Generic Operational Characteristics of Piezoelectric Transformers

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Abstract - The universal attributes of piezoelectric transformers (PT) were derived by an approximate analysis that yielded closed form equations relating the normalized load resistance to the voltage gain, output power per unit drive and efficiency. It is suggested that the closed form formulas developed in this study could be invaluable when studying, specifying and designing practical PTs applications.

I. INTRODUCTION

Piezoelectric Transformers (PT) have some advantages over electromagnetic transformers in specific applications [1-5]. Notwithstanding the fact that practical applications of PTs have been described in the literature, clear delineation of the engineering characterization of these devices is still missing. For example, the questions of trade-offs between voltage gain and efficiency, maximum per unit output power and the effect of the quality factor of the PT on its performance - have not been thorough analyzed as yet (at least in the open literature). These issues were probed in this study and the theoretical results, verified by measurements and simulations, provide tools for evaluating the expected performance of a PT in a given application. Although this paper is concerned with PTs in inverter (AC-AC) applications, the study can be used to assess the PT performance in converter applications (AC-DC) by applying the equivalent AC resistance approach (R_{ac}) [6] or the RC model for a converter with a capacitive filter [7, 8].

II. AN INTUITIVE ANALYSIS

The general equivalent circuit of a PT when operating around one of its mechanical resonant frequencies is depicted in Fig. 1a. In this figure we replaced the output transformer shown in earlier papers, by two dependent sources: $\frac{v_{Co}}{n}$ and $\frac{i_{Lr}}{n}$. This presentation is valid even when the output is exposed to a DC voltage. The electromagnetic transformer presentation (used by other authors) would be undesirable in such a case, especially when the equivalent circuit is studied by circuit simulation. This is due to the fact that the windings of an electromagnetic transformer represent a short circuit to DC voltage.

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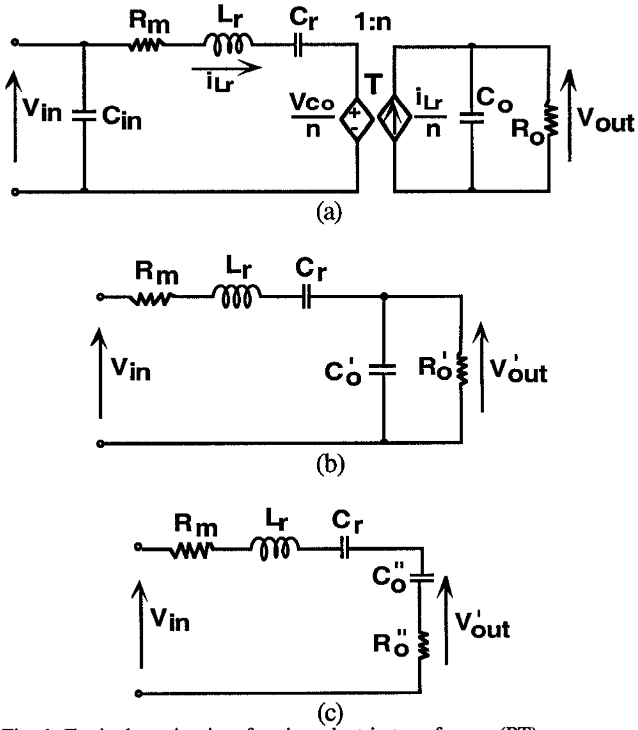

Fig. 1. Equivalent circuits of a piezoelectric transformer (PT): (a) general model; (b) after reflecting the output capacitance and load resistance to the primary; (c) after parallel to series transformation.

From the power transfer point of view the basic equivalent circuit can be simplified to that of Fig. 1b in which the network at the secondary is reflected to the primary. Note that the input capacitance (C_{in} of Fig. 1a) is eliminated in Fig. 1b since it does not affect the power transfer of the PT. The values of the reflected resistance (R_o'), reflected capacitance (C_o') and reflected output voltage (V_{out}') will be :

$$R_o' = \frac{R_o}{n^2} \quad (1)$$

$$C_o' = n^2 C_o \quad (2)$$

$$V_{out}' = \frac{V_{out}}{n} \quad (3)$$

where R_o is the load resistance, C_o is the output capacitance, V_{out} is the output voltage and n is the mechanical output transfer ratio.

Further simplification can be achieved by converting the parallel network R_o',C_o' to a series network (Fig. 1c) in which the series resistance R_o'' and series capacitance C_o'' are defined as :

$$R_o'' = \frac{R_o'}{1+(\omega C_o' R_o')^2} \quad (4)$$

$$C_o'' = C_o' \frac{1+(\omega C_o' R_o')^2}{(\omega C_o' R_o')^2} \quad (5)$$

where ω is the operating frequency.

Examination of the dependence of R_o'' and C_o'' on R_o' reveals some interesting and important features. As R_o' varies from 0 to ∞, R_o'' varies from zero back to zero with a maximum R_{om}'' at :

$$R_{om}' = \frac{1}{\omega C_o'} \quad (6)$$

On the other hand, over the entire range of R_o', series capacitance C_o'' varies from infinity back to the value of C_o'.

Based on this simple observation some general conclusions can already be drawn:

1. For a given reflected load R_o', maximum output voltage will be obtained at the resonant frequency ω_m (Fig. 1):

   $$\omega_m = \frac{1}{\sqrt{L_r C_{eq}}} \quad (7)$$

   where C_{eq} is the series value of C_r and C_o'':

   $$C_{eq} = \frac{C_r C_o''}{C_r + C_o''} \quad (8)$$

2. The range of the series resonant frequency is dictated by the range of C_o'':

   $$\omega_{rs} < \omega_m < \omega_{ro} \quad (9)$$

   where ω_{rs} is the resonant frequency at short circuit (R_o=0):

   $$\omega_{rs} = \frac{1}{\sqrt{L_r C_r}} \quad (10)$$

   and ω_{ro} is the series resonant frequency at open circuit ($R_o=\infty$):

   $$\omega_{ro} = \frac{1}{\sqrt{L_r \frac{C_r C_o'}{C_r + C_o'}}} \quad (11)$$

3. For any given load R_o, output voltage can be controlled by shifting the frequency above or below ω_m. This is, in fact, the method used in inverters and converters operating in frequency-shift control mode.
4. For any given load (R_o) the fraction of power transferred to the load at the resonant frequency will depend on the ratio of R_o'' to R_m (Fig. 1c).
5. Maximum power will be delivered to the load when $R_o''=R_m$. Since R_o'' is convex, two R_o' (and hence two R_o) satisfy the maximum power condition.
6. At maximum output power ($R_o''=R_m$) the PT efficiency will be 0.5.
7. Maximum efficiency is obtained at the peak of R_o''.
8. Since maximum efficiency point corresponds to the maximum R_o'', it also corresponds to a local minimum of output power (per a given input voltage).

III. DETAILED ANALYSIS

A. Operating Frequency and Output to Input Voltage Ratio

The output to input voltage ratio k_{21} (Fig. 1) was found to be:

$$k_{21} = \frac{V_{out}'}{V_{in}} = \frac{1}{\sqrt{Y}} \quad (12)$$

where

$$Y=\left\{1-c\left[\left(\frac{\omega}{\omega_{rs}}\right)^2-1\right]+\frac{R_m}{R_o'}\right\}^2+\left\{\frac{\omega_{rs}}{\omega}\frac{c}{Q}\left[\left(\frac{\omega}{\omega_{rs}}\right)^2-1\right]+\frac{\omega}{\omega_{rs}}\frac{c}{Q_m}\right\}^2 \quad (13)$$

$$c = \frac{C_o'}{C_r} \quad (14)$$

$$Q = \omega_{rs} C_o R_o \quad (15)$$

$$Q_m = \frac{1}{\omega_{rs} C_r R_m} \quad (16)$$

Q is the electrical quality factor and Q_m is the mechanical quality factor.

Equation (12) implies that k_{21} has a maximum value (k_{21m}) when Y has a minimum value. Therefore, the frequency ratio $\frac{\omega_m}{\omega_{rs}}$ corresponding to k_{21m} can be found by setting derivative of the function (13) to zero. This was carried out two ways: by an exact and by an approximate analysis.

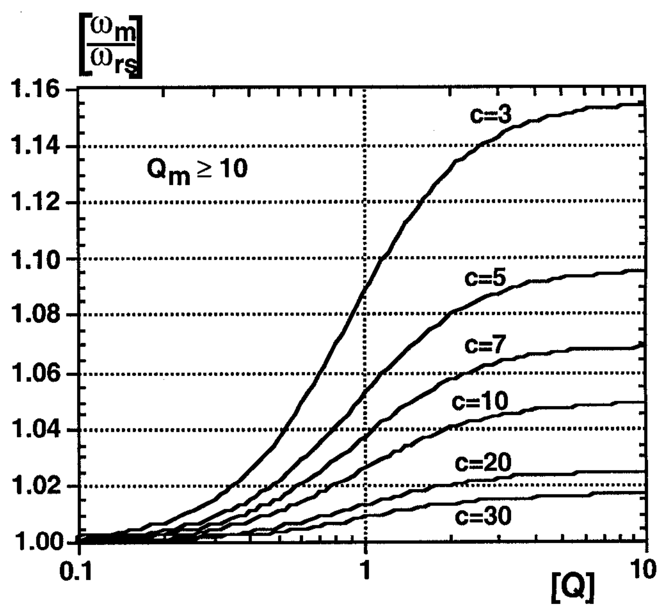

Fig. 2. Normalized operating frequency wm corresponding to the maximum output to input voltage ratio as a function of the electrical quality factor Q and capacitances ratio c for mechanical quality factor Qm≥10 (wrs is the resonant frequency in short circuit mode).

Exact analysis was based on the solution of the derivative of the third order equation (13) which can be presented by the canonical form:

$$x^3+[\frac{1}{2Q^2}+\frac{1}{2Q_m^2}-1-\frac{1}{c})]x^2-\frac{1}{2Q^2}=0 \qquad (17)$$

where

$$x=(\frac{\omega_m}{\omega_{rs}})^2 \qquad (18)$$

The frequency ratio $\frac{\omega_m}{\omega_{rs}}$ corresponding to the maximum value of k_{21} (k_{21m}), as a function of Q, Q_m and c was found from (17) and (18) applying the "Mathematica" software package [9]. This dependence is plotted in Fig. 2. It shows that an increase of Q from zero to infinity shifts ω_m from ω_{rs} to ω_{ro} as expected. The curves are valid for a large range of Q_m (10 to 1000) in which range the discrepancy is less than 0.22%.

By inserting the values of $\frac{\omega_m}{\omega_{rs}}$ into (13) and applying (12) we found the relationships between k_{21m} and circuit parameters Q, Q_m and c (Fig. 3).

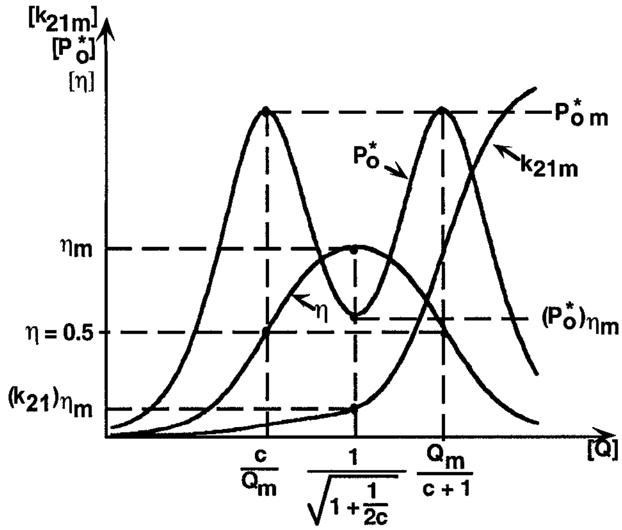

Fig. 3. Maximum value of the output to input voltage ratio k_{21m}, output power in per unit system P_o^* and efficiency η as a function of the electrical quality factor Q for c=const and Q_m=const.

The results of the exact analysis, given above, are presented by tables and graphs. The results of the approximate analysis given below, are presented in closed form which is more convenient for design procedures. The approximation is based on the fact that the derivatives some terms in (17) with respect to x are smaller than others and hence could be considered constant.

Taking into account the fact that near resonance $\{(\frac{\omega}{\omega_{rs}})^2-1\}$ is changing much more rapidly as a function of $(\frac{\omega}{\omega_{rs}})$ than does $(\frac{\omega}{\omega_{rs}})$, we replace here the first order multipliers $(\frac{\omega}{\omega_{rs}})$ and $(\frac{\omega_{rs}}{\omega})$ in (13) by as yet unknown constants:

$$g=\frac{\omega}{\omega_{rs}} \text{ and } \frac{1}{g}=\frac{\omega_{rs}}{\omega} \qquad (19)$$

Hence, (13) can be transformed into a following form:

$$Y=\{1-c[(\frac{\omega}{\omega_{rs}})^2-1]+\frac{R_m}{R_o'}\}^2+\{\frac{c}{gQ}[(\frac{\omega}{\omega_{rs}})^2-1]+\frac{gc}{Q_m}\}^2 \qquad (20)$$

Taking the derivative of (20) and equating it to zero, we solved the equation for the frequency ratio $\frac{\omega_m}{\omega_{rs}}$ corresponding to the maximum value of output to input voltage ratio k_{21m}:

$$\frac{\omega_m}{\omega_{rs}}=\sqrt{1+\frac{C_r}{C_o'}\frac{(\frac{\omega_m}{\omega_{rs}})^2Q^2}{1+(\frac{\omega_m}{\omega_{rs}})^2Q^2}} \qquad (21)$$

or

$$\frac{\omega_m}{\omega_{rs}}=\sqrt{1+\frac{C_r}{C_o'}\sin^2\varphi_m} \qquad (22)$$

where φ_m is the phase angle of the parallel circuit $R_o'C_o'$ (Fig. 1b) at the frequency ω_m corresponding to the maximum value of the output to input voltage ratio k_{21m}:

$$\varphi_m=\tan^{-1}(Q\frac{\omega_m}{\omega_{rs}})=\tan^{-1}(\omega_m C_o' R_o') \qquad (23)$$

Applying (14) we define from (21) the expression of $\frac{\omega_m}{\omega_{rs}}$ in a convenient form for calculation:

$$\frac{\omega_m}{\omega_{rs}}=\sqrt{0.5(1+\frac{1}{c}-\frac{1}{Q^2})+\sqrt{0.25(1+\frac{1}{c}-\frac{1}{Q^2})^2+\frac{1}{Q^2}}} \qquad (24)$$

Note that the values of $\frac{\omega_m}{\omega_{rs}}$ obtained by the approximate analysis are independent of $\frac{R_m}{R_o'}$ (the terms including $\frac{R_m}{R_o'}$ in (20) were reduced during the mathematical transformation).

Inserting (22) into (13) and applying (12), (14)-(16) and (23), we define now expressions, that are convenient for calculation, for the maximum value of the output to input voltage transfer ratio k_{21m}:

$$k_{21m}=\frac{1}{\cos\varphi_m+\frac{R_m}{R_o'\cos\varphi_m}}=\frac{1}{\cos\varphi_m+\frac{c}{Q_mQ\cos\varphi_m}} \qquad (25)$$

Detailed comparison between simulation, exact analysis and approximate formulas for Q in the range of 0.01-100, Q_m in the range of 10-1000 and c in the range 0.5-50 reveal that the maximum discrepancy is smaller than 4.5%. In most operational regions, however, the agreement was found to be better than 0.1%. We believe therefore that the approximate (closed form) equations derived in this study are more than sufficient from the engineering point of view.

B. Output Power and Efficiency

The output power P_o can be calculated from the following expression:

$$P_o = \frac{(k_{21m}V_{in})^2}{R_o'} \qquad (26)$$

or in per unit system (taking into account (14) and (15)):

$$P_o^* = \frac{P_o}{P_{bas}} = \frac{ck_{21m}^2}{Q} \qquad (27)$$

where P_{bas} is the base power unit:

$$P_{bas} = V_{in}^2\sqrt{C_r/L_r} \qquad (28)$$

Efficiency can be found from [5]:

$$\eta = \frac{R_o''}{R_o''+R_m} \qquad (29)$$

where R_o'' is the reflected load resistance in the equivalent series circuit $R_o''C_o''$ (Fig. 1c).

Applying (4) and (14)-(16) we transform (29) to obtain the expression for η in a form that is convenient for calculation:

$$\eta = \frac{1}{1+\frac{c}{Q_m}\left[\frac{1}{Q}+\left(\frac{\omega}{\omega_{rs}}\right)^2 Q\right]} \qquad (30)$$

The values of P_o^* and η, calculated from (27), (25) and (30) as a function of the electrical quality factor Q for c=const and Q_m=const, are plotted in Fig. 3. These graphs reveal three extremes. Two of them correspond to the equal-height peaks of the output power $P_o^*=P_{om}^*$. The third extreme point corresponds to the maximum efficiency η_m and a local minimum per unit output power $(P_o^*)_{\eta_m}$.

To derive the location of these extreme points we find first the relationship between the output power P_o^* and efficiency η. Applying equations (27) and (29) of P_o^* and η and taking into account (23) and (25) we obtain:

$$P_o^* = \frac{c}{Q}\,\frac{1+Q^2\left(\frac{\omega_m}{\omega_{rs}}\right)^2}{1+\frac{2R_m}{R_o''}+\left(\frac{R_m}{R_o''}\right)^2} \qquad (31)$$

$$\frac{R_m}{R_o''} = \frac{1}{\eta}-1 \qquad (32)$$

from where

$$\frac{P_o^*}{\eta^2} = c\left[\frac{1}{Q}+Q\left(\frac{\omega_m}{\omega_{rs}}\right)^2\right] \qquad (33)$$

Inserting (33) into (30) we find:

$$\frac{P_o^*}{Q_m} = \eta(1-\eta) \qquad (34)$$

This relationship is plotted on Fig. 4. It is clear that maximum output power P_{om}^* corresponds to the efficiency η=0.5 and hence:

$$P_{om}^* \approx 0.25\,Q_m \qquad (35)$$

Inserting η=0.5 into (30) we find the following equation for the power peak points:

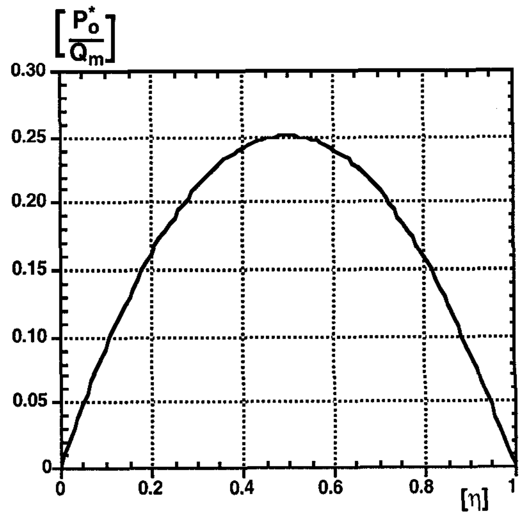

Fig. 4. Output power in per unit system P_o^* as a function of the efficiency η (Q_m is the mechanical quality factor).

$$Q^2 - \frac{Q_m}{c\left(\frac{\omega_m}{\omega_{rs}}\right)^2}Q + \frac{1}{\left(\frac{\omega_m}{\omega_{rs}}\right)^2} = 0 \qquad (36)$$

This equation has two solutions:

$$Q_1 = \frac{1}{\frac{Q_m}{2c}+\sqrt{\left(\frac{Q_m}{2c}\right)^2-\left(\frac{\omega_m}{\omega_{rs}}\right)^2}} \qquad (37)$$

$$Q_2 = \left[\frac{Q_m}{2c}+\sqrt{\left(\frac{Q_m}{2c}\right)^2-\left(\frac{\omega_m}{\omega_{rs}}\right)^2}\,\right]\left(\frac{\omega_{rs}}{\omega_m}\right)^2 \qquad (38)$$

We simplify these solutions taking into account the following conditions:

- $\left(\frac{Q_m}{2c}\right)^2$ is usually much larger than $\left(\frac{\omega_m}{\omega_{rs}}\right)^2$;

- the value of Q_2 is practically very large and therefore $\left(\frac{\omega_m}{\omega_{rs}}\right)^2 \approx 1+\frac{1}{c}$ (see (21) and Fig. 2).

Under these conditions we obtain:

$$Q_1 \approx \frac{c}{Q_m} \qquad (39)$$

$$Q_2 \approx \frac{Q_m}{c+1} \qquad (40)$$

Now we consider the extreme point corresponding to the maximum value of the efficiency η_m. Analysis of (30) shows that $\eta=\eta_m$ when

$$Q\frac{\omega_m}{\omega_{rs}} = 1 \qquad (41)$$

i. e. when $\varphi_m=\frac{\pi}{4}$ (see (23)). The values of $\frac{\omega_m}{\omega_{rs}}$ and Q corresponding to η_m are defined from (22) and (41):

$$\frac{\omega_m}{\omega_{rs}} = \sqrt{1+\frac{1}{2c}} \tag{42}$$

$$Q = \frac{1}{\sqrt{1+\frac{1}{2c}}} \tag{43}$$

Inserting (42) and (43) into (30) we obtain the expression of the maximum efficiency:

$$\eta_m = \frac{1}{1+\frac{2c}{Q_m}\sqrt{1+\frac{1}{2c}}} \tag{44}$$

Inserting (44) into (34) we obtain the output power at the point corresponding to the maximum value of the efficiency η_m:

$$(P_o{}^*)_{\eta_m} = \frac{2c\sqrt{1+\frac{1}{2c}}}{\left(1+\frac{2c\sqrt{1+\frac{1}{2c}}}{Q_m}\right)^2} \tag{45}$$

In the case that $Q_m >> 2c$ the equation can be simplified to:

$$(P_o{}^*)_{\eta_m} \approx 2c\sqrt{1+\frac{1}{2c}} \tag{46}$$

C. Output to Input Voltage Ratio and Efficiency

Dependence between the output to input voltage ratio k_{21m} and efficiency η is found in the same manner as above. From (23) and (25) we define:

$$k_{21m} = \frac{\sqrt{1+\left(Q\frac{\omega_m}{\omega_{rs}}\right)^2}}{1+\frac{c}{Q_m Q}\left[1+\left(Q\frac{\omega_m}{\omega_{rs}}\right)^2\right]} \tag{47}$$

Inserting (47) into (30) we obtain:

$$\eta = \frac{k_{21m}}{\sqrt{1+\left(Q\frac{\omega_m}{\omega_{rs}}\right)^2}} \tag{48}$$

The value of k_{21m} corresponding to the efficiency peak point ($\eta=\eta_m$), defined as $(k_{21m})_{\eta_m}$, is found from (41) and (48):

$$(k_{21m})_{\eta_m} = \sqrt{2}\,\eta_m \tag{49}$$

or taking into account (44)

$$(k_{21m})_{\eta_m} \approx \frac{\sqrt{2}}{1+\frac{2c\sqrt{1+\frac{1}{2c}}}{Q_m}} \tag{50}$$

IV. PT WITH A MATCHING INDUCTOR CONNECTED IN PARALLEL TO THE OUTPUT TERMINALS

The current of the output capacitance C_o (Fig. 1a), is lowering the efficiency since it passes through R_m. Hence, the overall efficiency of a PT can be improved by compensating the reactive current of C_o by an inductor L_{mt} connected in parallel to the output terminals of PT (Fig. 5).

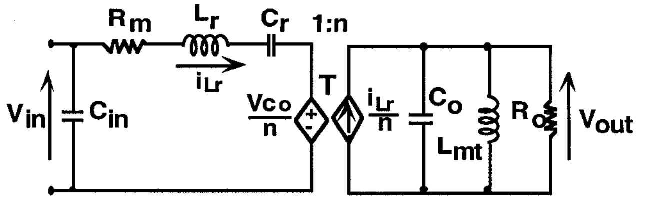

Fig. 5. Connection of a matching inductor L_{mt} in parallel to the output terminals of PT.

For a perfect compensation :

$$L_{mt} = \frac{1}{\omega^2 C_o} \tag{51}$$

where $\omega=\omega_{rs}$ is the operating frequency. The losses in the resistor R_m are lower in this case and therefore the efficiency of PT will be higher. In such a case, the equivalent resistances R_o'' and R_o' (Figs. 1c and 1b) have identical values and hence equation (29) of the PT efficiency is reduced to a following form:

$$\eta_L = \frac{1}{1+\frac{R_m}{R_o'}} \tag{52}$$

The ratio between the efficiency values in the two cases: without and with the matching inductor L_{mt} (η, and η_L) is found from (52), (29) and (4):

$$\frac{\eta}{\eta_L} = \frac{1+\frac{R_m}{R_o'}}{1+\frac{R_m}{R_o'}[1+(\omega C_o'R_o')^2]} = \frac{1}{1+\frac{(\omega C_o'R_o')^2}{1+\frac{R_o'}{R_m}}} \tag{53}$$

This relationship is shown in Fig. 6. We see that in general $\eta<\eta_L$, but for small R_m/R_o' and $\omega C_o'R_o'$ values we find $\eta\approx\eta_L$.

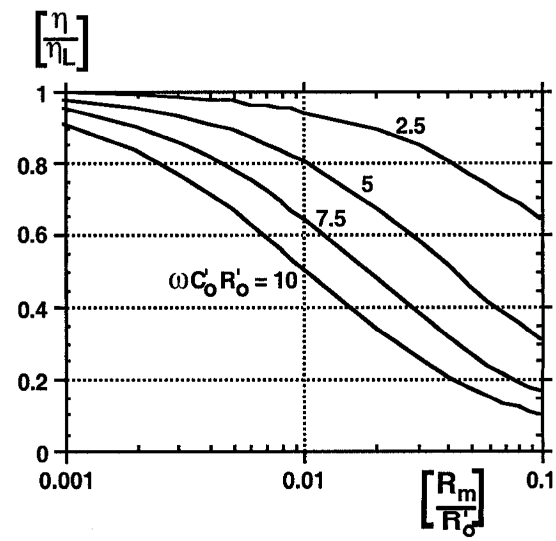

Fig. 6. Ratio between efficiency values when the matching inductor L_{mt} is missing (η) and when this inductor is present (η_L) as a function of R_m/R_o' and $\omega C_o'R_o'$.

Note that the matching inductor L_{mt}, operating in the resonant mode with C_o, decreases the output to input voltage transfer ratio k_{21m}:

$$(k_{21m})_L \approx \eta_L \quad (54)$$

To conserve the same output voltage V_{out}, input voltage V_{in} of PT must be increased.

V. EXPERIMENTAL

Philips piezoelectric transformer (RT 35x8x2 PXE43-S) with thickness polarization [5] was investigated experimentally. Its main parameters were measured to be (Fig. 1a): L_r=170mH, C_r=14.7pF, R_m=100.5Ω, C_o=500pF, n=0.988, $f_{rs}=\omega_{rs}/2\pi$=100.68kHz, $f_{ro}=\omega_{ro}/2\pi$ =102.18kHz. The experiments were carried out for a load resistance R_o range of 10 Ω to 540 kΩ. The input voltage V_{in} was a sine wave of 1 Vrms. It was found that the performance of this PT is practically independent of the magnitude of input voltage. For each measurement point, the frequency was adjusted to obtain the maximum output voltage V_{out_m}. Excellent agreement was found between the experimental results and the corresponding values predicted by the equations derived in this study for the voltage gain and output power (Fig. 7).

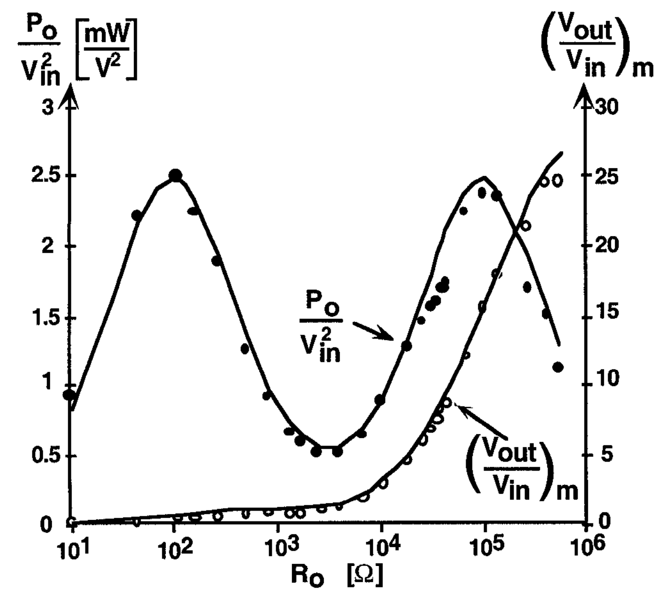

Fig. 7. Maximum value of the output voltage $V_{out\ m}$ and the output power P_o as a function of load resistance R_o: circles - experimental results; lines - theoretical prediction.

VI. DISCUSSION AND CONCLUSIONS

The generic requations developed in this study reveal some universal relationships between key parameters of a PT. These physical trade-off can be used two ways: to optimize the design of a PT for a given application and/or specifying the desired PT to fill given tasks. From the engineering point of view it is clear that the range for high efficiency operation for a PT is:

$$\frac{c}{Q_m} < Q < \frac{Q_m}{c+1} \quad (55)$$

Over this range the voltage transfer ratio is bound as follows:

$$0.5 < k_{21m} < \frac{0.5Q_m}{\sqrt{c(c+1)}} \quad (56)$$

Connection of a matching inductor L_{mt} in parallel to the output terminals of PT will increase the efficiency, but will decrease the output to input voltage ratio. The results displayed in Fig. 6 can be used to estimate the improvement in efficiency that can be gained by incorporating the compensating inductor. Trade-off between cost and performance can be examined.

The main conclusion of the paper is that the characteristics of a PT are described to a very good approximation by the equations developed in this study. These closed form formulas could be invaluable when studying, specifying and designing practical PTs applications.

REFERENCES

[1] C. Y. Lin and F. C. Lee, "Design of a piezoelectric transformer converter and its matching networks", *IEEE PESC'94 Record*, pp. 607-612.

[2] M. Imori, T. Taniguchi, H. Matsumoto, T. Sakai, "A photomultiplier high voltage power supply incorporating a piezoelectric ceramic transformer", *IEEE Trans. Nuclear Science*, vol. 43, no. 3, pt. 2, pp. 1427-1431, June 1996.

[3] P. J. M. Smidt and J. L. Duarte, "Powering neon lamps through piezoelectric transformers", *IEEE PESC'96 Record*, pp. 310-315.

[4] Y. Fuda, K. Kumasaka, M. Katsuno, H. Sato, Y. Ino, "Piezoelectric transformer for cold cathode fluorescent lamp inverter", *Jpn. J. Appl. Phys.*, vol. 36, pt. 1, no. 5B, pp. 3050-3052, May 1997.

[5] Philips,"Piezoelectric transformers", *Application note Philips Magnetic Products.* Date of release: 2/97.

[6] R. L. Steigerwald, "A comparison of half-bridge resonant converter topologies", *IEEE Trans. Power Electron.*, vol. 3, pp. 174-182, Apr. 1988.

[7] G. Ivensky, A. Kats, S. Ben-Yaakov, "A RC-model of parallel and series-parallel resonant dc-dc converters with capacitive output filter", *IEEE Trans. Power Electron.*, vol. 14, pp. 515-521, May 1999.

[8] G. Ivensky, M. Shvartsas, S. Ben-Yaakov, "Analysis and modeling of a piezoelectric transformer in high output voltage applications", *IEEE APEC'00 Record*, pp. 1081-1087.

[9] S. Wolfram, "Mathematica. A system for doing mathematics by computer", *Addison-Wesley Publishing Company, Inc.*, 1988.