# Generation of Highly Nonlinear and Dynamic AES Substitution-Boxes (S-Boxes) Using Chaos-Based Rotational Matrices 

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#### Abstract

This work reports a novel chaos-based affine transformation generation method, which is based on rotational matrices to design strong key-based S-boxes. Chaotic logistic map's nonlinear trajectories are used to generate rotational matrices under given design conditions. Thus, the inherent logic is to generate key-based S-boxes, as strong as AES S-box, in terms of cryptographic properties using chaos in affine transformation. The randomness of chaotic sequences is tested using the National Institute of Standard and Technology (NIST) Statistical Test Suit (STS) 800-22 that validates the generated sequences for S-box design. The results show that methodology adapted to design proposed key-based dynamic S-boxes entails near-optimal cryptographic properties so that proposed S-boxes are as stronger as AES S-box.


INDEX TERMS Affine transformation, chaotic logistic map, S-box, NIST test.

## I. INTRODUCTION

Cryptography plays a critical role in secure transmission of information. With the increasing demand for communication systems, the role of encryption becoming more critical and cryptographers continue to work on new algorithms to ensure secure transmission of confidential information. Likewise, cryptanalysts are hard at work to find new ways of breaking those algorithms. Advanced Encryption Standard (AES) is the standard algorithm approved by the National Institute of Standards and Technology (NIST). Although no attack still exist that can break AES, there are some attacks like side-channel attack [1], which exploits the incomplete diffusion feature in AES [2] and SQUARE [3]. The meet-in-the-middle attack exploits the weakness in the key scheduling [4] in AES. The advanced capabilities of cryptanalysis demand cryptographers to modify cryptographic algorithms. According to Shannon, modern block ciphers

[^0]are based on confusion and diffusion components. Confusion is achieved in encryption algorithms by substitution-box (S-box). The primary function of $S$-box is to change the position of the input block of data $x$ with a new position of data block named as $y$, in a nonlinear fashion. There are many applications of S-box in image encryption [5], [6], low profile mobile applications [7], multimedia encryption [8], watermarking [9], and steganography [10]. In encryption, S-box is commonly used in modern block ciphers like Data Encryption Standard (DES) and AES. It is responsible for providing confusion property in algorithms. The strength of the algorithm depends on the strength of the S-box [11], [12]. NIST defined the criteria for finding the strength of an S-box. These include bit independence criterion (BIC), nonlinearity (NL), strict avalanche criterion (SAC), linear probability (LP), and differential probability (DP). Generally, S-box with low DP value and high non-linearity is desirable.

The non-linearity of an S-box causes uncertainty in the output, which offers resistance against linear and differential cryptanalysis attacks [13]. AES S-box is considered highly
strong as it is based on finite field and $G F\left(2^{8}\right)$ inverse operations that play critical role to made S-box highly non-linear, thus resulting in higher security.

Researchers have proposed different methods to design S-box to achieve high nonlinearity and low DP value. Chaos, due to strong similarities with cryptography, is considered as a good candidate in designing S-box. Chaos, with inherent properties like sensitive dependence on initial condition, mixing, and ergodicity, has attracted different researchers [14]. The authors proposed S-boxes using one dimensional [15], [16], two dimensional [17], and three dimensional [18]-[20] chaotic maps. Tian and Lu [21] proposed an S-box design based on a six-dimensional hyperchaotic map and an artificial bee colony algorithm. Tanyildizi and Özkaynak [22] utilized seven (07) different optimization algorithms to determine the most suitable initial condition and control parameter values corresponding to four chaotic algorithms. In addition, they proposed a new method for S-box generation design based on the optimized chaotic maps. However, their proposed S-box is not as strong as AES S-box. The authors in [23] examined the properties of a three-dimensional plasma system from a cryptographic perspective and proposed a new S-box generation algorithm based on a 3D plasma chaotic system. In [24], the authors designed a new S-box for wireless sensor networks using Linear Congruence Generator (LCG) in combination with compound chaotic map, Baker's map, and sinusoidal chaotic map. Khan et al. [25] proposed a new cryptographic method for substitution-permutation network design, where the results of gold sequences and chaotic logistic tent maps are used in linear fraction transformation to generate a new S-box. Lu et al. [26] proposed a new compound technique for S-box design using tent-logistic chaotic map, which involves a novel linear mapping scheme for the generation of the initial S-box. In [6], authors proposed a chaotic Jaya optimization algorithm for image encryption, the S-box achieved good cryptographic properties. A few researchers also incorporate other methods [5], [7] with chaos to make S-box more robust. For instance, combining chaos with algebra makes S-box more suitable for cryptography.

Elliptic curve [27], cubic fractional transformation [28], and algebraic properties like group and rings [29]-[31] are supposed to improve the DP value of S-box, but those do not give significant results. Most of these S-boxes give DP value of 10/256. In [32], an S-box is designed using Gaussian distribution and linear fractional transformation by applying the Box-Muller transform, polarization decision, and central limit algorithm. The research work in [33] proposed a way of generating highly non-linear $n \times n$ S-boxes for $3 \leq \mathrm{n} \leq 7$. For each iteration, the best S-box candidate is chosen based on heuristic optimization. This approach is good for non-linear key-dependent S-boxes. In [34], the authors proposed a technique for designing S-box via single expression algebra instead of using matrix algebra that reduced the computational complexity for S-Box construction.

Researchers have also worked on AES in terms of performance and security, both in hardware and software. Tillich and Herbst [35], Rupanagudi et al. [36], Moh'd [37], Talha [38], and Shreedhar et al. [39] proposed a method for improving the performance of AES in hardware in terms of area and clock speed. Intel's corporation proposed new AES instruction set for improving its performance and security for their processors [40]. Sahoo et al. [41] proposed new affine transformation for improving performance of AES in software. Pachori et al. [42] proposed java-based AES parallel processing for improving processing speed. In [41], the author modified the S-box of AES by modifying the affine transformation to minimize the time complexity of AES.

Considerable research has been done to improve AES security in software. De Los Reyes et al. [43], [44] worked on AES key scheduling to increase diffusion and confusion rate and enhance the security of AES. They also proposed different versions of AES to increase its key length and security [45]. The authors in [46] proposed two different S-boxes instead of one AES S-box by modifying the affine transformation matrices that are used alternatively in the algorithm. In [47], [48] the authors proposed a variable mapping S-boxes for AES. Using secret key as well as using different irreducible polynomials produce dynamic S-boxes. A different S-box is designed in each round in byte substitution layer only, while all other layers of AES (shift row, mix column and key addition layer) remain the same. This enhances the security of AES but generates only limited number of S-boxes. The research works in [49]-[56] proposed dynamic S-boxes based on secret keys for enhancing the security of AES.

S-boxes are designed to resist differential attacks. The attacker tries to investigate the differential properties of S-box to analyze the ciphertext. Stronger the S-box cryptographically, higher will be the possibility to resist attacks. If there are different S-boxes in every round of algorithm, it will be more difficult for an attacker to investigate each S-box. This will provide an extra security layer. In this paper, authors proposed a novel key-based dynamic S-box design technique that is as strong as static AES S-box, with DP value of $4 / 256$. S-boxes are designed using $\operatorname{GF}\left(2^{8}\right)$ inverse and modified affine transformation. In affine transformation, using chaos modifies rotational matrix. Rotational matrix selection is based on specific parameters. A change in $8 \times 8$ rotation matrix can give rise to a different S -box that is as strong as AES S-box.

The rest of the paper is organized as follows. In Section II, an overview of the AES S-box generation method is described. In Section III, NIST tests are applied to the chosen chaotic map as a random number generator, and the obtained results are discussed. Section IV provides the proposed S-box design technique. Section V presents the detailed results regarding the proposed S-boxes. In Section VI, the proposed S-boxes are compared with state of the art S-boxes, and it is verified that proposed dynamic S-boxes are as strong as AES S-box. Finally, the conclusions are provided in Section VII.

## II. PRELIMINARIES

This section briefly describes the standard AES S-box generation method, chaotic logistic map (CLM), and its NIST test that shows the effectiveness of the CLM in proposed S-box generation.

## A. THE AES S-BOX

The AES S-Box is the only non-linear element in AES encryption, which bijectively maps every input element to exactly one output element so that the reverse process stays possible during decryption. The S-box is generated by calculating the multiplicative inverse of a given number in Rijndael's finite field, i.e. $G F\left(2^{8}\right)$, using a fixed generator polynomial $G(x)=x^{8}+x^{4}+x^{3}+x+1$. No inverse exists for the input element zero in Rijndael's finite field; hence, zero is mapped to itself. In all other cases, the multiplicative inverse $A_{i}^{-1}$ of an input element $A_{i}$ belongs to the elements of $G F\left(2^{8}\right)$ with a fixed irreducible polynomial $G(x)=x^{8}+$ $x^{4}+x^{3}+x+1$. Table 1 provides the multiplicative inverse for all 8 -bit numbers in $G F\left(2^{8}\right)$. Once the multiplicative inverse is computed, it is transformed to $S_{i}$ using the affine transformation given in (1) as follows:

$$
\begin{equation*}
S_{i} \equiv R \cdot A_{i}^{-1}+C \bmod 2 \tag{1}
\end{equation*}
$$

where, $R$ is a rotational matrix, $C$ is an additive constant, and $S_{i}$ is the output of the AES S-box that corresponds to input $A_{i}$. Equation (1) can be expanded by placing the values of $R$ and $C$ as in (2), where, $S_{i}=\left[s_{7}, \ldots, s_{0}\right]$ represents the 8 -bit output of the AES S-box. Table 2 presents the final AES $S$-box generated by (2).

$$
\left.\begin{array}{r}
\left(\begin{array}{l}
s_{0} \\
s_{1} \\
s_{2} \\
s_{3} \\
s_{4} \\
s_{5} \\
s_{6} \\
s_{7}
\end{array}\right)=\left(\begin{array}{llllllll}
1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1
\end{array}\right)\left(\begin{array}{c}
a_{0}^{\prime} \\
a_{1}^{\prime} \\
a_{2}^{\prime} \\
a_{3}^{\prime} \\
a_{4}^{\prime} \\
a_{5}^{\prime} \\
a_{6}^{\prime} \\
a_{7}^{\prime}
\end{array}\right) \\
 \tag{2}\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
0 \\
0 \\
0 \\
1 \\
1 \\
0
\end{array}\right) \bmod 2
$$

## B. CHAOTIC LOGISTIC MAP (CLM)

The logistic equation is proposed by Pierre Francois Verhulst [57] in 1838 and introduced as a chaotic map first time by May [58]. The mathematical equation can be written as:

$$
\begin{equation*}
q_{i+1}=u q_{i}\left(1-q_{i}\right), \quad \text { where } 0<u<4, q_{i} \in[0,1] \tag{3}
\end{equation*}
$$

The behavior of (3) is chaotic for the interval $3.57 \leq u \leq 4$.

Before ( $0,3.57$ ), its behavior is deterministic and nonchaotic. Figure 1(a) shows the bifurcation diagram that elaborates its behavior, whereas Figure 1(b) shows the Lyapunov exponent of a chaotic logistic map.

In this research work, we use chaotic logistic map as random number generator (RNG) to obtain a random key value in the range $(0,255]$. The logistic map is easy to implement and computationally feasible. The random key value obtained from the logistic map is used for producing certain affine transformations based on rotational matrices. Any rotational matrix can be generated with this behavior, and there is an equal proportion in the selection of any rotational matrix. Hence, any random S-box can be generated under the defined settings, based on the affine transformation matrix with strong cryptographic properties.

## III. NIST STS 800-22 TESTS ON LOGISTIC MAP

To validate the randomness of a bitstream generated by any RNG, NIST statistical test suite (STS) 800-22 [59] is used. It entails fifteen (15) different tests that investigate the security of an RNG design. All tests must be passed by any RNG to ensure the security of the random number sequence produced. As discussed before, chaotic logistic map was used in the proposed scheme to generate a random key value for designing an S-box. Hence, chaotic logistic map (given in (3)) was initially tested as RNG. In this aspect, the chaotic map was iterated for $10^{6}$ times (with initial seed value of $q_{0}=0.33$ and control parameter $u=4$ ), which produced 1 M bits using a threshold value of 0.5 . This bitstream was then passed to NIST STS 800-22 tool for validating the randomness of the binary sequence. The results (Table 3) showed that all the tests were passed. Hence, the use of chaotic logistic map as RNG for the proposed scheme is validated as fulfills the NIST criteria. It is essential to understand here that NIST STS for RNG validation does not have any direct relation with the S-box generation and its cryptographic properties. However, it provides help in ensuring the randomness of the key value obtained using the chaotic logistic map. The key is used to generate a random rotational matrix for affine transformation, which in turn produces an S-box. Hence, in the case of a random key value, it becomes unpredictable for the attackers to find out which $S$-box is going to be used in the next round of AES. In this way, NIST STS for RNG helps in selecting a random key for generating a random S-box, thus increasing the strength of AES with undetectable effect in the performance of algorithm.

## IV. PROPOSED METHOD FOR S-BOX GENERATION

The proposed methodology for S-box generation is comprised of four stages, as shown in Figure 2. These stages include computation of the $G F\left(2^{8}\right)$ inverse, key generation using chaotic logistic map, computation of affine transformation rotational matrix $R_{k}$, and finally generation of a new $S$-box based on affine transformation operation. In the first stage, the same methodology is used as in AES S-box to generate the multiplicative inverse of an input byte $A_{i}$.

TABLE 1. Multiplicative inverse of input bytes $X Y$ in $G F\left(\mathbf{2}^{\mathbf{8}}\right)$.

| Y |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
|  | 0 | 0 | 1 | 141 | 246 | 203 | 82 | 123 | 209 | 232 | 79 | 41 | 192 | 176 | 225 | 229 | 199 |
|  | 1 | 116 | 180 | 170 | 75 | 153 | 43 | 96 | 95 | 88 | 63 | 253 | 204 | 255 | 64 | 238 | 178 |
|  | 2 | 58 | 110 | 90 | 241 | 85 | 77 | 168 | 201 | 193 | 10 | 152 | 21 | 48 | 68 | 162 | 194 |
|  | 3 | 44 | 69 | 146 | 108 | 243 | 57 | 102 | 66 | 242 | 53 | 32 | 111 | 119 | 187 | 89 | 25 |
|  | 4 | 29 | 254 | 55 | 103 | 45 | 49 | 245 | 105 | 167 | 100 | 171 | 19 | 84 | 37 | 233 | 9 |
|  | 5 | 237 | 92 | 5 | 202 | 76 | 36 | 135 | 191 | 24 | 62 | 34 | 240 | 81 | 236 | 97 | 23 |
|  | 6 | 22 | 94 | 175 | 211 | 73 | 166 | 54 | 67 | 244 | 71 | 145 | 223 | 51 | 147 | 33 | 59 |
| X | 7 | 121 | 183 | 151 | 133 | 16 | 181 | 186 | 60 | 182 | 112 | 208 | 6 | 161 | 250 | 129 | 130 |
|  | 8 | 131 | 126 | 127 | 128 | 150 | 115 | 190 | 86 | 155 | 158 | 149 | 217 | 247 | 2 | 185 | 164 |
|  | 9 | 222 | 106 | 50 | 109 | 216 | 138 | 132 | 114 | 42 | 20 | 159 | 136 | 249 | 220 | 137 | 154 |
|  | A | 251 | 124 | 46 | 195 | 143 | 184 | 101 | 72 | 38 | 200 | 18 | 74 | 206 | 231 | 210 | 98 |
|  | B | 12 | 224 | 31 | 239 | 17 | 117 | 120 | 113 | 165 | 142 | 118 | 61 | 189 | 188 | 134 | 87 |
|  | C | 11 | 40 | 47 | 163 | 218 | 212 | 228 | 15 | 169 | 39 | 83 | 4 | 27 | 252 | 172 | 230 |
|  | D | 122 | 7 | 174 | 99 | 197 | 219 | 226 | 234 | 148 | 139 | 196 | 213 | 157 | 248 | 144 | 107 |
|  | E | 177 | 13 | 214 | 235 | 198 | 14 | 207 | 173 | 8 | 78 | 215 | 227 | 93 | 80 | 30 | 179 |
|  | F | 91 | 35 | 56 | 52 | 104 | 70 | 3 | 140 | 221 | 156 | 125 | 160 | 205 | 26 | 65 | 28 |

## TABLE 2. The AES S-box.

| Y |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
|  | 0 | 99 | 124 | 119 | 123 | 242 | 107 | 111 | 197 | 48 | 1 | 103 | 43 | 254 | 215 | 171 | 118 |
|  | 1 | 202 | 130 | 201 | 125 | 250 | 89 | 71 | 240 | 173 | 212 | 162 | 175 | 156 | 164 | 114 | 192 |
|  | 2 | 183 | 253 | 147 | 38 | 54 | 63 | 247 | 204 | 52 | 165 | 229 | 241 | 113 | 216 | 49 | 21 |
|  | 3 | 4 | 199 | 35 | 195 | 24 | 150 | 5 | 154 | 7 | 18 | 128 | 226 | 235 | 39 | 178 | 117 |
|  | 4 | 9 | 131 | 44 | 26 | 27 | 110 | 90 | 160 | 82 | 59 | 214 | 179 | 41 | 227 | 47 | 132 |
|  | 5 | 83 | 209 | 0 | 237 | 32 | 252 | 177 | 91 | 106 | 203 | 190 | 57 | 74 | 76 | 88 | 207 |
|  | 6 | 208 | 239 | 170 | 251 | 67 | 77 | 51 | 133 | 69 | 249 | 2 | 127 | 80 | 60 | 159 | 168 |
| X | 7 | 81 | 163 | 64 | 143 | 146 | 157 | 56 | 245 | 188 | 182 | 218 | 33 | 16 | 255 | 243 | 210 |
|  | 8 | 205 | 12 | 19 | 236 | 95 | 151 | 68 | 23 | 196 | 167 | 126 | 61 | 100 | 93 | 25 | 115 |
|  | 9 | 96 | 129 | 79 | 220 | 34 | 42 | 144 | 136 | 70 | 238 | 184 | 20 | 222 | 94 | 11 | 219 |
|  | A | 224 | 50 | 58 | 10 | 73 | 6 | 36 | 92 | 194 | 211 | 172 | 98 | 145 | 149 | 228 | 121 |
|  | B | 231 | 200 | 55 | 109 | 141 | 213 | 78 | 169 | 108 | 86 | 244 | 234 | 101 | 122 | 174 | 8 |
|  | C | 186 | 120 | 37 | 46 | 28 | 166 | 180 | 198 | 232 | 221 | 116 | 31 | 75 | 189 | 139 | 138 |
|  | D | 112 | 62 | 181 | 102 | 72 | 3 | 246 | 14 | 97 | 53 | 87 | 185 | 134 | 193 | 29 | 158 |
|  | E | 225 | 248 | 152 | 17 | 105 | 217 | 142 | 148 | 155 | 30 | 135 | 233 | 206 | 85 | 40 | 223 |
|  | F | 140 | 161 | 137 | 13 | 191 | 230 | 66 | 104 | 65 | 153 | 45 | 15 | 176 | 84 | 187 | 22 |



FIGURE 1. a) Bifurcation diagram and b) Lyapunov exponent for chaotic logistic map.


Affine Transformation
FIGURE 2. Block-level view of the proposed S-box generation method.

TABLE 3. NIST STS 800-22 tests results for logistic map.

| NIST Statistical Test | P-value | Status |
| :--- | :--- | :--- |
| Approximate Entropy | 0.935081 | SUCCESS |
| Block Frequency | 0.881134 | SUCCESS |
|  | 0.577018 (FORWARD) |  |
| Cumulative Sums | 0.622087 (REVERSE) |  |
| Fast Fourier Transform (FFT) | 0.365257 | SUCCESS |
| Frequency Test | 0.508261 | SUCCESS |
| Linear Complexity | 0.861830 | SUCCESS |
| Longest Run | 0.014259 | SUCCESS |
| Non-Overlapping Template | 0.56962 | SUCCESS |
| Overlapping Template | 0.245325 | SUCCESS |
| Random Excursions | 0.579367 | SUCCESS |
| Random Excursions Variant | 0.44762 | SUCCESS |
| Rank test | 0.673806 | SUCCESS |
| Runs test | 0.644401 | SUCCESS |
| Serial test | 0.762361 | SUCCESS |
| Universal test | 0.214860 | SUCCESS |

In the second stage, four chaotic logistic maps are combined to generate a random key based on the proposed key scheme. This key is considered as the first row of the affine transformation matrix, which is used to generate the entire rotational matrix. Finally, the output of the first stage,
i.e. $A_{i}^{-1}$, is multiplied by a variable rotational matrix $R_{k}$ generated under specific settings, followed by the addition of an 8 -bit constant $C$ to obtain the final output.

Equation (4) shows the entire process of generating the S-box output using the proposed S-box.

$$
\begin{equation*}
S_{i} \equiv R_{k} \cdot A_{i}^{-1}+C \bmod 2 \tag{4}
\end{equation*}
$$

where, $R_{k}$ represents a $8 \times 8$ variable rotational matrix used for affine mapping and $k$ is the matric key that actually represents the dynamicity of the rotational matrix as compared the AES S-box that utilizes a static matrix $R$. The rotational matrix $R_{k}$ is entirely generated based on key value $k$. Hence, a large number of dynamic $S$-boxes can be produced by generating random keys. Figure 3 shows the detailed view of proposed S-box generation, and the following sections explain in detail the proposed algorithms for generating affine transformation matrix and the S-box.

## A. KEY GENERATION

The proposed algorithm for generating the key for the affine transformation matrix consists of the following steps:

Step-1: Choose initial seed values for four different chaotic maps i.e., $w_{0}, x_{0}, y_{0}, z_{0}$, with a range between [01].


FIGURE 3. Detailed methodology for proposed S-box generation design.

Step-2: As the logistic map is chaotic for the region [3.57 4], hence, take four different parameter values (i.e., $u_{w}, u_{x}, u_{y}, u_{z}$ ) corresponding to four chaotic maps with range [3.57 4]. Herein, we choose the control parameter $u=4$ to iterate the chaotic logistic map.

Step-3: Apply four chaotic maps (following (3)) shown to these four initial parameter pairs, i.e., $\left(w_{0}, u_{w}\right),\left(x_{0}, u_{x}\right)$, $\left(y_{0}, u_{y}\right),\left(z_{0}, u_{z}\right)$, and perform $n$ iterations to get $n+1$ output sequences for each pair between $[0,1]$.

Step-4: Add the last value from all four chaotic map sequences together to obtain $t$.
Step-5: Use equation $k=\bmod (|t \times 10000|$, 256) and obtain a decimal number $k$ within $(0,255]$.
Step-6: Convert this decimal number $k$ into an 8-bit binary number that represents the binary key value for the affine matrix generation.

The pseudo-code for the key generation algorithm is given in Algorithm 1.

```
Algorithm 1 Generation of Key
    Output: Key \(k\) for the affine transformation matrix \(R_{k}\)
    Procedure:
```

    \(\%\) initial seed value \(w_{0}, x_{0}, y_{0}, z_{0} \forall \in[0,1]\)
    1. Choose $w_{0}, x_{0}, y_{0}, z_{0}$
$\%$ logistic map is chaotic between 3.57 to 4
2. Choose $u_{w}, u_{x}, u_{y}, u_{z}$
3. for $n 1: m$ do $\quad \% m$ is the range
4. $w_{n+1}=u w_{n}\left(1-w_{n}\right)$
5. $x_{n+1}=u x_{n}\left(1-x_{n}\right)$
6. $\quad y_{n+1}=u y_{n}\left(1-y_{n}\right)$
7. $\quad z_{n+1}=u z_{n}\left(1-z_{n}\right)$
8. end for
9. $t=w_{n+1}+x_{n+1}+y_{n+1}+z_{n+1}$
10. $k=\bmod \left(\left|t^{*} 10,000\right|, 256\right) \quad \%$ convert real number into decimal
11. $k=\operatorname{dec} 2 \operatorname{bin}(k, 8) \%$ convert decimal into binary string
12. $k=\operatorname{str} 2 n u m(k) \%$ convert string to binary number

## B. AFFINE TRANSFORMATION MATRIX GENERATION

The proposed algorithm for generating the affine transformation matrix consists of the following steps:

Step-1: Take a bit matrix $R_{k}$ of size $8 \times 8$ and initialize it with all zero bits.

Step-2: Initialize the first row $\left(r_{1}\right)$ of the matrix $R_{k}$ with the key $k$ as an 8-bit binary integer obtained in step-6 of the $1^{\text {st }}$ algorithm as first row $\left(r_{1}\right)$ of $8 \times 8$ the rotational matrix.

Step-3: Rotate $r_{1}$ one-bit right to generate the second row ( $r_{2}$ ) for the matrix $R_{k}$. Likewise, rotate ( $r_{2}$ ) one-bit right again to generate an 8-bit binary number as the third row $\left(r_{3}\right)$.

Step-4: Repeat the step- 3 for all eight rows and generate a $8 \times 8$ rotational matrix.

Step-5: Find the determinant $\operatorname{det}\left(R_{k}\right)$ of the rotational bit matrix $R_{k}$.

Step-6: D iscard the matrix $R_{k}$ and go to the step-1 key generation of $1^{\text {st }}$ algorithm if its determinant-modulo 2 is equal to zero, i.e. $\operatorname{det}\left(R_{k}\right) \equiv 0 \bmod 2$. Otherwise, go to next step.

Step-7: Take the bit matrix $R_{k}$ that passes the step-6 as the affine transformation matrix and uses (4) to generate the S-box.

The pseudo-code for the proposed affine matrix algorithm is given in Algorithm 2.

## C. PROPOSED S-BOX GENERATION

Algorithm 3 presents the proposed $S$-box generation steps. After the affine transformation matrix $R_{k}$ is generated using Algorithm 2, the S -box is created in the following way:3

```
Algorithm 2 Generation of Affine Transformation Matrix
    Output: Affine Transformation Matrix \(R_{k}\)
    Procedure:
```

    \(\% 8 \times 8\) matrix with all elements equal to zero
    1. \(R_{k}=\operatorname{zeros}(8,8)\)
    \(\%\) make generated key \(1^{\text {st }}\) row of the rotational matrix
    2. \(\quad R_{k}(l, ;)=k\)
    3. for \(m=2: 8\) do \(\quad \%\) iterate for all rows in \(R_{k}\)
        \(R_{k}(m,:)=\operatorname{circshift}\left(R_{k}\left(r_{m},:\right), 1\right)\)
    end for
    \% det represents the determinant of a matrix
    6. \(\quad D=\operatorname{det}\left(R_{k}\right)\)
    \% mod represents the modulus operator
    7. if \(D \bmod 2 \neq 0\) then
        Set as an affine transformation matrix
    9. else
    10. Go to line 1 of the \(1^{\text {st }}\) algorithm
    endif
    ```
Algorithm 3 S-Box Generation Based on Affine Transforma-
tion Matrix
    Output: S-Box \(S_{i}\)
    Procedure:
    for \(A_{i}=0: 255\) do
        \% Multiplicative inverse of \(A_{i}\) in \(G F\left(2^{8}\right)\)
        Find \(A_{i}^{-1}\)
        \(\% S_{i}\) is the required S-box output for input \(A_{i}\)
        \(S_{i}=R_{k} \cdot A_{i}^{-1}+C \bmod 2\)
        end for
```

Step-1: Take an input byte $A_{i}$ and compute its multiplicative inverse $A_{i}^{-1}$ in $G F\left(2^{8}\right)$ using Table 1.

Step-2: Apply affine mapping on $A_{i}^{-1}$ using (4) and compute the S -box output $S_{i}$. Use $R_{k}$ obtained from Algorithm 2 as the affine transformation matrix in (4).

Step-3: Repeat step-1 and step-2 for all possible inputs $A_{i}$, i.e., 0 to 255 keeping the bit matrix $R_{k}$ constant and generate the final S-box.

Table 4 shows four different rotational matrices of size $8 \times$ 8 , and Table 5 to Table 8 specifies the corresponding S-boxes generated by these matrices using the proposed methodology. $R_{11}, R_{88}, R_{145}$, and $R_{203}$ are produced when the value of key generated is to be $11,88,145$, and 203 , respectively. The value of the subscript index $k$ with the matrix $R_{k}$ represents the key used to produce the rotational matrix. The sample rotational matrices $R_{11}, R_{88}, R_{145}$, and $R_{203}$ fulfill the specified criteria in Algorithm 2.

## V. RESULTS

S-boxes designed using the proposed methodology have good cryptographic properties like the AES S-box. This paper presents four different S-boxes (Table 5 - Table 8) generated corresponding to the matrices $R_{11}, R_{88}, R_{145}, R_{203}$ and validates their cryptographic properties.

TABLE 4. Rotational matrices generated with the proposed S-box design.
Matrix index $\quad$ Matrices

|  |
| :---: |
| $R_{11}$ |
|  |
| $R_{88}$ |
|  |

$$
\left[\begin{array}{llllllll}
0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0
\end{array}\right]
$$

$$
R_{145}\left[\begin{array}{llllllll}
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1
\end{array}\right]
$$

$$
\left[\begin{array}{llllllll}
1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 & 1
\end{array}\right]
$$

## A. S-BOX TESTING CRITERIA IN CRYPTOGRAPHY

The effectiveness of the proposed S-boxes is validated by analyzing the results of conventional S-box tests, i.e., NL, SAC, BIC, DP, and LP.

TABLE 5. S-box designed with rotational matrix $R_{11}$.

| Y |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
|  | 0 | 99 | 117 | 150 | 58 | 103 | 171 | 207 | 154 | 159 | 52 | 7 | 237 | 203 | 57 | 97 | 143 |
|  | 1 | 29 | 147 | 54 | 108 | 175 | 43 | 36 | 85 | 55 | 18 | 176 | 5 | 156 | 230 | 235 | 231 |
|  | 2 | 92 | 224 | 27 | 88 | 201 | 24 | 26 | 75 | 251 | 255 | 185 | 76 | 192 | 190 | 134 | 193 |
|  | 3 | 73 | 168 | 37 | 204 | 116 | 102 | 80 | 202 | 98 | 142 | 161 | 246 | 39 | 65 | 33 | 164 |
|  | 4 | 252 | 138 | 162 | 70 | 95 | 214 | 0 | 130 | 200 | 124 | 32 | 56 | 223 | 239 | 137 | 197 |
|  | 5 | 209 | 111 | 45 | 113 | 14 | 249 | 10 | 25 | 178 | 4 | 141 | 78 | 145 | 199 | 50 | 96 |
|  | 6 | 118 | 67 | 120 | 182 | 64 | 222 | 180 | 220 | 22 | 132 | 31 | 94 | 250 | 51 | 183 | 74 |
| X | 7 | 227 | 169 | 107 | 38 | 2 | 133 | 87 | 40 | 191 | 69 | 140 | 23 | 188 | 210 | 126 | 68 |
|  | 8 | 82 | 129 | 151 | 104 | 125 | 127 | 15 | 243 | 131 | 205 | 71 | 42 | 44 | 79 | 109 | 242 |
|  | 9 | 72 | 184 | 236 | 218 | 60 | 244 | 48 | 105 | 61 | 90 | 219 | 216 | 232 | 100 | 206 | 149 |
|  | A | 196 | 173 | 101 | 215 | 186 | 123 | 106 | 86 | 213 | 93 | 46 | 122 | 41 | 77 | 160 | 8 |
|  | B | 139 | 47 | 208 | 253 | 20 | 11 | 245 | 83 | 228 | 172 | 49 | 62 | 53 | 35 | 28 | 229 |
|  | C | 233 | 17 | 115 | 144 | 16 | 212 | 119 | 177 | 12 | 195 | 189 | 59 | 136 | 166 | 66 | 91 |
|  | D | 217 | 1 | 110 | 30 | 163 | 6 | 3 | 179 | 81 | 226 | 181 | 194 | 247 | 254 | 9 | 174 |
|  | E | 221 | 157 | 248 | 165 | 153 | 167 | 63 | 84 | 211 | 34 | 238 | 21 | 121 | 135 | 198 | 241 |
|  | F | 13 | 155 | 112 | 152 | 148 | 146 | 89 | 128 | 114 | 225 | 187 | 170 | $` 19$ | 158 | 240 | 234 |

TABLE 6. S-box designed with rotational matrix R88.

| Y |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
|  | 0 | 99 | 211 | 204 | 169 | 67 | 37 | 6 | 172 | 132 | 217 | 64 | 23 | 38 | 177 | 115 | 4 |
|  | 1 | 144 | 228 | 201 | 27 | 5 | 33 | 89 | 210 | 193 | 232 | 253 | 80 | 156 | 79 | 39 | 71 |
|  | 2 | 154 | 127 | 160 | 186 | 54 | 184 | 168 | 34 | 167 | 135 | 181 | 26 | 126 | 141 | 76 | 118 |
|  | 3 | 50 | 61 | 81 | 30 | 219 | 75 | 250 | 46 | 107 | 12 | 117 | 207 | 65 | 114 | 113 | 93 |
|  | 4 | 159 | 44 | 109 | 74 | 130 | 206 | 120 | 108 | 62 | 155 | 121 | 185 | 134 | 7 | 52 | 86 |
|  | 5 | 246 | 3 | 17 | 243 | 8 | 183 | 40 | 176 | 237 | 88 | 20 | 10 | 244 | 70 | 233 | 123 |
|  | 6 | 203 | 98 | 187 | 205 | 122 | 142 | 221 | 158 | 200 | 92 | 128 | 138 | 175 | 225 | 197 | 42 |
| X | 7 | 103 | 53 | 35 | 73 | 104 | 84 | 194 | 57 | 133 | 82 | 28 | 192 | 157 | 238 | 139 | 90 |
|  | 8 | 234 | 116 | 196 | 59 | 147 | 131 | 0 | 231 | 100 | 22 | 66 | 41 | 25 | 2 | 19 | 239 |
|  | 9 | 58 | 189 | 31 | 174 | 153 | 223 | 249 | 51 | 145 | 170 | 166 | 190 | 63 | 91 | 14 | 212 |
|  | A | 94 | 21 | 83 | 198 | 173 | 163 | 43 | 202 | 214 | 146 | 9 | 171 | 49 | 18 | 125 | 56 |
|  | B | 36 | 1 | 254 | 151 | 216 | 32 | 215 | 226 | 95 | 29 | 241 | 137 | 209 | 97 | 152 | 87 |
|  | C | 55 | 240 | 227 | 252 | 248 | 222 | 195 | 245 | 24 | 102 | 149 | 161 | 60 | 77 | 106 | 162 |
|  | D | 182 | 112 | 11 | 136 | 101 | 72 | 96 | 229 | 242 | 111 | 213 | 110 | 199 | 143 | 48 | 13 |
|  | E | 150 | 148 | 191 | 85 | 180 | 69 | 129 | 218 | 230 | 105 | 15 | 208 | 179 | 68 | 78 | 247 |
|  | F | 16 | 164 | 251 | 188 | 220 | 236 | 178 | 124 | 235 | 119 | 165 | 45 | 224 | 140 | 255 | 47 |

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TABLE 7. S-box designed with rotational matrix $\boldsymbol{R}_{\mathbf{1 4 5}}$.

| Y |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
|  | 0 | 99 | 64 | 68 | 166 | 70 | 223 | 129 | 43 | 71 | 91 | 61 | 58 | 164 | 125 | 241 | 211 |
|  | 1 | 113 | 40 | 201 | 215 | 250 | 123 | 207 | 105 | 128 | 197 | 218 | 175 | 156 | 171 | 141 | 226 |
|  | 2 | 106 | 28 | 198 | 79 | 54 | 29 | 143 | 0 | 25 | 60 | 217 | 254 | 53 | 39 | 208 | 124 |
|  | 3 | 146 | 4 | 134 | 90 | 9 | 15 | 5 | 237 | 42 | 154 | 7 | 63 | 20 | 216 | 163 | 107 |
|  | 4 | 231 | 191 | 220 | 38 | 177 | 22 | 195 | 245 | 127 | 67 | 234 | 52 | 21 | 168 | 100 | 89 |
|  | 5 | 232 | 12 | 204 | 101 | 62 | 139 | 27 | 84 | 72 | 230 | 65 | 108 | 186 | 203 | 236 | 184 |
|  | 6 | 155 | 74 | 102 | 109 | 145 | 92 | 255 | 206 | 224 | 66 | 227 | 248 | 80 | 165 | 36 | 73 |
| X | 7 | 199 | 77 | 41 | 93 | 81 | 11 | 251 | 160 | 110 | 253 | 8 | 169 | 181 | 51 | 209 | 180 |
|  | 8 | 151 | 46 | 13 | 242 | 10 | 152 | 119 | 83 | 188 | 19 | 111 | 50 | 133 | 37 | 158 | 26 |
|  | 9 | 219 | 144 | 115 | 121 | 17 | 173 | 126 | 187 | 88 | 221 | 48 | 235 | 86 | 157 | 200 | 159 |
|  | A | 16 | 104 | 212 | 95 | 2 | 189 | 96 | 178 | 205 | 35 | 23 | 244 | 233 | 183 | 78 | 137 |
|  | B | 246 | 94 | 161 | 174 | 114 | 82 | 228 | 222 | 57 | 33 | 55 | 131 | 18 | 49 | 56 | 112 |
|  | C | 31 | 30 | 247 | 243 | 87 | 132 | 210 | 147 | 172 | 238 | 252 | 239 | 45 | 249 | 3 | 148 |
|  | D | 162 | 138 | 69 | 170 | 149 | 116 | 24 | 1 | 76 | 142 | 182 | 167 | 118 | 117 | 192 | 179 |
|  | E | 135 | 213 | 194 | 34 | 240 | 176 | 202 | 32 | 122 | 120 | 225 | 59 | 47 | 153 | 130 | 193 |
|  | F | 229 | 98 | 44 | 185 | 214 | 97 | 6 | 103 | 190 | 85 | 75 | 150 | 140 | 14 | 136 | 196 |

TABLE 8. S-box designed with rotational matrix $\boldsymbol{R}_{\mathbf{2 0 3}}$.

| Y |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
|  | 0 | 99 | 244 | 221 | 183 | 73 | 208 | 9 | 163 | 3 | 220 | 186 | 77 | 35 | 40 | 118 | 171 |
|  | 1 | 83 | 125 | 201 | 130 | 250 | 149 | 116 | 165 | 67 | 178 | 179 | 175 | 156 | 134 | 114 | 12 |
|  | 2 | 123 | 185 | 108 | 81 | 54 | 243 | 230 | 102 | 218 | 240 | 109 | 211 | 232 | 216 | 117 | 98 |
|  | 3 | 115 | 79 | 254 | 150 | 126 | 195 | 5 | 169 | 233 | 33 | 145 | 46 | 235 | 39 | 212 | 49 |
|  | 4 | 111 | 11 | 14 | 146 | 228 | 127 | 15 | 95 | 188 | 42 | 94 | 162 | 161 | 88 | 148 | 72 |
|  | 5 | 202 | 29 | 170 | 222 | 100 | 207 | 78 | 121 | 166 | 37 | 190 | 198 | 104 | 93 | 227 | 252 |
|  | 6 | 107 | 50 | 0 | 140 | 173 | 43 | 153 | 62 | 152 | 96 | 70 | 110 | 80 | 105 | 6 | 236 |
| X | 7 | 38 | 197 | 55 | 97 | 26 | 234 | 176 | 10 | 82 | 13 | 52 | 18 | 205 | 85 | 63 | 135 |
|  | 8 | 16 | 192 | 87 | 168 | 160 | 181 | 238 | 142 | 213 | 28 | 24 | 31 | 32 | 76 | 8 | 4 |
|  | 9 | 249 | 231 | 199 | 1 | 136 | 59 | 246 | 34 | 2 | 68 | 139 | 20 | 237 | 214 | 131 | 66 |
|  | A | 194 | 239 | 92 | 245 | 242 | 159 | 189 | 58 | 224 | 241 | 53 | 21 | 128 | 89 | 27 | 91 |
|  | B | 129 | 191 | 64 | 229 | 141 | 196 | 177 | 154 | 147 | 101 | 124 | 157 | 86 | 193 | 217 | 25 |
|  | C | 103 | 45 | 203 | 226 | 167 | 106 | 225 | 57 | 113 | 119 | 71 | 61 | 30 | 36 | 184 | 206 |
|  | D | 158 | 133 | 151 | 204 | 132 | 48 | 144 | 44 | 143 | 172 | 19 | 253 | 164 | 122 | 209 | 112 |
|  | E | 180 | 22 | 69 | 187 | 60 | 174 | 23 | 47 | 223 | 75 | 210 | 7 | 138 | 255 | 215 | 155 |
|  | F | 251 | 41 | 84 | 182 | 200 | 247 | 219 | 74 | 65 | 51 | 120 | 90 | 56 | 137 | 17 | 248 |

TABLE 9. Cryptographic properties of proposed S-boxes.

| Initial <br> value | NL | SAC | BIC | DP | LP |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 112 | 0.500977 | 112 | $4 / 256$ | 0.0625 |
| 88 | 112 | 0.500977 | 112 | $4 / 256$ | 0.0625 |
| 145 | 112 | 0.49585 | 112 | $4 / 256$ | 0.0625 |
| 203 | 112 | 0.495605 | 112 | $4 / 256$ | 0.0625 |

## 1) NON-LINEARITY (NL)

The importance of non-linearity in the cryptosystem was first introduced by Staffelbach and Meier in 1980s and later by Nyberg in early 1990s after the discovery of linear and differential cryptanalysis [60]. S-box is the only non-linear component in any cryptographic algorithm. The non-linearity of an S-box is its ability to resist against linear and differential cryptanalysis, which is measured using Walsh's spectrum [8]. Higher non-linearity value means strong resistance against linear and differential attacks [61]. Mathematically, it can be defined as:

$$
\begin{equation*}
S_{f}(w)=\sum_{S_{f}(w)}(-1)^{f(x) \oplus x . w} \tag{5}
\end{equation*}
$$

The non-linearity of an n-bit Boolean function $f$ is calculated as:

$$
\begin{equation*}
N L(f)=2^{n-1}-\frac{1}{2}\left(\max _{z \in\{0,1\}^{n}}\left|W_{f}(z)\right|\right) \tag{6}
\end{equation*}
$$

Higher the value of non-linearity, higher will be the security of S-box. The highest value of non-linearity that is achieved with AES S-box is 112. It is the standard value till now for all other S-boxes and the $S$-boxes with this non-linearity value are generally considered as secure. The proposed S-boxes of this study obtained the highest value of nonlinearity, i.e., 112, which has been achieved till today, as shown in Table 9.

## 2) STRICT AVALANCHE CRITERIA (SAC)

Tavares and Webster introduced strict avalanche criteria (SAC) [62] for the first time. According to the SAC, a single bit change in input will affect half of the output bits and the value of the SAC should be near 0.5 . The proposed S-boxes fulfill the SAC satisfactory, as presented in Table 9.

## 3) BIT INDEPENDENCE CRITERION (BIC)

Bit independence criterion says that for the given change in the input bits there will be the change in the output independently. If the S-box satisfies the BIC, all the functions will be non-linear and fulfill the SAC also [62]. In our case, this criterion is also fulfilled by the proposed S -boxes, as presented in Table 9.

## 4) DIFFERENTIAL PROBABILITY (DP)

Differential probability is one of the most common methods to decrypt the ciphertext. It provides the difference in the original and the cipher message [28]. Mathematically it can

TABLE 10. Key values for S-box design with $\mathbf{N L = 1 1 2}$ and $\mathbf{D P}=\mathbf{4 / 2 5 6}$.

| Initial keys | DP Value |
| :--- | :--- |
| $1,2,4,7,8,11,13,14,16,19,21,22,25,26,28,31,32,35$, |  |
| $37,38,41,42,44,47,49,50,52,55,56,59,61,62,64,67$, |  |
| $69,70,73,74,76,79,81,82,84,87,88,91,93,94,97,98$, |  |
| $100,103,104,107,109,110,112,115,117,118,121,122$, |  |
| $124,127,128,131,133,134,137,138,140,143,145,146$, | $4 / 256$ |
| $148,151,152,155,157,158,161,162,164,167,168,171$, |  |
| $173,174,176,179,181,182,185,186,188,191,193,194$, |  |
| $196,199,200,203,205,206,208,211,213,214,217,218$, |  |
| $220,223,224,227,229,230,233,234,236,239,241,242$, |  |
| $244,247,248,251,253,254$ |  |

be defined as an input difference $\Delta x_{i}$ that should map to an output difference $y_{i}$ [63]. It is defined in (7) as:

$$
\begin{equation*}
D P^{s}(\Delta x \rightarrow \Delta y)=\left(\frac{\#\{x \in X \mid S(x) \oplus S(x \oplus \Delta x)=\Delta y\}}{2^{m}}\right) \tag{7}
\end{equation*}
$$

where, X is the set of all possible values of input and $2^{m}$ is the number of elements. The maximum achievable value of DP is $4 / 256$, and our $S$-boxes fulfilling this criterion also, as shown in Table 9.

## 5) LINEAR PROBABILITY (LP)

If an S-box has a small linear probability, it is considered as very strong against linear cryptanalysis [28]. Linear probability (LP) [64] of an S-box can be defined as:

$$
\begin{equation*}
L P=\max _{A_{x}, B_{x} \neq 0}\left|\frac{\#\left\{x \in Z \mid x \cdot A_{x}=S(x) \cdot B_{x}\right\}}{2^{n}}-\frac{1}{2}\right| \tag{8}
\end{equation*}
$$

where, $A_{x}$ and $B_{x}$ are the input and output values respectively while $\mathrm{Z}=\{1,2,3 \ldots 255\}$. As shown in Table 9, the LP value achieved with our proposed S-boxes is 0.0625 , which is the same as the standard AES S-box.

## B. CRYPTOGRAPHIC PROPERTIES OF <br> THE PROPOSED S-BOXES

As the research on S-box design and construction is growing and becoming increasingly vital in the field of cryptography, numerous researchers have designed tools for testing and verifying the performance of an S-box [63], [65], [66]. These tools are based on the NIST criteria for S-box performance analysis and entail common S-box tests as defined by the NIST (including NL, SAC, BIC, DP, and LP as given in Eq. (5) - (8)). The purpose of designing these tools is to simplify the research process and provide an ease to the researchers in testing and verifying the S-box performance. For verifying the proposed S-boxes in this study, authors used the same $S$-box testing tool as presented in [63]. Table 9 shows the cryptographic properties of the proposed S-boxes, which are generated as a result of matrices $R_{11}$ $R_{88}, R_{145}$ and $R_{203}$. It can be analyzed from table 9 that all $S$-boxes have excellent cryptographic properties and are as stronger as the AES S-box. Table 10 presents all the key values that successfully generate the corresponding rotational

TABLE 11. Performance comparison with other states of the art S-boxes.

| Study | Proposed Techniques to design S-box | NL | SAC | BIC | DP |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $[16]$ | S-box based on 1-D chaotic map and beta hill climbing | 110.25 | 0.500 | 104 | $10 / 256$ | 0.125 |
| $[17]$ | S-box design based on proposed 2-D bakers chaotic map | 104 | 0.4965 | 102.9 | $10 / 256$ | 0.1289 |
| $[20]$ | Construction of S-boxes based on TD ERC sequence | 104 | 0.507 | 102.9 | $12 / 256$ | 0.086 |
| $[21]$ | S-box based on hyperchaotic map and bee colony algorithm | 108 | 0.5073 | 104 | $10 / 256$ | 0.1523 |
| $[22]$ | Construction of S-box based on optimized chaotic maps | 106.75 | 0.5015 | 104.07 | $10 / 256$ | 0.1367 |
| $[23]$ | S-box generation based on 3-D plasma chaotic system | 106 | 0.4978 | 103.92 | $10 / 256$ | 0.1298 |
| $[24]$ | S-box using LCG and combination of different chaotic maps | 107.75 | 0.4976 | 105.07 | $10 / 256$ | 0.1250 |
| $[25]$ | S-box based on gold sequence and chaotic logistic tent system | 112 | 0.5065 | 109 | $6 / 256$ | 0.1090 |
| $[6]$ | S-box design for image encryption based on hybrid chaotic map | 106.2 | 0.5009 | 103.6 | $10 / 256$ | 0.132 |
| $[69]$ | Effect of chaotic system in S-box performance characteristic | 105.25 | 0.5037 | 102.64 | $10 / 256$ | 0.125 |
| $[27]$ | Substitution box based on elliptic curves | 100 | 0.5007 | 104.1 | $10 / 256$ | 0.1250 |
| $[28]$ | S-box design using cubic fractional transformation | 107 | 0.497 | 103.5 | $10 / 256$ | 0.156 |
| $[29]$ | S-box generation using projective general linear group | 105 | 0.5021 | 106 | $10 / 256$ | 0.1250 |
| $[31]$ | Group theoretic approach to construct S-box | 104.8 | 0.493 | 105.1 | $10 / 256$ | 0.125 |
| $[32]$ | S-box designing using Gaussian distribution method | 111 | 0.5036 | 110 | $6 / 256$ | 0.0781 |
| $[48]$ | Design S-boxes using GF $\left.2^{8}\right)$ different polynomial | 105.5 | 0.507 | 106 | $6 / 256$ | 0.140 |
| $[53]$ | Dynamic AES-128 with key-dependent S-box | 104.3 | 0.497 | 103.4 | $12 / 256$ | 0.133 |
| AES | S-box designed using AES | 112 | 0.504 | 112 | $4 / 256$ | 0.0625 |
| $\mathrm{~S}_{1}$ | Proposed S-box with first row of rotational matrix as 1 | 112 | 0.503 | 112 | $4 / 256$ | 0.0625 |
| $\mathrm{~S}_{88}$ | Proposed S-box with first row of rotational matrix as 88 | 112 | 0.501 | 112 | $4 / 256$ | 0.0625 |
| $\mathrm{~S}_{145}$ | Proposed S-box with first row of rotational matrix as 145 | 112 | 0.496 | 112 | $4 / 256$ | 0.0625 |
| $\mathrm{~S}_{203}$ | Proposed S-box with first row of rotational matrix as 203 | 0.495 | 112 | $4 / 256$ | 0.0625 |  |

matrices for affine transformations. These rotational matrices produce the S-boxes having NL and DP values 112 and 4/256, respectively.

## VI. COMPARISON AND DISCUSSION

The performance of the proposed S-box compared to other state-of-the-art S-boxes, based on their cryptographic properties, is given in Table 11. The proposed S-boxes are cryptographically more effective than some of the existing S-boxes. When comparing the proposed S -boxes with the standard AES S-box, it can be seen that their cryptographic values are the same as the AES S-box, which means that these S-boxes are as strong as the AES S-box.

In [67], [68], the authors have investigated the effect of different types of practical attacks on the AES algorithm, and validated the efficacy of AES as compared to other encryption algorithms. However, AES produces only a static S-box that can be vulnerable to cryptanalyst study to crack the ciphertext. The advantage of the proposed method over AES algorithm is that the former generates a large number of dynamic S-boxes. Furthermore, the proposed S-boxes are generated based on the equivalence classes of AES, hence they provide the same security and equally strong cryptographic properties as the AES S-box. Any of the proposed S-box can be used in each round of AES encryption. The selection of the S-box is based on the key scheduling algorithm that uses a chaotic logistic map. Chaos is generally important to produce entropy in the sequence but cannot directly generate the S-boxes with high NL and low DP values [69].

For generating a cryptographically strong S-box using chaos, some additional non-linear phenomenon is incorporated in the system, which can be observed from the existing studies reported in Table 11. In our proposed scheme, we used affine transformation in addition to chaotic logistic map for generating the strong S-boxes. The main reason of using chaos in the proposed scheme is to obtain a pseudorandom sequence that is further used to generate an unpredictable and secure key for creating a new S-box (based on the rotational matrix) in each round of encryption. As a result, it becomes challenging for an attacker to analyze which S-box has been used in each round of the algorithm. Hence, it will be near impossible to decrypt the ciphertext. Thus, the addition of dynamic S-boxes provides an additional layer of security using the proposed scheme, which tremendously enhances the security of a cryptosystem.

In comparison to AES S-box, our proposed method entails two additional steps, which include key generation and affine matrix generation. A chaotic logistic map is used for generating the 8 -bit random key value, which is computationally very cheap and fast [14]. Moreover, the generation of the affine transformation matrix is only based on rotating the obtained key value. Hence, the additional transformation cost tends to be very insignificant. Table 12 shows the overall computational time required for generating a large number of random key values and the corresponding affine matrices. The computational time is computed in MATLAB using a core-i7 Intel processor ( $3.2 \mathrm{GHz}, 8 \mathrm{~GB}$ RAM). It can be analyzed that the average additional time required for these

TABLE 12. Additional cost for keys and affine matrices generation.

| No. of Keys and Affine <br> Matrices | Computational Time <br> $(\mathrm{sec})$ |
| :---: | :---: |
| 1 | 0.006722 |
| 2 | 0.007955 |
| 4 | 0.008723 |
| 8 | 0.012374 |
| 16 | 0.012773 |
| 32 | 0.013795 |
| 64 | 0.004586 |
| 128 | 0.014601 |
| 256 | 0.014706 |

transformations in generating a single S-box is very trivial (i.e., 6.7 milliseconds) and hence negligible. As a result, the proposed S-box generation design is computationally viable and comparable to AES S-box as well.

## VII. CONCLUSION

In this study, dynamic AES S-boxes are generated using proposed chaos based rotation matrices. The standard AES S-box is adequate for AES security, but with the enhancement in computing resources and cryptanalysis techniques, there is a need to modify the AES algorithm. As a result, we proposed a significant modification in affine transformation of the AES S-box by proposing chaos based rotation metrics that generates strong AES S-boxes. Dynamic S-boxes nullifies the effects of algebraic attacks. The selection of S-box is based on a chaotic map-based key scheduling algorithm that makes it undetermined which S-box will be the targeted one. In this way, the security of a cryptosystem is greatly enhanced. The proposed S-boxes are tested based on the state of the art S-box tests, which demonstrate the effectiveness of the proposed S-box design.

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## REFERENCES

[1] S. Guo, X. Zhao, F. Zhang, T. Wang, Z. J. Shi, F.-X. Standaert, and C. Ma, "Exploiting the incomplete diffusion feature: A specialized analytical sidechannel attack against the AES and its application to microcontroller implementations," IEEE Trans. Inf. Forensics Security, vol. 9, no. 6, pp. 999-1014, Jun. 2014.
[2] J. Daemen and V. Rijmen, The Design of Rijndael: AES—The Advanced Encryption Standard. New York, NY, USA: Springer, 2013.
[3] J. Nakahara, Jr., P. S. L. Barreto, B. Preneel, J. Vandewalle, and H. Y. Kim, "SQUARE attacks on reduced-round PES and IDEA block ciphers," in Proc. 23rd Symp. Inf. Theory Benelux Date, 2001, p. 68.
[4] J. Huang, H. Yan, and X. Lai, "Transposition of AES key schedule," in Information Security and Cryptology (Lecture Notes in Computer Science: Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics). Berlin, Germany: Springer, 2017.
[5] M. Khan and Z. Asghar, "A novel construction of substitution box for image encryption applications with Gingerbreadman chaotic map and S8 permutation," Neural Comput. Appl., vol. 29, no. 4, pp. 993-999, Feb. 2018.
[6] M. A. B. Farah, A. Farah, and T. Farah, "An image encryption scheme based on a new hybrid chaotic map and optimized substitution box," Nonlinear Dyn., pp. 1-24, Dec. 2019. [Online]. Available: https://link.springer.com/article/10.1007/s11071-019-05413-8
[7] I. Hussain, A. Anees, M. Aslam, R. Ahmed, and N. Siddiqui, "A noise resistant symmetric key cryptosystem based on S8 S-boxes and chaotic maps," Eur. Phys. J. Plus, vol. 133, no. 4, Apr. 2018.
[8] M. A. Khan, A. Ali, V. Jeoti, and S. Manzoor, "A chaos-based substitution box (S-box) design with improved differential approximation probability (DP)," Iranian J. Sci. Technol., Trans. Elect. Eng., vol. 42, no. 2, pp. 219-238, Jun. 2018.
[9] S. S. Jamal, M. U. Khan, and T. Shah, "A watermarking technique with chaotic fractional S-box transformation," Wireless Pers. Commun., vol. 90, no. 4, pp. 2033-2049, Oct. 2016.
[10] A. A. Abd EL-Latif, B. Abd-El-Atty, and S. E. Venegas-Andraca, "A novel image steganography technique based on quantum substitution boxes," Opt. Laser Technol., vol. 116, pp. 92-102, Aug. 2019.
[11] A. ALabaichi, R. Mahmod, and F. Ahmad, "Analysis of some security criteria for S-boxes in blowfish algorithm," Int. J. Digit. Content Technol. its Appl., vol. 7, no. 12, p. 8, 2013.
[12] F. H. B. T. H. J. M. Ali and F. Hani, "A new 128-bit block cipher," Univ. Putra Malaysia, Seri Kembangan, Malaysia, Tech. Rep. FSKTM 2009 5, 2009.
[13] H. M. Heys, "A tutorial on linear and differential cryptanalysis," Cryptologia, vol. 26, no. 3, pp. 189-221, Jul. 2002.
[14] L. Kocarev, "Chaos-based cryptography: A brief overview," IEEE Circuits Syst. Mag., vol. 1, no. 3, pp. 6-21, Mar. 2001.
[15] A. Belazi, M. Khan, A. A. A. El-Latif, and S. Belghith, "Efficient cryptosystem approaches: S-boxes and permutation-substitution-based encryption," Nonlinear Dyn., vol. 87, no. 1, pp. 337-361, Jan. 2017.
[16] A. A. Alzaidi, M. Ahmad, M. N. Doja, E. Al Solami, and M. M. S. Beg, "A new 1D chaotic map and $\beta$-Hill climbing for generating substitutionboxes," IEEE Access, vol. 6, pp. 55405-55418, 2018.
[17] G. Tang, X. Liao, and Y. Chen, "A novel method for designing S-boxes based on chaotic maps," Chaos, Solitons Fractals, vol. 23, no. 2, pp. 413-419, Jan. 2005.
[18] I. Hussain, T. Shah, M. A. Gondal, and H. Mahmood, "Efficient method for designing chaotic S-boxes based on generalized Baker's map and TDERC chaotic sequence," Nonlinear Dyn., vol. 74, nos. 1-2, pp. 271-275, Oct. 2013.
[19] G. Chen, Y. Chen, and X. Liao, "An extended method for obtaining S-boxes based on three-dimensional chaotic Baker maps," Chaos, Solitons Fractals, vol. 31, no. 3, pp. 571-579, Feb. 2007.
[20] A. Hussain Alkhaldi, I. Hussain, and M. A. Gondal, "A novel design for the construction of safe S-boxes based on TDERC sequence," Alexandria Eng. J., vol. 54, no. 1, pp. 65-69, Mar. 2015.
[21] Y. Tian and Z. Lu, "S-box: Six-dimensional compound hyperchaotic map and artificial bee colony algorithm," J. Syst. Eng. Electron., vol. 27, no. 1, pp. 232-241, 2016.
[22] E. Tanyildizi and F. Ozkaynak, "A new chaotic S-box generation method using parameter optimization of one dimensional chaotic maps," IEEE Access, vol. 7, pp. 117829-117838, 2019.
[23] A. K. Farhan, R. S. Ali, H. Natiq, and N. M. G. Al-Saidi, "A new S-box generation algorithm based on multistability behavior of a plasma perturbation model," IEEE Access, vol. 7, pp. 124914-124924, 2019.
[24] L. Yi, X. Tong, Z. Wang, M. Zhang, H. Zhu, and J. Liu, "A novel block encryption algorithm based on chaotic S-box for wireless sensor network," IEEE Access, vol. 7, pp. 53079-53090, 2019.
[25] M. F. Khan, A. Ahmed, K. Saleem, and T. Shah, "A novel design of cryptographic SP-network based on gold sequences and chaotic logistic tent system," IEEE Access, vol. 7, pp. 84980-84991, 2019.
[26] Q. Lu, C. Zhu, and G. Wang, "A novel S-box design algorithm based on a new compound chaotic system," Entropy, vol. 21, no. 10, p. 1004, Oct. 2019.
[27] U. Hayat, N. A. Azam, and M. Asif, "A method of generating $8 \times 8$ substitution boxes based on elliptic curves," Wireless Pers. Commun., vol. 101, no. 1, pp. 439-451, Jul. 2018.
[28] A. Zahid, M. Arshad, and M. Ahmad, "A novel construction of efficient substitution-boxes using cubic fractional transformation," Entropy, vol. 21, no. 3, p. 245, Mar. 2019.
[29] I. Hussain, T. Shah, H. Mahmood, and M. A. Gondal, "A projective general linear group based algorithm for the construction of substitution box for block ciphers," Neural Comput. Appl., vol. 22, no. 6, pp. 1085-1093, May 2013.
[30] S. Farwa, T. Shah, and L. Idrees, "A highly nonlinear S-box based on a fractional linear transformation," SpringerPlus, vol. 5, no. 1, p. 1658, Dec. 2016.
[31] I. Hussain, T. Shah, M. A. Gondal, W. A. Khan, and H. Mahmood, "A group theoretic approach to construct cryptographically strong substitution boxes," Neural Comput. Appl., vol. 23, no. 1, pp. 97-104, Jul. 2013.
[32] M. F. Khan, A. Ahmed, and K. Saleem, "A novel cryptographic substitution box design using Gaussian distribution," IEEE Access, vol. 7, pp. 15999-16007, 2019.
[33] M. Ahmad, M. Alauddin, and H. D. AlSharari, "Heuristic approach for nonlinear $\mathrm{n} \times \mathrm{n}(3 \leq \mathrm{n} \leq 7)$ substitution-boxes," in Proc. Adv. Intell. Syst. Comput. Data Eng. Intell. Comput., 2018, pp. 347-355.
[34] N. Munir and M. Khan, "A generalization of algebraic expression for nonlinear component of symmetric key algorithms of any characteristic P," in Proc. Int. Conf. Appl. Eng. Math. (ICAEM), pp. 48-52, Sep. 2018.
[35] S. Tillich and C. Herbst, "Boosting AES performance on a tiny processor core," in Topics in Cryptology-CT-RSA (Lecture Notes in Computer Science: Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics). Berlin, Germany: Springer, 2008.
[36] S. Rao Rupanagudi, V. Vidya J, V. G. Bhat, P. Padmavathi, G. Darshan, S. K. Gurikar, S. Darshan, and N. Sindhu, "A further optimized mix column architecture design for the advanced encryption standard," in Proc. 11th Int. Conf. Knowl. Smart Technol. (KST), Jan. 2019, pp. 181-185.
[37] A. Moh'd, Y. Jararweh, and L. Tawalbeh, "AES-512: 512-bit advanced encryption standard algorithm design and evaluation," in Proc. 7th Int. Conf. Inf. Assurance Secur. (IAS), Dec. 2011, pp. 292-297.
[38] S. M. U. Talha, M. Asif, H. Hussain, A. Asghar, and H. Ameen, "Efficient advance encryption standard (AES) implementation on FPGA using Xilinx system generator," in Proc. 6th Int. Conf. Intell. Adv. Syst. (ICIAS), Aug. 2016, pp. 1-6.
[39] A. Shreedhar, K.-S. Chong, N. K. Z. Lwin, N. A. Kyaw, L. Nalangilli, W. Shu, J. S. Chang, and B.-H. Gwee, "Low gate-count ultra-small area nano advanced encryption standard (AES) design," in Proc. IEEE Int. Symp. Circuits Syst. (ISCAS), May 2019, pp. 1-5.
[40] S. Gueron, "Intel's new AES instructions for enhanced performance and security," in Fast Software Encryption (Lecture Notes in Computer Science: Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics). Springer, 2009. [Online]. Available: https://link.springer.com/chapter/10.1007/978-3-642-03317-9_4
[41] O. B. Sahoo, D. K. Kole, and H. Rahaman, "An optimized S-box for advanced encryption standard (AES) design," in Proc. Int. Conf. Adv. Computing Commun. (ICACC), 2012, pp. 154-157.
[42] V. Pachori, G. Ansari, and N. Chaudhary, "Improved performance of advance encryption standard using parallel computing," Int. J. Eng. Res. Appl., vol. 2, no. 1, pp. 967-971, 2012.
[43] E. M. De Los Reyes, A. M. Sison, and R. Medina, "Modified AES cipher round and key schedule," Indonesian J. Electr. Eng. Informat., vol. 7, no. 1, pp. 29-36, 2019.
[44] H. M. Hussien, Z. Muda, and S. M. Yasin, "Enhance the robustness of secure Rijndael key expansion function based on increment confusion," in Proc. 6th Int. Conf. Comput. Inform., 2017.
[45] A. M. Sagheer, S. S. Al-Rawi, and O. A. Dawood, "Proposing of developed advance encryption standard," in Proc. Develop. E-Syst. Eng., Dec. 2011, p. 1.
[46] N. Tiwari and A. Kumar, "Security effect on AES in terms of avalanche effect by using alternate S-box," in Proc. Int. Conf. Intell. Data Commun. Technol. Internet Things, 2018, pp. 1-14.
[47] F. Mohammad, A. E. Rohiem, and A. Elbayoumy, "A novel S-box of AES algorithm using variable mapping technique," in Proc. Int. Conf. Aerosp. Sci. Aviation Technol., vol. 13, May 2009, pp. 1-10.
[48] S. Mahmood, S. Farwa, M. Rafiq, S. M. J. Riaz, T. Shah, and S. S. Jamal, "To study the effect of the generating polynomial on the quality of nonlinear components in block ciphers," Secur. Commun. Netw., vol. 2018, pp. 1-8, Apr. 2018.
[49] A. Alabaichi and A. I. Salih, "Enhance security of advance encryption standard algorithm based on key-dependent S-box," in Proc. 5th Int. Conf. Digit. Inf. Process. Commun. (ICDIPC), Oct. 2015, pp. 44-53.
[50] F. J. D'Souza and D. Panchal, "Advanced encryption standard (AES) security enhancement using hybrid approach," in Proc. Int. Conf. Comput., Commun. Autom. (ICCCA), May 2017, pp. 647-652.
[51] J. Juremi, R. Mahmod, and S. Sulaiman, "A proposal for improving AES S-box with rotation and key-dependent," in Proc. Int. Conf. Cyber Secur., Cyber Warfare Digit. Forensic (CyberSec), Jun. 2012, pp. 38-42.
[52] V. Ramaswamy, "Making AES stronger?: AES with key dependent S-box," Int. J. Comput. Sci. Netw. Secur, vol. 8, no. 9, pp. 388-398, 2008.
[53] E. M. Mahmoud, "Dynamic AES-128 with key-dependent S-box," Int. J. Eng. Res. Appl., to be published.
[54] N. Stoianov, "One approach of using key-dependent S-BOXes in AES," in Proc. Int. Conf. Multimedia Commun., Services Secur., 2011, pp. 317-323.
[55] G. Jacob, A. Murugan, and I. Viola, "Towards the generation of a dynamic key-dependent S-box to enhance security," IACR Cryptol. ePrint Arch., vol. 2015, p. 92, Feb. 2015.
[56] K. Kazlauskas and J. Kazlauskas, "Key-dependent S-box generation in AES block cipher system," Informatica, vol. 20, no. 1, pp. 23-24, 2009.
[57] N. Bacaër and N. Bacaër, "Verhulst and the logistic equation (1838)," in A Short History of Mathematical Population Dynamics. London, U.K.: Springer, 2011.
[58] R. M. May, "Simple mathematical models with very complicated dynamics," in Universality in Chaos: A Reprint Selection, 2nd ed. Nature, 2017. [Online]. Available: https://www.nature.com/articles/261459a0
[59] A. Rukhin, J. Soto, and J. Nechvatal, "A statistical test suite for random and pseudorandom number generators for cryptographic applications," in Proc. NIST, 2010, pp. 1-164.
[60] C. Blondeau and K. Nyberg, "Perfect nonlinear functions and cryptography," Finite Fields Their Appl., vol. 32, pp. 120-147, Mar. 2015.
[61] S. Gao, W. Ma, and J. Zhu, "Nonlinearity profile test for an S-box," in Future Wireless Networks and Information Systems (Lecture Notes in Electrical Engineering). Berlin, Germany: Springer, 2012.
[62] A. F. Webster and S. E. Tavares, "On the design of S-boxes," in Advances in Cryptology-CRYPTO (Lecture Notes in Computer Science: Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics). Berlin, Germany: Springer, 1986.
[63] Y. Wang, Q. Xie, Y. Wu, and B. Du, "A software for S-box performance analysis and test," in Proc. Int. Conf. Electron. Commerce Bus. Intell., Jun. 2009, pp. 125-128.
[64] E. Biham and A. Shamir, "Differential cryptanalysis of DES-like cryptosystems," J. Cryptol., vol. 4, no. 1, pp. 3-72, 1991.
[65] F. Özkaynak, "An analysis and generation toolbox for chaotic substitution boxes: A case study based on chaotic labyrinth rene thomas system," Iranian J. Sci. Technol., Trans. Elect. Eng., to be published.
[66] S. Picek, L. Batina, D. Jakobović, B. Ege, and M. Golub, "S-box, SET, match: A toolbox for S-box analysis," in Information Security Theory and Practice. Securing the Internet of Things (Lecture Notes in Computer Science: Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics). Springer, 2014. [Online]. Available: https://link.springer.com/chapter/10.1007/978-3-662-43826-8_10
[67] A. Biryukov and J. Großschädl, "Cryptanalysis of the full AES using GPUlike special-purpose hardware," Fundam. Informaticae, vol. 114, nos. 3-4, pp. 221-237, 2012.
[68] S. Ali, X. Guo, R. Karri, and D. Mukhopadhyay, "Fault attacks on AES and their countermeasures," in Secure System Design and Trustable Computing. Cham, Switzerland: Springer, 2015, pp. 163-208.
[69] F. Özkaynak, "On the effect of chaotic system in performance characteristics of chaos based S-box designs," Phys. A, Stat. Mech. Appl., Jan. 2020, Art. no. 124072. [Online]. Available: https://www.sciencedirect.com/ science/article/pii/S0378437119322514


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