Geodesic deviation equation for relativistic tops and the detection of gravitational waves

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In this contribution, we review the derivation of the relativistic top geodesic deviation equation. This equation results in a generalization of the geodesic deviation equation for a pair of nearby point particles. This property is taken into account in investigating the detection of gravitational waves, and we show how our generalized formula for a relativistic top can be used to study the gravitational wave backgrounds. Besides of these facts, we argue that our formulation may be of special interest for detecting the inflationary gravitational waves via the polarization of the cosmic background radiation.

Keywords: Geodesic equation; relativistic top; gravitational waves.

En esta contribución, revisamos la derivación de la ecuación de desviación de geodésicas para trompos relativistas. Esta ecuación es una generalización de la ecuación de geodésicas para un par de objetos puntuales cercanos. Esta propiedad es tomada en cuenta para investigar la detección de ondas gravitacionales, y mostramos cómo nuestra fórmula generalizada para un trompo relativista puede ser usada para estudiar los fondos de ondas gravitacionales. Además de estos hechos, argumentamos que nuestra formulación puede ser de especial interés para detectar las ondas gravitacionales inflacionarias a través de la polarización en la radiación cósmica de fondo.

Descriptores: Ecuación de geodésicas, trompo relativista; ondas gravitacionales.

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1. Introduction

The importance of the geodesic deviation equation (GDE) for spinless particles is evident when we study gravitational wave phenomena and their detection [1, 2]. In fact, all currently operating projects for the detection of gravitational waves, including LIGO [3], VIRGO [4] and LISA [5], have among their root physical bases such an equation. In this contribution, we review a recently published work [6], which is based on previous works [7] and [8], where it was proposed that the relativistic top equations of motion (RTEM) [9] can be used instead of the GDE for the same purpose of detecting gravitational radiation.

The main contribution of Ref. 6 is the derivation of a relativistic top deviation equation (RTDE), which is reduced to the GDE when the spin tensor associated with the top vanishes. Among the main ideas for application of this equation is the study of gravitational radiation generated by binary pulsars and their spin interaction with gravitational waves produced by a companion object such as a massive black hole (see [6] and references therein). Instead of focusing our attention on the black hole curvature, we think of the black hole gravitational waves as being responsible for the timing effect of the binary pulsars. Our work may be useful, among other things, for distinguishing these two possibilities. Actually, our formulation is so general that the companion of the binary pulsar can be any other source of gravitational waves such as supernovae or vibrating neutron star systems.

Another very interesting setting where our proposal of Ref. 6 may find application is the search for the stochastic gravitational wave background (SGWB) as proposed some time ago by Detweiler [10], who showed that measurements of signal arrival time from the pulsar may be used to investigate properties of this background. The main strategy for the detection of the SGWB is to consider a number of pulsars separated at different parts in the sky. It is clear then that our RTDE formulation may be useful for this proposal.

Ultimately, the RTDE may also have an interesting application in connection with the so called inflationary gravitational waves (see Ref. 11 and references therein). As is known, the polarization of the cosmic microwave radiation [12] may solve the problem of detecting the gravitational waves produced during the inflationary scenario. Just before the universe became transparent to radiation, the plasma motion caused by the gravitational waves may be generated by different sources. In particular, the effect predicted by the RTDE may be of physical interest in this scenario.

Our plan for presenting this review is the following. In Sec. 2, we briefly review one of the possible mechanisms to obtaining the GDE, and we apply similar techniques to obtain the RTDE formulation. In Sec. 3, we explain how the RTDE can be applied to the detection of gravitational waves. Finally, in Sec. 4, we make some final comments.

2. The geodesic deviation equation and the relativistic top

Several methods can be used to obtain the GDE. Some of them are, in fact, quite brief. For our purpose, however, it turns out to be more convenient to follow the one in Ref. 13.

Consider a point particle whose trajectory is described by the coordinates $x^\mu(\tau)$, where τ is the proper time parameter. The geodesic equation is

$$\frac{d^2x^\mu}{d\tau^2}+\Gamma^\mu_{\alpha\beta}(x)\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}=0. \tag{1}$$

Here, $\Gamma^\mu_{\alpha\beta}(x)$ stands for the Christoffel symbols.

A nearby point particle must also satisfy a geodesic equation. If we use the coordinates $x'^\mu(\tau)$ to describe the position of such a nearby point particle, we have

$$\frac{d^2x'^\mu}{d\tau^2}+\Gamma'^\mu_{\alpha\beta}(x')\frac{dx'^\alpha}{d\tau}\frac{dx'^\beta}{d\tau}=0. \tag{2}$$

By "nearby" we mean that the coordinates $x'^\mu(\tau)$ can be written as

$$x'^\mu=x^\mu+\xi^\mu(x), \tag{3}$$

with ξ^μ a very small quantity.

Then we expand to the first order in ξ^μ

$$\Gamma'^\mu_{\alpha\beta}(x+\xi)=\Gamma^\mu_{\alpha\beta}(x)+\Gamma^\mu_{\alpha\beta},_\lambda\,\xi^\lambda, \tag{4}$$

with

$$\Gamma^\mu_{\alpha\beta},_\lambda=\frac{\partial\Gamma^\mu_{\alpha\beta}}{\partial x^\lambda}.$$

Thus, using (3) and (4), we find that Eq. (2) for ξ^μ becomes

$$\frac{d^2\xi^\mu}{d\tau^2}+2\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{d\xi^\beta}{d\tau}+\Gamma^\mu_{\alpha\beta},_\lambda\,\xi^\lambda\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}=0. \tag{5}$$

Through some algebraic manipulations, this equation can be written in a totally covariant expression (see Ref. 6)

$$\frac{D^2\xi^\mu}{D\tau^2}=-R^\mu_{\alpha\lambda\beta}\frac{dx^\alpha}{d\tau}\xi^\lambda\frac{dx^\beta}{d\tau}, \tag{6}$$

which is, of course, the famous geodesic deviation equation (GDE) for a pair of nearby freely falling particles in a gravitational field background.

Now, we turn to analyzing the equations of motion for a relativistic top moving in a gravitational field background, namely

$$\frac{D^2x^\mu}{D\tau^2}=-\frac{1}{2}R^\mu_{\alpha\lambda\beta}\frac{dx^\alpha}{d\tau}S^{\lambda\beta}, \tag{7}$$

and

$$\frac{DS^{\mu\nu}}{D\tau}=0. \tag{8}$$

Here $S^{\mu\nu}$ is the internal angular momentum (or the spin tensor) per unit mass of the top satisfying the Pirani constraint [14]

$$S^{\mu\nu}\frac{dx_\nu}{d\tau}=0$$

(see Refs. 15 to 17). It is worth mentioning that formulae (7) and (8) can be derived by a number of different methods [9]. An important observation is that (7) can be understood as the analogue of the geodesic equation (1), and in fact it reduces to (1) when the spin tensor $S^{\mu\nu}$ vanishes.

After comparing equations (6) and (7), we note a great similarity. But in fact they are very different in the sense that, while equation (6) refers to a pair of nearby point particles, (7) is associated with just one physical system: a relativistic top. Nevertheless, this similarity was used as an inspiration to propose that, just as (6) is used to detect gravitational waves, equation (7) can be used for the same purpose. In order to further understand the real differences between the two point particle system and the relativistic top, it turns out to be necessary to derive the analogue of (6) for a pair of nearby relativistic tops. For this purpose, we suppose that a nearby top satisfies the corresponding equations of motion,

$$\frac{d^2x'^\mu}{d\tau^2}+\Gamma'^\mu_{\alpha\beta}(x')\frac{dx'^\alpha}{d\tau}\frac{dx'^\beta}{d\tau}=-\frac{1}{2}R'^\mu_{\alpha\lambda\beta}(x')\frac{dx'^\alpha}{d\tau}S'^{\lambda\beta}. \tag{9}$$

Consider now a perturbation of the form

$$x'^\mu=x^\mu+\xi^\mu(x) \tag{10}$$

and

$$S'^{\mu\nu}=S^{\mu\nu}+S^{\mu\nu},_\alpha\,\xi^\alpha(x). \tag{11}$$

Now, after some algebra, (see Ref. 6 for a full explanation and also see Ref. 18), we obtain our master equation:

$$\frac{D^2\xi^\mu}{D\tau^2}=-R^\mu_{\alpha\lambda\beta}\frac{dx^\alpha}{d\tau}\xi^\lambda\frac{dx^\beta}{d\tau}-\frac{1}{2}[R^\mu_{\alpha\lambda\beta}\frac{D\xi^\alpha}{D\tau}S^{\lambda\beta}+R^\mu_{\alpha\lambda\beta}\frac{dx^\alpha}{d\tau}S^{\lambda\beta};_\gamma\,\xi^\gamma+R^\mu_{\alpha\lambda\beta};_\sigma\,\xi^\sigma\frac{dx^\alpha}{d\tau}S^{\lambda\beta}], \tag{12}$$

which is the covariant form of the relativistic top deviation equation (RTDE). Clearly, (12) reduces to (6) when the spin tensor $S^{\lambda\beta}$ vanishes. One of the attractive features of (12) is that the spin angular momentum $S^{\lambda\beta}$ of the top appears to be coupled to gravity via the curvature Riemann tensor $R^\mu_{\alpha\lambda\beta}$ and its gradient. It seems reasonable to think that this characteristic can provide a better description of the properties of the underlying intrinsic curvature of the geometry in question. In particular, we shall see in the next section that the RTDE may be used to study the different properties of a gravitational wave background.

3. The relativistic top deviation equation and gravitational waves

In this section we summarize the consequences of equation (12) in the case of gravitational waves. But for completeness we shall start by reviewing briefly how formula (6) is used for this particular case.

As usual, consider a gravitational wave in a flat background where the metric $g_{\mu\nu}$ can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{13}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $\mid h_{\mu\nu} \mid \ll 1$. In the transverse-traceless gauge the conditions

$$h_{0\mu} = 0\,, \quad h_{ij,j} = 0\,, \quad h^{\mu}_{\mu} = 0\,, \tag{14}$$

are obeyed, with the indices $i, j, \ldots, etc$ running from 1 to 3. The Einstein gravitational field equations imply that h_{ij} satisfies the wave equation

$$\Box^2 h_{ij} = 0, \tag{15}$$

where $\Box^2 = \partial^{\mu}\partial_{\mu}$ is the D'Alambertian. In the gauge given in (14), the space-time components of the Riemann tensor R_{i0j0} have the simple form

$$R_{i0j0} = -\frac{1}{2} h_{ij,00}\,. \tag{16}$$

Using (14), one discovers that h_{ij} can be written as

$$h_{ij} = A_{+} e^{+}_{ij} + A_{\times} e^{\times}_{ij}\,, \tag{17}$$

where A_{+} and $A_{\times}$ are two independent dimensionless amplitudes and e^{+}_{ij} and $e^{\times}_{ij}$ are polarization tensors. For a wave traveling in the z- direction, the only nonvanishing components of e_{ij} are

$$e^{+}_{xx} = -e^{+}_{yy} \text{ and } e^{\times}_{xy} = e^{\times}_{yx}, \tag{18}$$

and in this case A_{+} and $A_{\times}$ turn out to be functions only of $t - z$.

In a proper reference frame we have $x^0 = \tau$, $x^i = 0$, so that $dx^0/d\tau = 1$ and $dx^i/d\tau = 0$. In this reference frame we find that (6) becomes

$$\frac{d^2\xi^i}{dt^2} = -R^{i}_{0j0}\xi^j. \tag{19}$$

For a wave propagating in the z- direction, we have

$$\frac{d^2\xi^z}{dt^2} = 0,, \tag{20}$$

as well as

$$\frac{d^2\xi^a}{dt^2} = \frac{1}{2} h^{a}_{b},_{00}\,\xi^b\,, \tag{21}$$

where now, the Latin indices $a, b, \ldots, etc$ run from 1 to 2. The formula (21) tells us that only separations of two nearby point particles in the transverse direction are meaningful.

Let us now address the problem at hand, namely, the fact that we are interested in applying similar methods as to the above in the case of a system with two nearby relativistic tops. For this purpose let us consider the formula (12) in a proper reference frame. Using the fact that $S^{0\mu} = 0$, due to the Twlczyjew-Pirani constraint $S^{\mu\nu}(dx_{\nu}/d\tau) = 0$ and the symmetries of the Riemann curvature tensor we find that the time component of (12) is

$$\frac{d^2\xi^0}{dt^2} = -\frac{1}{2} R^{0}_{jkl}\frac{d\xi^j}{dt} S^{kl}, \tag{22}$$

which for the particular case of a gravitational plane wave propagating in the $z-$direction is

$$\frac{d^2\xi^0}{dt^2} = -R^{0}_{azb}\frac{d\xi^a}{dt} S^{zb}, \tag{23}$$

where we used the fact that the only nonvanishing components of the Riemann curvature tensor are

$$R_{zazb} = R_{0a0b} = -R_{0azb} = -\frac{1}{2} h_{ab,00}\,. \tag{24}$$

The z component of (12) turns out to be

$$\frac{d^2\xi^z}{dt^2} = -R^{z}_{azb}\frac{d\xi^a}{dt} S^{zb}\,, \tag{25}$$

where we used (24), and the x and y components are

$$\begin{aligned}\frac{d^2\xi^a}{dt^2} &= -R^{a}_{0b0}\xi^b \\ &-[R^{a}_{0bz}\frac{d\xi^0}{dt} S^{bz} + R^{a}_{0bz} S^{bz},_{0}\,\xi^0 + R^{a}_{0bz},_{0}\,\xi^0 S^{bz}] \\ &-[R^{a}_{zbz}\frac{d\xi^z}{dt} S^{bz} + R^{a}_{0bz} S^{bz},_{z}\,\xi^z + R^{a}_{0bz},_{z}\,\xi^z S^{bz}] \\ &-R^{a}_{0bz} S^{bz},_{a}\,\xi^a.\end{aligned} \tag{26}$$

Note that in (23), (25) and (26) the S^{ab} component of the spin angular momentum does not appear and only the S^{zb} component remains. To better understand this, let us define

$$S^i = \frac{1}{2}\varepsilon^{ijk} S_{jk}\,, \tag{27}$$

where ε^{ijk} is the Levi-Civita tensor with ε^{xyz}=1. From (27), we note that the z component of the intrinsic angular momentum does not play any role in our equations (23), (25) and (26). This means that no effect is expected when the top is oriented along the direction of propagation of the gravitational wave .

The second interesting observation is that, if S^{zb} is nonvanishing, then $d^2\xi^0/dt^2 \neq 0$ and $d^2\xi^z/dt^2 \neq 0$, in contrast to the case of a pair of nearby point particles in which both of these terms vanish. The third important observation is that the $d^2\xi^a/dt^2$ equation contains a large number of terms in addition to the usual one, $R^{a}_{0b0}\xi^b$. Clearly, this means that the solution of (26) will not be as simple as in the case, of nearby point particles.

Let us concentrate on the terms in (26) not involving derivatives of S^{zb} and the Riemann tensor, which presumably represent small order corrections. In this case (26) is reduced to

$$\frac{d^2\xi^a}{dt^2} = -R^a_{0b0}\xi^b - R^a_{0bz}\frac{d\xi^0}{dt}S^{bz} - R^a_{zbz}\frac{d\xi^z}{dt}S^{bz}. \quad (28)$$

Using (24), we find that (23), (25) and (28) become

$$\frac{d^2\xi^0}{dt^2} = -\frac{1}{2}h_{ab,00}\frac{d\xi^a}{dt}S^{zb}, \quad (29)$$

$$\frac{d^2\xi^z}{dt^2} = \frac{1}{2}h_{ab,00}\frac{d\xi^a}{dt}S^{zb}, \quad (30)$$

and

$$\frac{d^2\xi^a}{dt^2}=\frac{1}{2}h^a_b{,}_{00}\,\xi^b-\frac{1}{2}h^a_b{,}_{00}\,\frac{d\xi^0}{dt}S^{bz}+\frac{1}{2}h^a_b{,}_{00}\,\frac{d\xi^z}{dt}S^{bz}, \quad (31)$$

respectively. From (29) and (30) we discover that we can set $\xi^0 = -\xi^z + cte$. Therefore (31) becomes

$$\frac{d^2\xi^a}{dt^2} = \frac{1}{2}h^a_b{,}_{00}\,\xi^b - h^a_b{,}_{00}\,\frac{d\xi^z}{dt}S^{bz}. \quad (32)$$

Note that if at an initial time the spin of the top S^{bz} has orientation such that

$$\xi^b - 2\frac{d\xi^z}{dt}S^{bz} = 0, \quad (33)$$

then there is no transverse motion $\xi^a = const.$ and therefore (30) allows the solution

$$\xi^z = const. \quad (34)$$

What this means is that, if the spin of the top is oriented along the vector separation ξ^a of the two tops, then the gravitational wave does not produce any effect on the system. This seems to be a new, interesting result since in the ordinary case of GDE the wave is always transverse in its physical effects.

Let us now look for a solution to (32) of the form

$$\xi^a = \xi^a_0 + \frac{1}{2}h^a_b\xi^b_0 - h^a_b(\frac{d\xi^z}{dt}\mid_0)S^{bz}, \quad (35)$$

where $\xi^a_0 = const$. We observe that

$$\frac{d\xi^a}{dt} = \frac{1}{2}h^a_b{,}_0\,\xi^b_0 - h^a_b{,}_0\,(\frac{d\xi^z}{dt}\mid_0)S^{bz}.$$

Substituting (35) into (30) we find that, to the first order in h, (30) becomes

$$\frac{d^2\xi^z}{dt^2} \approx 0, \quad (36)$$

Therefore, if initially

$$\frac{d\xi^z}{dt}\mid_0 = 0$$

to the first order of approximation, the solution (35) reduces to the ordinary case of a pair of nearby point particles.

4. Comments

We have been pursuing the possibility of using relativistic tops as detectors of gravitational waves. In two previous works [7,8], an isolated top was considered, making it difficult to compare our results with the case of DGE. In order to overcome this difficulty and to gain further progress towards our goal, in this article we have derived the RTDE equation for a pair of nearby tops. We have shown that the RTDE reduces to the GDE when the spin tensor vanishes.

By considering a plane gravitational wave, we find the solution of the RTDE for two simple cases. In the first case we discover that, if the internal angular momentum of the top is oriented along the vector separation of the two tops, the gravitational wave does not produce any effect on the physical system. In the second, more general, case we find that the nearby top will have an effect different from the case of GDE only to a second order of approximation in the perturbation h_{ab}. At first sight, it would appear that these two cases show that the RTDE formulation, although it may be theoretically interesting, does not seem to offer a promising route for experiments. However, rough estimates can show us that this turns out not to be the case (see Ref. 6 for details).

A possible, interesting extension of the present work is to apply to the nongeodesic equations of motion of spinning particles in a teleparallel gravitational background (GETGB) [19, 20] a method similar to the one used to obtain the RTDE. The GETGB equations are an extension of the RTEM in the sense that they include torsion interactions in addition to the gravity spin interaction. Recently, Garcia [21] has revisited the motion of spinless particles in a teleparallel gravitational wave background and proposed an experimental mechanism for detecting torsion via the GETGB model. The complete picture should be to detect both gravitational waves, and torsional waves and therefore it may be interesting for further research to find the nondeviation geodesic equations associated with the GETGB model.

Finally, we would like to comment on the possibility of using the RTDE in connection with the inflationary gravitational wave scenario and world brane cosmology. In the first case, just before the universe becomes transparent, the gravitational waves, produced during inflation, interacts with the plasma, producing polarization patterns of the cosmic microwave background (CMB). This kind of phenomenon is especially interesting since recent experiments seems to have measured the variation of the polarization pattern of the CMB. The RTDE model can be interesting in this direction if the spin tensor $S^{\lambda\beta}$ is identified as the fermi spin of elementary particles. According to the RTDE, a gravitational wave should cause two kinds of motions in the constituent fermionic particles of such a plasma. The first one is the motion of the particles caused by the forces due to GDE, and the second is the motion in the plasma caused by the spin-gravity interaction. Therefore one should expect that the spin-gravity interaction will also leave a “print” in the polarization pattern of the CMB.

In the case of the world brane universes, only gravity can move in the extra dimensions; all the matter and other forces are confined to the branes. Only gravitational waves (or gravitons) travel from brane to brane carrying some energy information away from the branes. Presumably, such a gravitational wave affects objects held together by gravity, such as stars and galaxies, by distances shorter than millimeters. But, according to our above rough estimate, such short-distance changes are also predicted by the RTDE model.

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