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GENERATION OF A SMALL-SCALE QUASI-STATIC MAGNETIC FIELD AND FAST PARTICLES DURING THE COLLISION OF ELECTRON-POSITRON PLASMA CLOUDS

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ABSTRACT

We present the results of analytical studies and 2D3V PIC simulations of electron-positron plasma cloud collisions. We concentrate on the problem of quasi-static magnetic field generation. It is shown from linear theory, using relativistic two-fluid equations for electron-positron plasmas, that the generation of a quasi-static magnetic field can be associated with the counterstreaming instability. A two-dimensional relativistic particle simulation provides good agreement with the above linear theory and shows that, in the nonlinear stage of the instability, about 5.3% of the initial plasma flow energy can be converted into magnetic field energy. It is also shown from the simulation that the quasi-static magnetic field undergoes a collisionless change of structure, leading to large-scale, long-living structures and the production of high-energy particles. These processes may be important for understanding the production of high-energy particles in the region where two pulsar winds collide.

Subject headings: acceleration of particles — magnetic fields — plasmas

1. INTRODUCTION

The recent discovery of binary millisecond pulsars (Becker et al. 1996; Lorimer et al. 1996) has raised an interesting question as to what happens in the interaction region where two pulsar winds collide. Pulsar winds are believed to consist of electron-positron plasmas. It is known that counterstreaming plasmas are subject to a host of instabilities. In the case of relativistic plasmas, the excitation of electromagnetic modes is as important as that of electrostatic modes because the factor v/c is on the order of 1. Moreover, in the case of colliding relativistic electron-positron streams in which the net electric charge and current vanish, we expect to see electromagnetic mode excitation that is similar to the Weibel instability in a plasma with anisotropic temperature (Weibel 1959). Recently, the Weibel instability has been the subject of detailed investigations in the study of quasi-static magnetic field generation in laser plasmas (see Askar'yan et al. 1994; Bulanov et al. 1996; and Califano, Pegoraro, & Bulanov 1997). It has been shown that in electron-ion plasmas, the Weibel instability in the nonlinear stage leads to the formation of large-scale, long-living electron vortices associated with the magnetic fields.

In this Letter, we investigate the physical processes during the collision of counterstreaming electron-positron plasmas by using a relativistic two-fluid model and a relativistic particle simulation. In § 2, we present a linear theory derived from the relativistic two-fluid model and show the excitation of strong electromagnetic perturbations associated with quasi-static magnetic fields. In § 3, we show the results from a two-dimensional relativistic particle simulation. In the linear regime, there is good agreement between linear theory and simulations. In the nonlinear stage, the magnetic field can undergo a collisionless change of structure, and a large-scale structure of the magnetic field is produced. In the final section, we summarize our results.

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2. LINEAR THEORY OF ELECTROMAGNETIC INSTABILITY

The dynamics of counterstreaming cold electron-positron plasmas with velocities $\boldsymbol{v}_{ia}$, where $i = p$ (positron) and $i = e$ (electron) and $a = 1, 2$ are the two counterstreaming components of particles, is described by the following equations:

$$\frac{\partial \boldsymbol{P}_{ea}}{\partial t} = -\boldsymbol{v}_{ea} \cdot \nabla \boldsymbol{P}_{ea} - (\boldsymbol{E} + \boldsymbol{v}_{ea} \times \boldsymbol{B}), \quad (1)$$

$$\frac{\partial \boldsymbol{P}_{pa}}{\partial t} = -\boldsymbol{v}_{pa} \cdot \nabla \boldsymbol{P}_{pa} + (\boldsymbol{E} + \boldsymbol{v}_{pa} \times \boldsymbol{B}), \quad (2)$$

$$\frac{\partial (n_{pa} - n_{ea})}{\partial t} + \nabla \cdot (n_{pa}\boldsymbol{v}_{pa} - n_{ea}\boldsymbol{v}_{ea}) = 0, \quad (3)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}, \quad (4)$$

$$\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \sum_a (\boldsymbol{J}_{pa} + \boldsymbol{J}_{ea}), \quad (5)$$

$$\nabla \cdot \boldsymbol{E} = \sum_a (n_{pa} - n_{ea}), \quad (6)$$

where $\boldsymbol{v}_{ea} = \boldsymbol{P}_{ea}(1 + \boldsymbol{P}_{ea}^2)^{-1/2}$, $\boldsymbol{v}_{pa} = \boldsymbol{P}_{pa}(1 + \boldsymbol{P}_{pa}^2)^{-1/2}$, $\boldsymbol{J}_{ea} = -\boldsymbol{n}_{ea}\boldsymbol{v}_{ea}$, and $\boldsymbol{J}_{pa} = n_{pa}\boldsymbol{v}_{pa}$.

The densities and velocities are normalized by a characteristic density n and the speed of light c, respectively, and the time is normalized by the plasma frequency $\omega_{pe} = (4\pi ne^2/m)^{1/2}$.

We consider a case in which the counterstreaming plasmas have equal velocity in the y-direction and density such as $v_{0,e1} = v_{0,p1}$, $n_{0,e1} = n_{0,p1}$, $v_{0,e2} = v_{0,p2}$, and $n_{0,e2} = n_{0,p2}$. Linearizing the above set of equations and assuming a dependence of the form $F(x, y, t) = f \exp[i(k_x x + k_y y - \omega t)]$, we obtain the following dispersion equation:

$$(1 - \Omega_{2p}^{-2} - \Omega_{2e}^{-2})[(1 + \Omega_{4p}^{-2} + \Omega_{4e}^{-2})k_y^2 - \omega^2(1 - \Omega_{1p}^{-2} - \Omega_{1e}^{-2}) \\ -2\omega k_y(\Omega_{3p}^{-2} + \Omega_{3e}^{-2})] + k_x^2[(1 + \Omega_{4p}^{-2} + \Omega_{4e}^{-2})(1 - \Omega_{1p}^{-2} - \Omega_{1e}^{-2}) \\ + (\Omega_{3p}^{-2} + \Omega_{3e}^{-2})^2] = 0, \quad (7)$$

where

$$\Omega_{1e}^{-2} = \sum_a \frac{n_{0,ea}}{\Gamma_{ea}\Omega_{ea}^2}, \quad \Omega_{2e}^{-2} = \sum_a \frac{n_{0,ea}}{\Gamma_{ea}^3\Omega_{ea}^2},$$

$$\Omega_{3e}^{-2} = \sum_a \frac{n_{0,ea}v_{0,ea}}{\Gamma_{ea}\Omega_{ea}^2}, \quad \Omega_{4e}^{-2} = \sum_a \frac{n_{0,ea}v_{0,ea}^2}{\Gamma_{ea}\Omega_{ea}^2},$$

$$\Omega_{1p}^{-2} = \sum_a \frac{n_{0,pa}}{\Gamma_{pa}\Omega_{pa}^2}, \quad \Omega_{2p}^{-2} = \sum_a \frac{n_{0,pa}}{\Gamma_{pa}^3\Omega_{pa}^2},$$

$$\Omega_{3p}^{-2} = \sum_a \frac{n_{0,pa}v_{0,pa}}{\Gamma_{pa}\Omega_{pa}^2}, \quad \Omega_{4p}^{-2} = \sum_a \frac{n_{0,pa}v_{0,pa}^2}{\Gamma_{pa}\Omega_{pa}^2},$$

and where the following notations are used: $\Omega_{pa} = \omega - k_y v_{0,pa}$, $\Gamma_{pa} = (1 - v_{0,pa}^2)^{-1/2}$, $\Omega_{ea} = \omega - k_y v_{0,ea}$, and $\Gamma_{ea} = (1 - v_{0,ea}^2)^{-1/2}$.

We note here that the above dispersion relation is an extension of the case derived by Califano et al. (1997), who investigated the development of the Weibel instability in the study of the interaction between high-intensity laser pulses and plasmas.

We consider the collision of two symmetric plasma flows with velocities $v_0 = 0.56c$ and $-0.56c$ and with equal density. Figure 1 shows the growth rate normalized by ω_{pe} versus the wavenumber k_x normalized by c/ω_{pe} for the propagation angle of 27°.5 from the y-direction. As seen in this figure, three different branches appear: a, b, and c. Branch a can exist for one-component counterstreams (electrons or positrons), while the other branches, b and c, disappear for one-component streams. Branch a is similar to the Weibel instability in laser plasmas (Califano et al. 1997). On the other hand, branches b and c are caused by the instability driven by the coupling of two-component counterstreams, and these two branches merge into one mode when $k_x \gg k_y$.

3. SIMULATION MODEL

The code used here is a 2D3V, fully relativistic electromagnetic particle-in-cell code, modified from the 3D3V TRISTAN code (Buneman 1993). The system size is $L_x = 65\Delta$ and $L_y = 1024\Delta$, where $\Delta(=1)$ is the grid size. Periodic boundary conditions are imposed on particles and fields. There are 1,331,200 electron-positron pairs filling the entire domain uniformly and keeping the domain charge neutral. Hence, the average particle number density is about 20 per cell. The initial state is such that in the region $y = (1\text{–}512)\Delta$, the plasma drift velocity $v_d = 0.56c$ (c is the light speed, which is taken as 0.5), with a shifted Maxwellian distribution with the thermal velocity $v_{\rm th} = 0.09366c$, and in the region $y = (512\text{–}1024)\Delta$, $v_d = -0.56$c. This corresponds to a Lorentz factor $\gamma = [1 - (v_d/c)^2]^{-1/2} = 1.2$. Other parameters are as follows: the mass ratio $m_p/m_e = 1$, $\omega_{pe}\Delta t = 0.052$, and the electron collisionless skin depth $d_e = c/\omega_{pe} = 9.6\Delta$. Thus, the computation box has a size equal to $\approx 6.6d_e$ in the x-direction and $\approx 106.6d_e$ in the y-direction. The initial electric and magnetic fields, which are normalized by c, are zero.

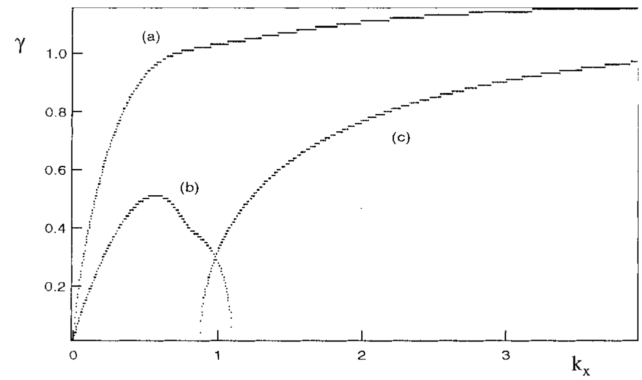

FIG. 1.—Linear growth rate normalized by ω_{pe} vs. the wavenumber k_x normalized by c/ω_{pe} for the propagation angle of 27°.5 with respect to the y-direction. The parameter used is $v_d = 0.56c$. Branch a is similar to the Weibel instability, while branches b and c are caused by the instability driven by the coupling of two-component counterstreams.

4. SIMULATION RESULTS

We investigated several cases of electron-positron plasma cloud collisions, and here we show only one case of them. The most important result is the generation of the magnetic field in the direction perpendicular to the counterstream direction from the initial state with no magnetic field.

Figure 2 shows the time development of the magnetic field (B_z) structures in the x-y plane. As seen in Figure 2a, initially (for $\omega_{pe}t = 10$), very coherent structures appear with positive and negative magnetic polarities. This corresponds to the filamentation of the plasma, with the creation of the current sheet system with the current sheets situated where the magnetic field changes sign. The characteristic scale of the current sheet is about $0.6d_e$ in the transverse direction along the x-axis and about $2\pi d_e$ in the longitudinal direction along the y-axis, which is also consistent with branch c of the linear theory shown in Figure 1.

As time goes on, the front of both streams propagates farther, and behind the front, the bending and coalescence of the filaments occur, as is seen in Figure 2b, where the magnetic field pattern is shown for $\omega_{pe}t = 31$. This process makes the characteristic scale length of the magnetic field larger. The coalescence process of filaments first becomes apparent after about $\omega_{pe}t = 31$. Asymptotically, at $\omega_{pe}t = 52.7$ in Figure 2c, we see the formation of large-scale, long-lived magnetic structures with size on the order of $2\pi c/\omega_{pe}$. The alternating polarity corresponds to electric current distributions in the x-y plane of the form of an antisymmetric vortex flow.

Figure 3 shows the time history of the electric (E_x^2, E_y^2) and magnetic (B_z^2) field energy normalized by $2VtL_x/L_xL_y$, where V is the plasma front velocity of the streams, which is assumed to be constant. Thus, they give the dependencies of the average values of the fields versus time. As seen in Figures 3a and 3b, almost the same amount of electrostatic wave energy is excited in the x- and y-directions. From a Fourier analysis of the wave patterns in E_x and E_y, we find that the electrostatic energy is associated with the Langmuir waves. The propagation direction in which the waves are excited is oblique and at an angle of about 45° from the x-direction.

As seen in Figure 3c, a strong magnetic B_z component can

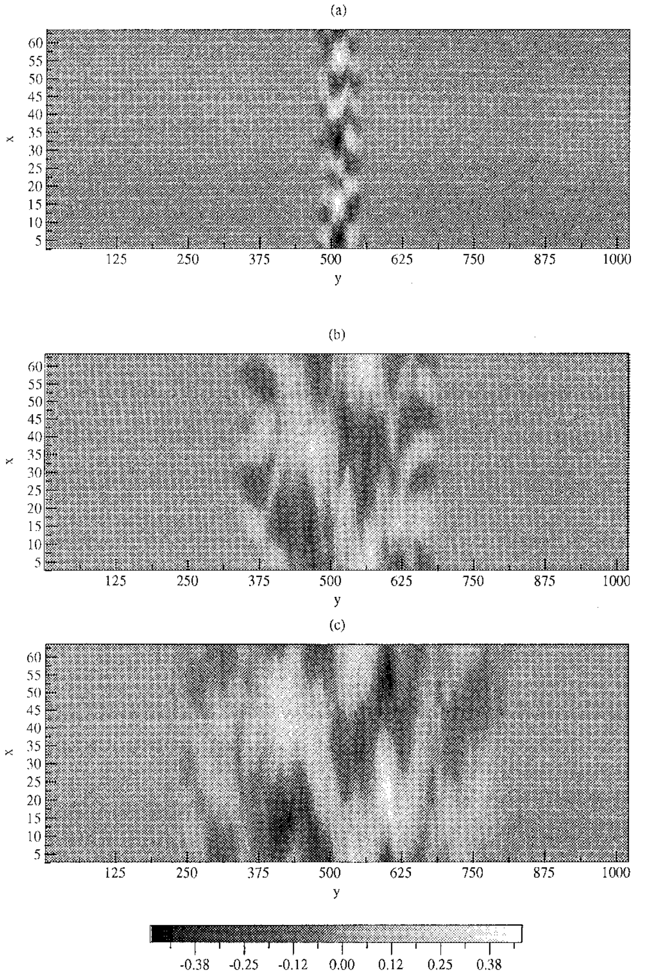

FIG. 2.—Time development of the magnetic field (B_z) structures in the *x*-*y* plane.

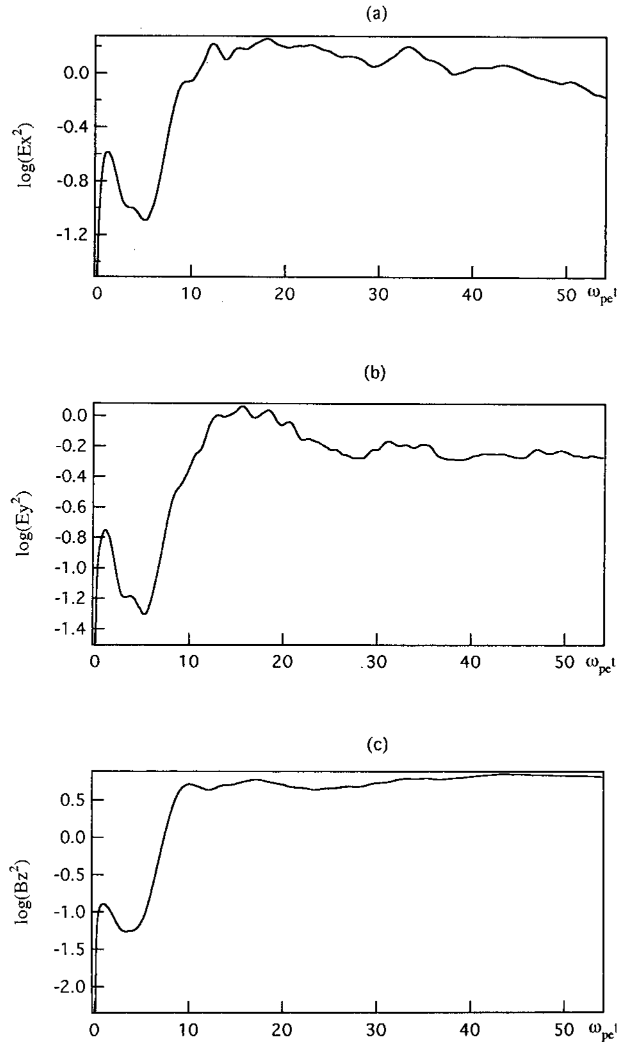

FIG. 3.—Time history of the electric field energy: (*a*) E_x^2, (*b*) E_y^2 and magnetic field energy, and (*c*) B_z^2, normalized by $2VtL_x/L_xL_y$, where V is the plasma front velocity of the streams, which is assumed to be constant.

be excited with almost the same growth rate as the Langmuir waves. The maximum magnetic energy is about twice the electric field energy. This is consistent with the Weibel mode development. The electric field amplitude can be estimated from equation (4) as $E \approx (\omega/kc)B$. If we take ω to be on the order of $\omega \approx \gamma \approx (v_0/c)\omega_{pe}$, we obtain for $v_0/c \approx 0.5$ a magnetic energy that is about twice the electric field energy. The growth rate observed from Figure 3*c* agrees with branch c, obtained from the linear theory given in the previous section, and is about $\gamma = (v_0/c)\omega_{pe}$. The magnitude of the growth rate calculated from the dispersion relation is about 0.69, with $k_x = 1.68$ and $k_y = 3.82$, while the value of growth rate calculated from the time history of magnetic field energy is about 0.67. By using the final value of the magnetic energy, we can estimate the energy conversion rate from the initial kinetic flow energy. We find that the energy conversion rate is about 5.3%.

Regarding the question of whether the magnetic field remains in a plasma when the clouds have passed or whether it propagates with the plasma waves, we refer to the paper by Bulanov et al. (1997), where a similar problem has been studied in the electron-ion plasma. Under similar conditions, it was demonstrated that the long-living magnetic structures, with the scale on the order of the collisionless skin depth, remain in a plasma with a typical velocity of propagation that is much less than the velocity of the regions where they have been generated.

If we study the time evolution of high-energy particle distribution, we find that particles appear with significant energy after $30\omega_{pe}t$. Therefore, we may conclude that the particles can be accelerated because of the collisionless change of the structure of the magnetic fields associated with the induction electric field. The collisionless change of the structure of the magnetic fields occurs via annihilation of magnetic fields with opposite polarity. This induces the electric field in the *x*-*y* plane, which in turn accelerates charged particles. We have seen no acceleration of the electrons and positrons along the *z*-direction, in accordance with this process. In Figure 4, we show the (*a*) electron and (*b*) positron energy spectrum in the final stage; (*c*) is the initial energy distribution of both particles.

5. SUMMARY

It is shown from linear theory, using relativistic two-fluid equations for electron-positron plasmas, that the generation of a quasi-static magnetic field is associated with the counter-streaming instability of electron-positron plasmas. A two-dimensional relativistic particle simulation is in good agreement with the above linear theory and, furthermore, in the nonlinear stage of the instability, shows that about 5.3% of the initial plasma flow energy can be converted to magnetic field energy.

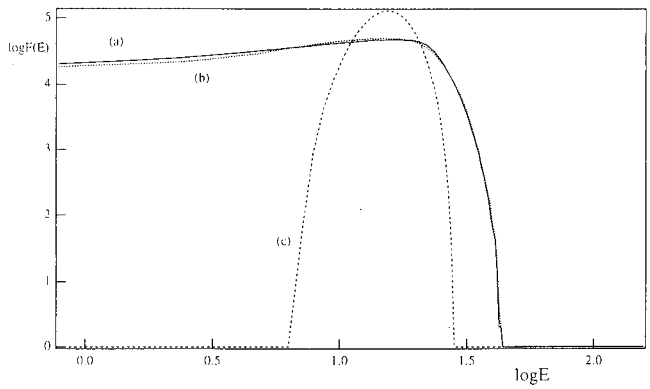

FIG. 4.—The (*a*) electron and (*b*) positron energy spectrum in the final state of the simulation. *E* normalized by the thermal energy is the total kinetic energy of particles. (*c*) The initial energy distribution of both particle species.

It is also shown from the simulation that the generated quasi-magnetic field undergoes a change in structure through coalescence of magnetic filaments, leading to large-scale structures and the production of high-energy particles. These processes may be important for understanding the production of high-energy particles in the region of two colliding pulsar winds.

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REFERENCES

Askar'yan, G. A., Bulanov, S. V., Pegoraro, F., & Pukhov, A. M. 1994, Soviet Phys.—JETP Lett., 60, 240
Becker, W., Trümper, J., Lundgren, S. C., Cordes, J. M., & Zepka, A. F. 1996, MNRAS, 282, L33
Bulanov, S. V., Esirkepov, T. Zh., Lontano, M., & Pegoraro, F. 1997, Plasma Phys. Rep., 23, 660
Bulanov, S. V., Lontano, M., Esirkepov, T. Zh., Pegoraro, F., & Pukhov, A. M. 1996, Phys. Rev. Lett., 76, 3562
Buneman, O. 1993, in Computer Space Plasma Physics, Simulation Techniques and Software, ed. H. Matsumoto & Y. Omura (Tokyo: Terra Scientific), 67
Califano, F., Pegoraro, F., & Bulanov, S. V. 1997, Phys. Rev. E, 56, 963
Lorimer, D. R., Lyne, A. G., Bailes, M., Manchester, R. N., D'Amico, N., Stappers, B. W., Johnston, S., & Camilo, F. 1996, MNRAS, 283, L1383
Weibel, E. W. 1959, Phys. Rev. Lett., 2, 83