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# Generative Adversarial Network-based Postfilter for STFT Spectrograms

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### Abstract

We propose a learning-based postfilter to reconstruct the high-fidelity spectral texture in short-term Fourier transform (STFT) spectrograms. In speech-processing systems, such as speech synthesis, voice conversion, and speech enhancement, the STFT spectrograms have been widely used as key acoustic representations. In these tasks, we normally need to precisely generate or predict the representations from inputs; however, generated spectra typically lack the fine structures close to the true data. To overcome these limitations and reconstruct spectra having finer structures, we propose a generative adversarial network (GAN)-based postfilter that is implicitly optimized to match the true feature distribution in adversarial learning. The challenge with this postfilter is that a GAN cannot be easily trained for very high-dimensional data such as the STFT. Therefore, we introduce a divide-and-concatenate strategy. We first divide the spectrograms into multiple frequency bands with overlap, train the GAN-based postfilter for the individual bands, and finally connect the bands with overlap. We applied our proposed postfilter to a DNN-based speech-synthesis task. The results show that our proposed postfilter can be used to reduce the gap between synthesized and target spectra, even in the highdimensional STFT domain.

**Index Terms**: postfilter, deep neural network, generative adversarial network, statistical parametric speech synthesis

# 1. Introduction

The aim of many speech-processing systems, including speech synthesis, conversion, enhancement, separation, and coding, is to produce speech with quality indistinguishable from clean and real speech. However, the quality of synthesized or processed speech is usually not as good as that of real speech. In this paper, we address the problem of restoring spectro-temporal fine details of a synthetic speech signal to make it sound like real speech.

Many methods for statistical parametric speech synthesis and voice conversion tend to produce over-smoothed spectra, which often result in muffled and buzzy synthesized speech. This is caused by a side effect of assuming a particular form of loss function (e.g., mean squared error) or distribution (e.g., Gaussian) for parameter training of the acoustic model. Conventionally, postfiltering methods based on the global variance [1, 2] or the modulation spectrum [3] have proved to be effective in improving the intelligibility of synthesized speech.

Meanwhile, speech enhancement and separation are typically carried out using a Wiener filter or a time-frequency mask. While a time-frequency mask allows aggressive suppression of noise components, it can also over-suppress and damage speech components. A Wiener filter provides a conservative way of separating out a speech signal from a mixture signal so that the sum of the separated signals is ensured to be equal to the mixture; however, it often produces artifacts perceived as timevarying tones known as musical noise. To reduce artifacts or musical noise in processed speech, postprocessing methods using cepstral smoothing techniques have been proposed [4].

In both the fields, the limitations of many current postprocessing methods are to rely on heuristics due to the difficulty of modeling the exact probability density of the spectrograms of real speech. This typically causes generated spectra to lack the fine structures close to the true data. Recently, some studies proposed learning-based postfilters [5, 6], but they designed them with a particular form of loss function or distribution, which still resulted in over-smoothing.

To overcome this limitation, in our previous study [7], we proposed the use of a generative adversarial network (GAN) [8], which makes it possible to generate random samples following the underlying data distribution without the need for the explicit form of its density, to construct a postfilter for acoustic-feature sequences generated using a deep neural network (DNN)-based text-to-speech (TTS) synthesizer. In the previous study, we showed the effectiveness of our postfilter for a sequence of lowdimensional vocoder features (typically around forty dimensions), such as the mel-cepstral features, as the inputs and outputs, but its effectiveness for a sequence of high-dimensional features, such as the short-time Fourier transform (STFT) spectra, has not been clarified.

Motivated by this background, in this paper, we explore a GAN postfilter that allows the handling of high-dimensional features, such as the STFT spectra, so that our postfilter can be applied to any speech-processing system (not limited to speech synthesis) that produces the spectrograms of speech. To the best of our knowledge, this is the first study to explore such a postfilter. This is particularly useful and convenient because once a magnitude spectrogram is obtained, we can use phase-reconstruction algorithms [9] to reconstruct a time-domain waveform signal.

In a previous study [10], we proposed a DNN-based TTS system that directly produces a sequence of STFT spectra. In the experiments, we applied our proposed postfilter to the STFT spectrograms generated using our DNN-TTS system. The experimental results revealed that the use of the proposed postfilter had a certain effect in reducing the gap between synthesized and target spectra, even in the high-dimensional STFT domain.

This paper is organized as follows. In Section 2, we explain the proposed GAN-based postfilter for the STFT spectrograms. In Section 3, we explain how we used it in our DNN-based TTS system. We present the experimental results in Section 4 and summarize our findings in Section 5.

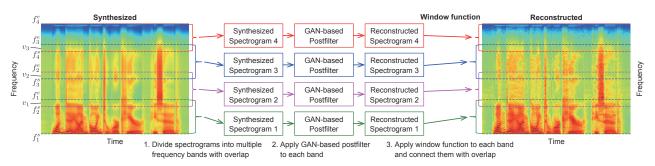


Figure 1: System overview of proposed GAN-based postfilter for high-dimensional STFT spectrograms.

# 2. GAN-based postfilter

Our proposed postfilter is built upon the GAN-based postfilter that we previously proposed [7]. In this section, we first briefly review a GAN then explain the formulation for using a GAN as a postfilter for vocoder parameters. Next, we introduce our proposed postfilter that can be applied to the high-dimensional STFT spectrograms.

#### 2.1. GAN

A GAN [8] is a framework for estimating a generative model by an adversarial process, and the goal is to learn a generator distribution  $P_G(x)$  that matches the true data distribution  $P_{\text{Data}}(x)$ . It is composed of two networks. One is a generator G that maps noise variables  $z \sim P_{\text{Noise}}(z)$  to the data space x = G(z). The other is a discriminator D that assigns probability p = D(x)when x is sampled from the  $P_{\text{Data}}(x)$  and assigns probability 1 - p when x is sampled from the  $P_G(x)$ . The D and G play a two-player minmax game, and the GAN objective is written as

$$\min_{G} \max_{D} \mathbb{E}_{x \sim P_{\text{Data}}(x)}[\log D(x)] \\
+ \mathbb{E}_{z \sim P_{\text{Noise}}(z)}[\log(1 - D(G(z)))].$$
(1)

This encourages D to find the binary classifier that gives the best possible discrimination between true and generated data and simultaneously encourages G to fit the  $P_{\text{Data}}(x)$ . Both G and D can be trained using back-propagation.

#### 2.2. GAN-based postfilter for vocoder parameters

By using the functionality of a GAN that is implicitly optimized to match the true feature distribution in adversarial learning, we previously proposed a GAN-based postfilter to reconstruct the true spectral texture generated using a vocoder [7]. We made three changes to the naive GAN architectures of conditional, residual, and convolutional to use a GAN for postfiltering.

**Conditional:** Our goal is to reconstruct spectral texture x from synthesized spectral texture y and random noise z. To achieve this, we use a conditional GAN (CGAN) [11, 12], which is an extension of a GAN, where G and D receive additional data y as input:

$$\min_{G} \max_{D} \mathbb{E}_{x,y \sim P_{\text{Data}}(x,y)} [\log D(x,y)]$$

$$+ \mathbb{E}_{z \sim P_{\text{Noise}}(z), y \sim P_{y}(y)} [\log(1 - D(G(z,y),y))].$$
(2)

Here, z represents the stochastic fluctuation in reconstructing a natural spectral texture from a synthesized one.

**Residual:** To shorten the entire process of generating the spectral texture, we design G as follows

$$G(x,y) = y + T(x,y),$$
(3)

#### where T represents residual texture [13].

**Convolutional:** Based on the observation that a spectral texture is structured in both time and frequency directions, we use convolutional architecture to determine the structure with reasonably small number of parameters. In particular, we design G as a fully convolutional network (FCN) [14] that allows input segments to take an arbitrary length.

#### 2.3. GAN-based postfilter for STFT spectrograms

The STFT spectrogram is not only high dimensional but also has a different structure depending on the frequency bands, e.g., a clear harmonic structure can be observed in the low-frequency band, while randomness increases in the high-frequency band. This makes it difficult to estimate the spectrogram distribution with the naive GAN-based postfilter described in Section 2.2. To mitigate this problem, we propose a method that converts the spectrograms on a band-by-band basis. With this method, we divide the spectrograms into multiple bands, train a single GAN-based postfilter for each of the individual bands, which is easier to model than the entire band, and finally concatenate them. The system overview is summarized in Figure 1. We describe the details of each step as follows.

**Dividing:** We first divide the spectrograms into N frequency bands, each of which ranges from the  $f_i^s$ -th to the  $f_i^e$ -th frequency, where N is the number of bands and  $i = \{1, ..., N\}$ . During division, we make the bands overlap with  $v_i$  bands, where  $v_i$  is the overlap length between the *i*-th band and (i+1)-th bands, i.e.,  $v_i = f_i^e - f_{i+1}^s$ . We use the overlap representation to smoothly concatenate the individual bands afterwards.

**Postfiltering:** For each band, we train the GAN-based postfilter that we describe in Section 2.2. In the divided band, a spectrogram is not only lower dimensional but also has a more homogeneous structure than that in the entire band; therefore, it is easier to model.

**Concatenating:** In order to connect the reconstructed spectrograms smoothly, we apply a window function (e.g., hanning, hamming, or blackman) to both ends of each band before connection, where the window width is  $2v_i$  and half of the window function is applied to each end. This method works well, as shown in the reconstructed spectrogram in Figure 1. In preliminary experiments, we also tested a model in which the spectrograms are divided and connected without overlap. In this model, the reconstructed spectrogram tends to have discontinuity between the bands, causing a popping sound.

## 3. Application to speech synthesis

In this section, we describe how we used the proposed GANbased postfilter in a DNN-based speech synthesis system. The improved components include direct STFT-spectra prediction

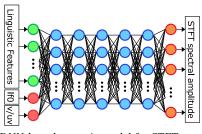


Figure 2: DNN-based acoustic model for STFT spectra. In this model,  $\log F_0$  and voiced/unvoiced values are used as input features as well as linguistic features.

from text, postfiltering of the predicted STFT spectra, and waveform generation using enhanced STFT spectra and phase recovery.

**Direct STFT-spectra prediction from text:** Figure 2 shows the acoustic model used in this study. This acoustic model directly predicts STFT spectra based on a feed-forward NN [10]. Also, in contrast to a conventional DNN-based acoustic model [15], we use  $F_0$  information as input features as well as linguistic features to predict STFT spectra, which include harmonics derived from  $F_0$ . To use the benefit of directly using the STFT spectra, the Kullback-Leibler divergence (KLD)-based criterion [16, 17] is used for training a DNN effectively. According to our previous experiment [10], this DNN-based TTS system leads to better quality of synthetic speech than that generated from a system using a vocoder.

**Postfiltering of predicted STFT spectrograms:** The predicted STFT spectra from the acoustic model is enhanced by a signal-processing-based postfilter first [18], followed by the proposed GAN-based postfilter described in the previous section. We apply the signal-processing-based postfilter for enhancing spectral peaks of predicted spectral amplitudes as follows. 1) The predicted STFT spectral amplitudes are converted into linear-scale cepstrum vectors that have the same dimensions as the STFT amplitudes, 2) the postfilter is applied to the cepstrum vectors for peak enhancement, and 3) the cepstrum vectors after postfiltering are converted back into the spectral amplitudes. These modified STFT spectra are then used as input features for the proposed GAN-based postfilter.

**Waveform generation based on phase recovery:** The speech-waveform generated from postfiltered STFT spectral amplitudes is based on a well-known phase-recovery algorithm proposed by Griffin and Lim [9]. The algorithm consists of the following iterative steps.

- 1. Generate initial speech waveforms using inverse STFT of predicted STFT spectral amplitudes A with or without a postfilter and random phase  $\theta$  at each frame, followed by overlap-add operations.
- 2. Window the speech waveforms and apply STFT at each frame to obtain new spectral amplitudes  $\widehat{A}$  and phase values  $\widehat{\theta}$ .
- 3. Replace the STFT  $\hat{A}$  with the original A at each frame.
- 4. Generate a new speech waveforms using inverse STFT of original STFT spectral A and updated  $\hat{\theta}$  followed by overlap-add operations.
- 5. Go back to step 2 until convergence.

This framework may be considered as another type of DNN-based speech synthesis system without a vocoder to avoid quality deterioration such as buzziness caused by using the vocoder. In this framework, the quality of synthetic speech

Table 1: Network architectures for proposed GAN-based postfilter.

<b>Generator</b> (Input: $F \times T$ spectrogram $+ F \times T$ noise)
$5 \times 5128$ conv., ReLU + input spectrogram $5 \times 5256$ conv., ReLU + input spectrogram
$5 \times 5128$ conv., ReLU + input spectrogram $5 \times 51$ conv.
<b>Discriminator</b> (Input: $F \times T_c$ spectrogram)
<b>Discriminator (Input:</b> $F \times T_c$ spectrogram) $5 \times 5.64$ conv. $\downarrow$ , LReLU
$5 \times 5.64 \text{ conv.} \downarrow$ , LReLU
$5 \times 5.64 \text{ conv.} \downarrow$ , LReLU $5 \times 5.128 \text{ conv.} \downarrow$ , BNorm, LReLU

totally relies on the prediction accuracy of the STFT spectra. We observed that refinement of the amplitudes using a signalprocessing-based postfilter improves the synthesized speech quality [10], but the filter is designed to enhance the peak frameby-frame; hence, the characteristics of STFT spectra, i.e., timefrequency dependency, are not appropriately considered. Therefore, the proposed GAN-based postfilter for STFT spectra is expected to further improve the synthesized speech quality.

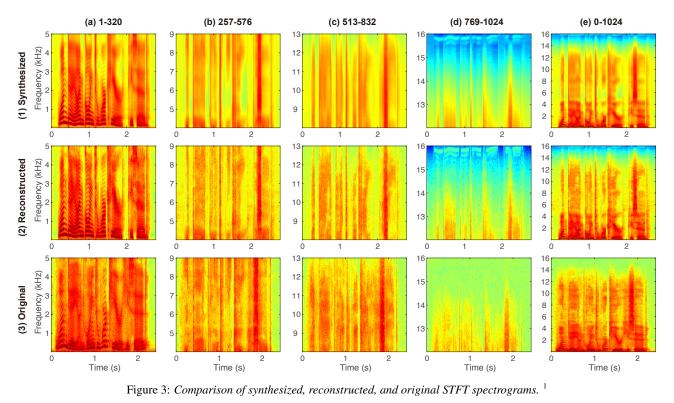
# 4. Experiments

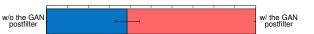
## 4.1. Experimental conditions

The database that was provided for the Blizzard Challenge 2011 [19], which contains approximately 17 hours of speech data composed of 12K utterances, was used for the experiment. All data were down-sampled from 48 to 32 kHz. Two hundred sentences not included in the database were used as a test set. The speech data were windowed at a frame rate of 160 points to extract their frequency spectra with 1025 STFT points. The linguistic features for the DNN-based acoustic model were composed of 396 dimensions. The log  $F_0$  and voiced/unvoiced values were also used as input features of the DNN-based acoustic model. Five-hidden-layers feed-forward neural networks with a sigmoid based activation function were used for the acoustic model.

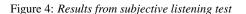
In the synthesis phase, we used  $\log F_0$  and voiced/unvoiced values predicted from a conventional vocoder-based system [20] as input features. In this conventional system, 49-dimensional bark-cepstral coefficients (plus the 0th coefficient) obtained from WORLD spectra,  $\log F_0$ , 25-dimensional band aperiodicity measures, their dynamic and acceleration coefficients, and voice/unvoiced values were modeled based on a DNN. Phoneme durations were estimated by a HMM-based speech synthesis system [21].

In the postfiltering phase, we divide the spectrogram into four frequency bands. We set the parameters as  $(f_1^s, f_1^e) =$  $(1, 320), (f_2^s, f_2^e) = (257, 576), (f_3^s, f_3^e) = (513, 832)$ , and  $(f_4^s, f_4^e) = (769, 1024)$ . We connected the bands with the hamming-window function where the window width was 128. Table 1 lists the network architectures for the GAN-based postfilter. The symbol  $\downarrow$  indicates down-sampling with stride 2. The terms *ReLU*, *LReLU*, and *BNorm* indicate rectified linear unit [22], leaky rectified linear unit [23, 24], and batch normalization [25], respectively. As the input of the *G*, we used the  $F \times T$  spectrograms and the same-sized noise where *F* is the frequency length and *T* is the frame length. The *F* was 320 for





1.0



0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

the first, second, third bands, and 256 for the fourth band. We designed the G as an FCN, so we could take inputs of arbitrary T. In the D, we used a fully connected architecture at the last layer, so we fixed the input size as  $F \times T_c$  with  $T_c = 64$ . During pre-processing, we normalized spectrograms to zero-mean and unit-variance for each dimension using the training sets. We trained our postfilter using an Adam optimizer [26] with a minibatch of size 16. The learning rate was set to 0.0002 for the D and 0.001 for the G and the momentum term was set to 0.5.

#### 4.2. Objective evaluation

Figure 3 shows the comparison of the synthesized, reconstructed, and original STFT spectrograms. From (a) to (d), we show the spectrograms in the individual frequency bands. These results indicate that the proposed GAN-based postfilter can not only emphasize the harmonic structure but also reproduce the detailed structures that are similar to those in the original spectrograms from the over-smoothed synthesized spectrograms. Figure 3(e) shows all spectrograms. Our postfilter enables the individual frequency bands to be connected smoothly.

#### 4.3. Subjective evaluation

We conducted a listening test to compare a DNN-based TTS system with the proposed GAN-based postfilter with another

DNN-based TTS system without the filter. The listening test that we used is a normal preferences test. The test was conducted in acoustically isolated quiet booths, and 18 native speakers of English have participated in the test. They were presented pairs of synthetic speech with or without the proposed filter in random order, read out one of 8 sentences randomly chosen from 200 test sentences, and were asked to choose a sample that had better audio quality per pair.

The results of the preference test are shown in Figure 4. The preference score of the DNN-based TTS system with the proposed postfilter was better than that of the system without the filter, and the difference was statistically significant according to *t*-test (p<0.01). Hence, we can conclude that the proposed GAN-based postfilter can improve the quality of synthetic speech based on STFT representation.

# 5. Conclusions

We proposed a learning-based postfilter to reconstruct the highfidelity spectral texture in STFT spectrograms. To achieve this, we first divide the spectrograms into multiple frequency bands with overlap, train our proposed GAN-based postfilter for the individual bands, and concatenate the bands using the window functions. In the experiments, we applied our postfilter to a DNN-based TTS system. Objective evaluation of the results showed that the proposed postfilter can reproduce the fine structures close to the true data without discontinuity. Moreover, subjective evaluation showed that our proposed postfilter significantly improves speech quality. For future work, we plan to apply our proposed postfilter to other tasks such as voice conversion and speech enhancement.

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<sup>&</sup>lt;sup>1</sup>In synthesizing speech, the utterance length is predicted in our acoustic model; therefore, it does not always match the original length. This means the time length of the original spectrograms does not exactly match those of the synthesized and reconstructed spectrograms. We manually adjusted them for ease of viewing in the figure.

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