Generation of a superposition of multiple mesoscopic states of radiation in a resonant cavity

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Using resonant interaction between atoms and the field in a high-quality cavity, we show how to generate a superposition of many mesoscopic states of the field. We study the quasiprobability distributions and demonstrate the nonclassicality of the superposition in terms of the zeros of the Q function as well as the negativity of the Wigner function. We discuss the decoherence of the generated superposition state. We propose homodyne techniques of the type developed by [Auffeves *et al.*, Phys. Rev. Lett. **91**, 230405 (2003)] to monitor the superposition of many mesoscopic states.

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I. INTRODUCTION

The interaction of a single atom with a high-quality cavity has yielded many important results which can be understood in terms of the Jaynes-Cummings model [1]. The advances in this field are extensively reviewed in the literature [2–5]. The generation of a superposition of mesoscopic coherent states has a fundamental place in quantum theory as such a state exhibits quantum interferences and the nonclassical character of the radiation field [6,7]. Eiselt and Risken [8] had discovered that if a cavity contains a coherent field with large photon numbers, say of the order of 10, then the state of the field for certain times splits into two parts. Each part can be characterized approximately by a coherent state. Several authors have studied many aspects of such splittings [9,10]. Auffeves *et al.* [10] made a successful observation of this splitting. They also devised a novel homodyne method to observe interferences. We note that previously such superpositions were produced using dispersive interactions in a high-quality cavity [11,12] or by using Raman transitions between the center-of-mass degrees of freedom of trapped ions [13].

Earlier studies of the superpositions of more than two coherent states have found many novel features of such states. For example, Zurek [14] noticed that such superpositions lead to structures in phase space which are smaller than Planck's constant. Clearly, we need efficient methods to produce such superpositions. One of the early suggestions [15] for the production of such states was through the passage of a field in a coherent state through a Kerr medium. However, Kerr nonlinearities are usually too small. Another possibility is via the dispersive interaction [12,16] in the cavity. In this paper, we present yet another possibility by using the resonant interaction between the atom and the cavity. We show how successive passage of atoms can be used to produce superpositions involving many coherent states. We specifically concentrate on a superposition of four coherent states.

The organization of the paper is as follows. In Sec. II, we present the details of our proposal to produce a superposition of four coherent states. We examine the Wigner function and the Q function for such states. We present a comparison of exact and approximate phase-space distribution functions. We further study zeros of the Q function which are a signature of the nonclassical properties of the field. In Sec. III, we show how the passage of the third atom can be used to monitor the superposition of four coherent states. In Sec. IV, we examine the scale over which such a superposition can decohere.

II. PREPARATION OF A SUPERPOSITION OF FOUR MESOSCOPIC STATES OF THE FIELD

In a recent experiment, Auffeves *et al.* [10] have observed a superposition of two distinguishable states of the field in a high-quality cavity using resonant interaction between an atom and the field inside the cavity. This observation is in agreement with the theoretical prediction of Eiselt and Risken [8]. When a two-level Rydberg atom interacts with a microwave field, it splits the field into two parts whose phases move in opposite directions. If the interaction time is chosen such that the phase difference between the split parts becomes π, then the cavity field can be projected into a superposition similar to a cat state, $|\alpha\rangle+|-\alpha\rangle$.

In this section, we show that the above method can be used for the preparation of a superposition of four mesoscopic states of the field. We consider a two-level Rydberg atom having its higher-energy state $|e\rangle$ and lower-energy state $|g\rangle$ and the cavity has a strong coherent field $|\alpha\rangle$. The atom passes through the cavity and interacts resonantly with the field. The Hamiltonian for the system in the interaction picture is written as

$$H=\hbar g(|e\rangle\langle g|a+a^{\dagger}|g\rangle\langle e|), \quad (1)$$

where g is the coupling constant for the atom with the cavity field, and a ($a^{\dagger}$) is the annihilation (creation) operator. The state of the atom-cavity system is written as

$$|\psi(t)\rangle=\sum_n [c_{en}(t)|e,n\rangle+c_{gn}(t)|g,n\rangle]. \quad (2)$$

Using Hamiltonian (1), the Schrödinger equation in terms of c_{en} and c_{gn} is

$$\dot{c}_{en-1}=-ig\sqrt{n}c_{gn}, \quad (3)$$

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