \nu_p = 0$$

and note this is the balance of changes of terms that results from $\overline{s}$ moving from $S_2(x)$ to $S_3(x)$ (note that this change requires replacing a term βx_b by $2\beta x_b$ and dropping a term $\dfrac{\alpha^{\overline{s}} - \nu_b}{2}$).

(iii) The expression is constant, when an element $\overline{s}$ goes from $S_3(x)$ into $S_4(x)$. To see this, take the equilibrium condition of player b when $y^{\overline{s}}$ becomes lower than x_b

$$-\alpha^{\overline{s}} + 2\beta x_b + \nu_b = 0$$

and note that this is the balance of changes of terms that results from $\overline{s}$ moving from $S_3(x)$ to $S_4(x)$.

(iv) The expression decreases when an element $\overline{s}$ goes from $S_1(x)$ into $S_2(x)$.

To see this, note that

$$\begin{aligned} & \textstyle\sum_{s\in S_1(x)\cup S_2(x)\cup S_3(x)}(-\alpha^s+\nu_b)+\sum_{s\in S_1(x)\setminus\{\overline{s}\}}\beta x_p \\ & \quad +\textstyle\sum_{s\in S_2(x)\cup\{\overline{s}\}}\left(\dfrac{\alpha^s-\nu_p}{2}\right) \\ & \quad +2(|S_1(x)|+|S_3(x)|-1)\beta x_b+(|S_2(x)|+1)\beta x_b] \\ & \quad -\textstyle\sum_{s\in S_1(x)\cup S_2(x)\cup S_3(x)}(-\alpha^s+\nu_b)-\sum_{s\in S_2(x)}\dfrac{\alpha^s-\nu_p}{2} \\ & \quad +\textstyle\sum_{s\in S_1(x)}\beta x_p+2(|S_1(x)|+|S_3(x)|)\beta x_b+|S_2(x)|\beta x_b \\ = & \quad \dfrac{\alpha^s-\nu_p}{2}-\beta x_p-\beta x_b<0 \end{aligned}$$

Applying the equilibrium condition of player p when $\lambda_p^{\overline{s}}$ becomes zero $(-\alpha^{\overline{s}}+2\beta x_p+\beta x_b+\nu_p=0)$, one sees that this expression is equal to $-\dfrac{\beta x_b}{2}<0$.

The above shows that $MC_b(x_b \mid x_p)$ is increasing with x_b as long as no element moves from S_1 into S_2. $OC_b(x_b \mid x_p)$ is thus convex in x_b in these zones. $MC_b(x_b \mid x_p)$ has downward jumps when elements move from $S_1(x)$ to $S_2(x)$. This happens when some λ_p^s becomes zero in the equilibrium condition of player p, that is when

$$\alpha^s-2\beta x_p-\beta x_b-\nu_p=0.$$

This proves the result.

Proof of Lemma 14

Suppose that the solution of $\min OC_b(x_b \mid x_p; s_p)$ is $x_b=b_{s_1}(x_p)$. By the convexity of $OC_b(x_b \mid x_p; s_1)$, $MC_b(x_b \mid x_p; s_1)$ must be nonpositive at $x_b=b_{s_1}(x_p)$. This implies that $MC_b(x_b \mid x_p; s_1-1)<MC_b(x_b \mid x_p; s_1)\leq 0$ and hence that player b will not select $b_{s_1}(x_p)$ as its reaction to x_p.

Proof of Proposition 3

By Lemma 13, $OC_b(x_b \mid x_p)$ is strictly convex in x_b for $x_b \leq b_{s_1}(x_p)$. The solution x_b of $\min_{x_b \leq b_{s_1}(x_p)} OC_b(x_b \mid x_p)$ is thus unique and hence continuous in x_p. Suppose also that it satisfies $x_b < b_{s_1}(x_p)$. Then it is the solution of

$$MC_b(x_b \mid x_p; s_1) = 0$$

which can be rewritten for a certain M

$$M + |S_1(x)|\beta x_p + (2|S_1(x)| + 2|S_3(x)| + |S_2(x)|)x_b = 0.$$

This implies

$$\frac{dx_b}{dx_p} = -\frac{|S_1(x)|}{2|S_1(x)| - 2|S_3(x)| + |S_2(x)|}$$

and hence the solution is piecewise affine with slope between 0 and -1.

Proof of Lemma 15

Let $x_b(x_p; s_1, s_2, s_3)$ designate a response function for an arbitrary $\sigma = (s_1, s_2, s_3)$ as defined above. $OC_b(x_b(x_p; s_1, s_2, s_3) \mid x_p)$ is a quadratic function of x_p that we denote

$$OC_b(x_p; s_1, s_2, s_3).$$

Let $\widetilde{x}_p$ be the largest value, before x_p^1 where there has been a change of s_2 or s_3 ($\widetilde{x}_p$ could be 0). Let $\tilde{s}_2$ and $\tilde{s}_3$ be the values of s_2 and s_3 valid in the interval $(\tilde{x}_p, x_p^1)$. By the definition of the reaction in $\widetilde{x}_p$, we have

$$OC_b(\widetilde{x}_p; s_1^0, \tilde{s}_2, \tilde{s}_3) \leq OC_b(\widetilde{x_p}; s_1, s_2, s_3) \quad \forall\ s_1, s_2, s_3$$

At $\overline{x}_p^1, \overline{x}_b^1$ one has by the definition of the two curves defining this intersection

$$OC_b(x_b(\overline{x}_p^1; s_1^0, \tilde{s}_2, \tilde{s}_3) \mid \overline{x}_p^1) = OC_b(\overline{x}_p; s_1^0 - 1, \tilde{s}_2 + 1, \tilde{s}_3 \mid \overline{x}_p^1)$$

and $$0 = MC_b(x_b(\overline{x}_p^1; s_1^0, \tilde{s}_2, \tilde{s}_3) \mid \overline{x}_p) > MC_b(x_b(\overline{x}_p^1; s_1^0 - 1, \tilde{s}_2 + 1, \tilde{s}_3 \mid \overline{x}_p).$$

Therefore $\overline{x}_b^1$ is not the optimal solution of $OC_b(x_b \mid \overline{x}_p^1; s_1^0 - 1, \tilde{s}_2 + 1, \tilde{s}_3)$ which implies

$$\begin{aligned} OC_b(\overline{x}_p^1; s_1^0, \tilde{s}_2, \tilde{s}_3) &> OC_b(\overline{x}_p^1; s_1^0 - 1, \tilde{s}_2 + 1, \tilde{s}_3) \\ &\geq OC_b(\overline{x}_p^1; s_1' s_2', s_3') \text{ for some } \sigma' = (s_1', s_2', s_3') \end{aligned}$$

Moreover, we note that if an $OC_b(\overline{x}_p^1; s_1', s_2', s_3')$ is smaller than $OC_b(\overline{x}_p; s_1^0, \tilde{s}_2, \tilde{s}_3)$ it must be that $s_1' < s_1^0$. Because $OC_b(x_p; s_1', s_2', s_3')$ and $OC_b(x_p; s^0, \tilde{s}_1, \tilde{s}_2)$ are quadratic functions, they can intersect in at most two points. Specifically a function $OC_b(x_p; s_1', s_2', s_3')$ that takes a lower value than $OC_b(x_p; s_1^0, \tilde{s}_2, \tilde{s}_3)$ at $\overline{x}_p^1$ must intersect $OC_b(x_p; s_1^0, s_2, s_3)$ before $\overline{x}_p^1$. Let x_p^1 the first of these intersection points and $OC_b(x_p; s_1^1, s_2^1, s_3^1)$ be the function that generates this intersection. At x_p^1 we have

$$0 = MC_b(x_b(x_p^1; s_1^0, \tilde{s}_2, \tilde{s}_3) \mid x_p^1) > MC_b(x_b(x_p^1; s_1^1, s_2^1, s_3^1) \mid x_p^1)$$
$$\text{(because } s_1^1 < s_1^0) \qquad \text{(A.8)}$$
$$OC_b(x_p; s_1^0, \tilde{s}_2, \tilde{s}_3) > OC_b(x_p; s_1^1, s_2^1, s_3^1) \text{ for } x_p > x_p^1$$
$$\text{(because of the properties of intersection of quadratic functions)} \qquad \text{(A.9)}$$

From (A.8) we derive that $x_b(x_p^1; s_1^1, s_2^1, s_3^1) > x_b(x_p^1; s_1^0, \tilde{s}_2, \tilde{s}_3)$. From (A.9) we see that $x_b(x_p; s_1^1, s_2^1, s_3^1)$ is the new reaction curve from x_p^1 on. We also note from before that $s_1^1 < s_1^0$.

Proof of Lemma 16

The proof follows the same reasoning as the proof of Lemma 15, starting from the point (x_p^k, x_b^k) with (x_p^k, s_2^k, s_2^k) instead of starting at $(0, x_b^0)$ with (s_1^0, s_2, s_3).

Notation

$s = 1, \cdots, S$	load segments
$s = 1$	peak segment
$s = S$	base segment
p	peak player
b	baseload player
S_p	last segment for which peak capacity is the lower-cost capacity
$i = p, b$	index of the players
K_i	investment cost
ν_i	operating cost
x_i	amount of investment by player i
$x = (x_p, x_b)$	
y_i^s	operating level of player i in segment s
p^s	price in segment s
α^s	intercept of the demand curve
ω_i^s	dual on the operating constraint for segment s
λ_i^s	dual on the capacity constraitn for segment s
$y_i^s(x)$	short term equilibrium asa function of capacity
$\omega_i^s(\alpha^s)$	dual on the operating constraint asa function of the demand-curve intercept
$\lambda_i^s(\alpha^s)$	dual on the capacity constraint asa function of the demand- curve intercept
$S_i(x)$	maximum segment index for which capacity of type i is binding
$B_i y_j(x)$	rates of change of the y_j with respect to the x_j's
$y_i^s(y_{-i}^s, x)$	short-run reaction curve given the capacities
$x_i(x_{-i})$	long-run reaction curve in the open-loop game
$y_i(x_{-i})$	short-run solution given the other player's capacity in the open- loop game