Generation of Equivalent Circuit from Finite Element Model Using Model Order Reduction

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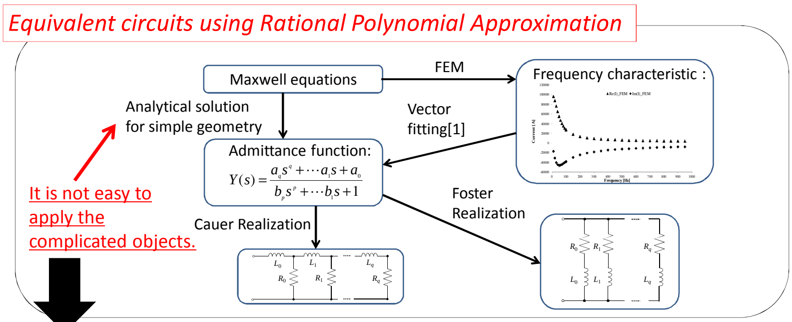
Introduction

Finite element method (FEM) has been widely used to develop and design electromagnetic devices.

We want to develop and design the electromagnetic devices **considering the control and driving circuits** connected to FE model.

The electromagnetic devices are often modeled as an **equivalent circuit** for design of the control and driving circuit.

Equivalent circuits using Rational Polynomial Approximation



Purpose

We propose a novel method to generate the equivalent circuit of the electromagnetic devices using model order reduction.

Padé approximation via the Lanczos process[2]

Laplace-transformed discrete Maxwell equations

$$s\mathrm{N}\boldsymbol{x}+\mathrm{K}\boldsymbol{x}=\boldsymbol{b}v$$
$$i=\boldsymbol{l}^t\boldsymbol{x}$$

N, K $\in R^{n\times n}$, $\boldsymbol{b}, \boldsymbol{l}, \boldsymbol{x} \in R^n$
v : voltage, i : current

Admittance function

$$\mathrm{Y}(s)=\boldsymbol{l}^t(\mathrm{K}+s\mathrm{N})^{-1}\boldsymbol{b}$$

$s \rightarrow s_0+\sigma$

$$\mathrm{Y}(s_0+\sigma)=\boldsymbol{l}^t(\mathrm{I}-\sigma\mathrm{A})^{-1}\boldsymbol{r}$$

$\mathrm{A}=-(\mathrm{K}+s_0\mathrm{N})^{-1}\mathrm{N}$
$\boldsymbol{r}=(\mathrm{K}+s_0\mathrm{N})^{-1}\boldsymbol{b}$

Spectral decomposition of A

$$\mathrm{Y}(s_0+\sigma)=\boldsymbol{l}^t(\mathrm{I}-\sigma\mathrm{S}\Lambda\mathrm{S}^{-1})^{-1}\boldsymbol{r}=\sum_{i=1}^{n}\frac{f_i g_i}{1-\sigma\lambda_i}$$

This formulation would be **unsuitable** for real uses because of heavy computational burden in solution of the eigenvalue problem.

Admittance function

$$\mathrm{Y}(s_0+\sigma)=\boldsymbol{l}^t(\mathrm{I}-\sigma\mathrm{A})^{-1}\boldsymbol{r}$$

Neumann series expansion

$$\mathrm{Y}(s_0+\sigma)=\boldsymbol{l}^t\left(\mathrm{I}+\sigma\mathrm{A}+\sigma^2\mathrm{A}^2+\cdots\right)\boldsymbol{r}=\sum_{i=0}^{\infty}m_i\sigma^i$$

Lanczos method

Reduced admittance function

$$\mathrm{Y}(s_0+\sigma)=\sum_{i=0}^{\infty}\boldsymbol{l}^t\boldsymbol{r}(\boldsymbol{e}_1^t\mathrm{T}_q^i\boldsymbol{e}_1)\sigma^i=\boldsymbol{l}^t\boldsymbol{r}\boldsymbol{e}_1^t(\mathrm{I}-\sigma\mathrm{T}_q)^{-1}\boldsymbol{e}_1$$

Spectral decomposition of T_q

$$\mathrm{Y}_q(s_0+\sigma)=\boldsymbol{l}^t\boldsymbol{r}\boldsymbol{e}_1^t(\mathrm{I}-\sigma\mathrm{S}_q\Lambda_q\mathrm{S}_q^{-1})^{-1}\boldsymbol{e}_1=\sum_{j=1}^{q}\frac{\boldsymbol{l}^t\boldsymbol{r}\mu_j\nu_j}{1-\sigma\lambda_j}$$

$\mathrm{T}_q \in R^{q\times q}$: Tridiagonal matrix (n>>q)

Lanczos method

We can obtain tridiagonal matrix T_q whose eigenvalues correspond to the significant eigenvalues of A.

---------*algorithm*---------
0) Set $\rho_1=\|\boldsymbol{r}\|_2$, $\eta_1=\|\boldsymbol{l}\|_2$, $\boldsymbol{v}_1=\boldsymbol{r}/\rho_1$, $\boldsymbol{w}_1=\boldsymbol{l}/\eta_1$, $\boldsymbol{v}_0=\boldsymbol{w}_0=0$ and $\delta_0=0$
For n=1,2…,q do
1) Compute $\delta_n=\boldsymbol{w}_n{}^t\boldsymbol{v}_n$
2) Set $\alpha_n=\boldsymbol{w}_n{}^t\mathrm{A}\boldsymbol{v}_n/\delta_n$, $\beta_n=\eta_n\delta_n/\delta_{n-1}$, $\gamma_n=\rho_n\delta_n/\delta_{n-1}$
3) Set $\boldsymbol{v}=\mathrm{A}\boldsymbol{v}_n-\alpha_n\boldsymbol{v}_n-\beta_n\boldsymbol{v}_{n-1}$, $\boldsymbol{w}=\mathrm{A}^t\boldsymbol{w}_n-\alpha_n\boldsymbol{w}_n-\gamma_n\boldsymbol{w}_{n-1}$
4) Set $\rho_{n+1}=\|\boldsymbol{v}\|_2$, $\eta_{n+1}=\|\boldsymbol{w}\|_2$, $\boldsymbol{v}_{n+1}=\boldsymbol{v}/\rho_{n+1}$, $\boldsymbol{w}_{n+1}=\boldsymbol{w}/\eta_{n+1}$

We need to solve the following equations to obtain $\mathrm{A}\boldsymbol{v}_n$ and $\mathrm{A}^t\boldsymbol{w}_n$.

$$\begin{cases}\mathrm{A}\boldsymbol{v}_n=-(\mathrm{K}+s_0\mathrm{N})^{-1}\mathrm{N}\boldsymbol{v}_n\\ \mathrm{A}^t\boldsymbol{w}_n=-\{(\mathrm{K}+s_0\mathrm{N})^{-1}\mathrm{N}\}^t\boldsymbol{w}_n\end{cases}$$

Tridiagonal matrix

$$\mathrm{T}_q=\begin{bmatrix}\alpha_1&\beta_2&0&\cdots&0\\ \rho_2&\alpha_2&\beta_3&\ddots&\vdots\\ 0&\rho_3&\ddots&\ddots&0\\ \vdots&\ddots&\ddots&\ddots&\beta_q\\ 0&\cdots&0&\rho_q&\alpha_q\end{bmatrix}$$

Generation of Equivalent Circuit

Reduced admittance function

$$Y(s_0+\sigma)=k_\infty+\sum_{\substack{j=1\\ \lambda_j\neq 0}}^{q}\frac{k_j}{\sigma-p_j}=\frac{1}{Z_\infty}+\frac{1}{Z_1}+\frac{1}{Z_2}+\cdots+\frac{1}{Z_q}$$

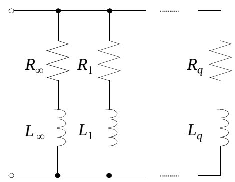
$$k_j=\frac{-\boldsymbol{l}^t\boldsymbol{r}\mu_j\nu_j}{\lambda_j},\ p_j=\frac{1}{\lambda_j}$$
$$k_\infty=\sum_{\substack{j=1\\ \lambda_j=0}}^{q}\boldsymbol{l}^t\boldsymbol{r}\mu_j\nu_j$$

when $\sigma=2\pi f_{\max}+\mathrm{j}\omega$

$$Z_j=\frac{-2\pi f_{\max}+\mathrm{j}\omega-p_j}{k_j}=R_j+\mathrm{j}\omega L_j$$

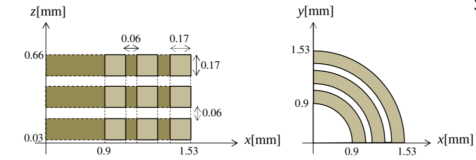
under the condition of $|p_j|>2\pi f_{\max}$

Foster Circuit

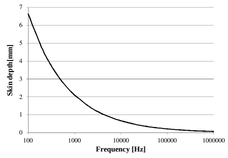


Numerical Results

Coil windings model



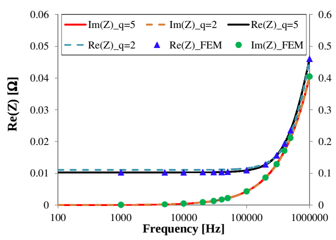
Skin depth



Analysis condition

Conductivity κ[S/m]	Relatively Permeability μ_r	Maximum frequency $f_{\max}$[MHz]	Number of elements (tetrahedral elements)	Number of nodes
5. 76×10^7	1	1	298201	52077

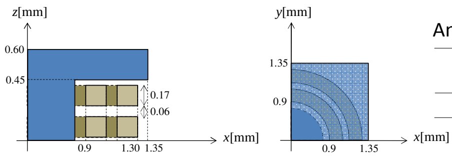
Impedance with respect to frequency

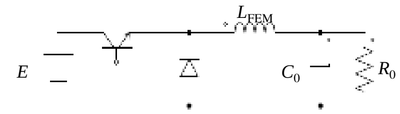


Circuit parameters

q	R_1[Ω]	R_2[Ω]	R_3[Ω]	R_4[Ω]	R_5[Ω]
5	1.03e-2	5.69	6.82	141	58.3
2	1.11e-2	3.51	----	----	----
q	L_1[H]	L_2[H]	L_3[H]	L_4[H]	L_5[H]
5	6.97e-8	8.02e-7	7.25e-7	1.31e-6	1.79e-6
2	6.89e-8	3.18e-7	----	----	----

Inductor model coupled with DC-DC converter





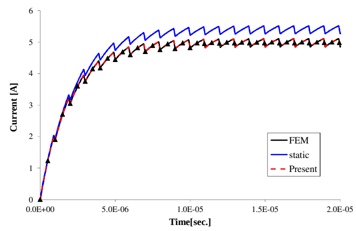
Analysis condition

Number of elements	Number of nodes
117548	20985

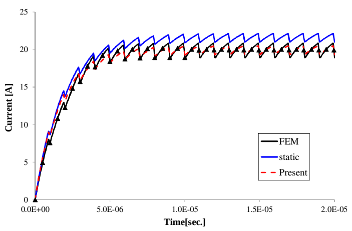
Driving condition

DC voltage E [V]	Driving frequency	Duty factor
0.3V, 1.2 V	1MHz	0.9

Currents in the case of E=0.3 V



Currents in the case of E=1.2 V



Computational time for generation of equivalent circuit

Coil Windings model	Inductor for DC-DC converter model
40 min. (q=5)	11 min. (q=5)

We use Xeon W5590/3.2GHz(12GB RAM)

Circuit analysis VS FE analysis

Coil Windings model		DC-DC converter model		
FEM*	Present	FEM(E=0.3V)	FEM(E=1.2V)	Present
230 min.	less than 1 sec.	240 min.	360 min.	less than 1 sec.

*the elapsed time of field computations by FEM at 13 sampling frequencies

References

[1] B. Gustavsen, A. Semlyen, "Rational Approximation of Frequency Domain Responses by Vector Fitting," *IEEE Trans. Power Delivery*, vol. 14, no. 3, pp. 1052-1061, 1999.

[2] P. Feldmann and R. A. Freund, "Efficient Linear Circuit Analysis by Padé Approximation via the Lanczos Process," *IEEE Trans. Computer-Aided Design*, vol. 14, no. 5, pp. 639–649, May 1995.