Geodetic Reference System 1980

by H. Moritz

1- *Definition*

The **Geodetic Reference System 1980** has been adopted at the XVII General Assembly of the IUGG in Canberra, December 1979, by means of the following :

"RESOLUTION N° 7

The International Union of Geodesy and Geophysics,

recognizing that the Geodetic Reference System 1967 adopted at the XIV General Assembly of IUGG, Lucerne, 1967, no longer represents the size, shape, and gravity field of the Earth to an accuracy adequate for many geodetic, geophysical, astronomical and hydrographic applications and

considering that more appropriate values are now available,

recommends

a) that the Geodetic Reference System 1967 be replaced by a new **Geodetic Reference System 1980**, also based on the theory of the geocentric equipotential ellipsoid, defined by the following conventional constants :

. equatorial radius of the Earth :
$a = 6378\ 137$ m,

. geocentric gravitational constant of the Earth (including the atmosphere) :
$GM = 3986\ 005 \times 10^{8}\ m^{3}\ s^{-2}$,

. dynamical form factor of the Earth, excluding the permanent tidal deformation :
$J_2 = 108\ 263 \times 10^{-8}$,

. angular velocity of the Earth :
$\omega = 7292\ 115 \times 10^{-11}$ rad s^{-1},

b) that the same computational formulas, adopted at the XV General Assembly of IUGG in Moscow 1971 and published by IAG, be used as for Geodetic Reference System 1967, and

c) that the minor axis of the reference ellipsoid, defined above, be parallel to the direction defined by the Conventional International Origin, and that the primary meridian be parallel to the zero meridian of the BIH adopted longitudes".

For the background of this resolution see the report of IAG Special Study Group 5.39 (**Moritz**, 1979, sec.2).
Also relevant is the following IAG resolution :

"RESOLUTION N° 1

The International Association of Geodesy,

recognizing that the IUGG, at its XVII General Assembly, has introduced a new Geodetic Reference System 1980,

recommends that this system be used as an official reference for geodetic work, and

encourages computations of the gravity field both on the Earth's surface and in outer space based on this system".

2- *The Equipotential Ellipsoid*

According to the first resolution, the Geodetic Reference System 1980 is based on the theory of the equipotential ellipsoid. This theory has already been the basis of the Geodetic Reference System 1967; we shall summarize (partly quoting literally) some principal facts from the relevant publication (IAG, 1971, Publ. Spéc. n° 3).

An equipotential ellipsoid or level ellipsoid is an ellipsoid that is defined to be an equipotential surface. If an ellipsoid of revolution (semimajor axis **a**, semiminor axis **b**) is given, then it can be made an equipotential surface

$$U = U_0 = \text{const.}$$

of a certain potential function U, called normal potential. This function U is uniquely determined by means of the ellipsoidal surface (semiaxes a, b), the enclosed mass M and the angular velocity ω, according to a theorem of Stokes-Poincaré, quite independently of the internal density distribution. Instead of the four constants a, b, M and ω, any other system of four independent parameters may be used as defining constants.

The theory of the equipotential ellipsoid was first given by **Pizzeti** in 1894; it was further elaborated by **Somigliana** in 1929. This theory had already served as a base for the International Gravity Formula adopted at the General Assembly in Stockholm in 1930.

Normal gravity $\gamma = |\mathrm{grad}\, U|$ at the surface of the ellipsoid is given by the closed formula of **Somilgiana**,

$$\gamma = \frac{a\gamma_e \cos^2\varphi + b\gamma_p \sin^2\varphi}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}} \quad ,$$

where the constants γ_e and γ_p denote normal gravity at the equator and at the poles, and φ denotes geographical latitude.

The equipotential ellipsoid furnishes a simple, consistent and uniform reference system for all purposes of geodesy: the ellipsoid as a reference surface for geometric use, and a normal gravity field at the earth's surface and in space, defined in terms of closed formulas, as a reference for gravimetry and satellite geodesy.

The standard theory of the equipotential ellipsoid regards the normal gravitational potential as a harmonic function outside the ellipsoid, which implies the absence of an atmosphere. (The consideration of the atmosphere in the reference system would require an ad-hoc modification of the theory, whereby it would lose its clarity and simplicity.)

Thus, in the same way as in the Geodetic Reference System 1967, the computation are based on the theory of the equipotential ellipsoid without an atmosphere. The reference ellipsoid is defined to enclose the whole mass of the earth, including the atmosphere; as a visualization, one might, for instance, imagine the atmosphere to be condensed as a surface layer on the ellipsoid. The normal gravity field at the earth's surface and in space can thus be computed without any need for considering the variation of atmospheric density.

If atmospheric effects must be considered, this can be done by applying corrections to the measured values of gravity; for this purpose, a table of corrections will be given later (sec.5).

3- *Computational Formulas*

An equipotential ellipsoid of revolution is determined by four constants. The IUGG has chosen the following ones:

a equatorial radius,
GM geocentric gravitational constant,
$\mathbf{J_2}$ dynamical form factor,
ω angular velocity.

The equatorial radius **a** is the semimajor axis of the meridian ellipse; the semiminor axis will be denoted by **b**. The geocentric gravitational constant **GM** is the product of the Newtonian gravitational constant, **G**, and the total mass of the earth, **M**. The constant J2 is given by :

$$J_2 = \frac{C-A}{Ma^2} \quad ,$$

where **C** and **A** are the principal moments of inertia of the level ellipsoid (**C**... polar, **A**... equatorial moment of inertia).

We shall also use the first excentricity **e**, defined by:

$$e^2 = \frac{a^2 - b^2}{a^2} \quad ,$$

and the second excentricity **e'**, defined by :

$$e'^2 = \frac{a^2 - b^2}{b^2}$$

Closed computational formulas are given in sec.3 of (IAG, 1971, Pub.Spéc. n° 3); we shall here reproduce this section practically unchanged.

The derivation of these formulas is found in the book (**Heiskanen** and **Moritz**, 1967) sections 2-7 to 2-10. Reference to this book is by page number and number of equation.

Computation of e^2

The fundamental derived constant is the square of the first excentricity, $\mathbf{e^2}$, as defined above.

From p. 73, equations (2-90) and (2-92'), we find :

$$J_2 = \frac{e^2}{3}\left(1 - \frac{2}{15}\frac{me'}{q_0}\right)$$

This equation can be written as :

$$e^2 = 3J_2 + \frac{2me'e^2}{15q_0}$$

with :

$$m = \frac{\omega^2 a^2 b}{GM}$$

(p. 69, eq. (2-70)) and with be' = ae it becomes :

$$e^2 = 3J_2 + \frac{4}{15}\,\frac{\omega^2 a^3}{GM}\,\frac{e^3}{2q_0}$$

This is the basic equation which relates e^2 to the data a, GM, J_2 and ω. It is to be solved iteratively for e^2, taking into account :

$$2\,q_0 = \left(1 + \frac{3}{e'^2}\right) \textit{arctan}\; e' - \frac{3}{e'}$$

$$= \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}\, n}{(2n+1)(2n+3)}\, e'^{2n+1}$$

with

$$e' = \frac{e}{\sqrt{1-e^2}} \qquad \text{(second excentricity)}$$

(p. 66, eq. (2-58), p. 72, second equation from top).

Geometric Constants

Now the other geometric constants of the reference ellipsoid can be computed by the well-known formulas:

$$b = a\sqrt{1-e^2} \qquad \text{(semiminor axis)},$$

$$f = \frac{a-b}{a} \qquad \text{(flattening)},$$

$$E = \sqrt{a^2-b^2} \qquad \text{(linear excentricity)},$$

$$c = \frac{a^2}{b} \qquad \text{(polar radius of curvature)}.$$

The arc of meridian from equator to pole (meridian quadrant) is given by :

$$Q = c \int_0^{\pi/2} \frac{d\varphi}{(1+e'^2\cos^2\varphi)^{3/2}}$$

where φ is the geographical latitude. This integral can be evaluated by a series expansion :

$$Q = c\,\frac{\pi}{2}\left(1 - \frac{3}{4}e'^2 + \frac{45}{64}e'^4 - \frac{175}{256}e'^6 + \frac{11025}{16384}e'^8\right)$$

Various mean radii of ellipsoid are defined by the following formulas :

arithmetic mean :

$$R_1 = \frac{a+a+b}{3} = a\left(1 - \frac{f}{3}\right) ;$$

radius of sphere of the same surface :

$$R_2 = c\left(\int_0^{\pi/2} \frac{\cos\varphi}{(1+e'^2\cos^2\varphi)^2}\, d\varphi\right)^{\frac{1}{2}}$$

$$= c\left(1 - \frac{2}{3}e'^2 + \frac{26}{45}e'^4 - \frac{100}{189}e'^6 + \frac{7034}{14175}e'^8\right)$$

radius of sphere of the same volume :

$$R_3 = \sqrt[3]{a^2 b}\ .$$

Physical Constants

The reference ellipsoid is a surface of constant normal potential, **U = U₀**. This constant U_0, the normal potential of the reference ellipsoid, is given by :

$$U_0 = \frac{GM}{E}\; \textit{arctan}\; e' + \frac{1}{3}\,\omega^2 a^2$$

$$= \frac{GM}{b}\left[1 + \sum_{n=1}^{\infty} (-1)^n \frac{e'^{2n}}{2n+1} + \frac{1}{3}m\right]$$

(p. 67, eq. (2-61)).

The normal gravitational potential **V** (gravity potential **U** minus potential of centrifugal force) can be developed into a series of zonal spherical harmonics :

$$V = \frac{GM}{r}\left[1 - \sum_{n=1}^{\infty} J_{2n}\left(\frac{a}{r}\right)^{2n} P_{2n}(\cos\theta)\right] ;$$

where **r** (radius vector) and θ (polar distance) are spherical coordinates. The coefficient $\mathbf{J_2}$ is a defining constant; the other coefficients are expressed in terms of $\mathbf{J_2}$ by :

$$J_{2n} = (-1)^{n+1} \frac{3e^{2n}}{(2n+1)(2n+3)}\left(1 - n + 5n\frac{J_2}{e^2}\right)$$

(p.73, eqs. (2-92) and (2-92')).

Normal gravity at the equator, γ_e, and normal gravity at the poles, γ_p, are given by the expressions :

$$\gamma_e = \frac{GM}{ab}\left(1 - m - \frac{m}{6}\,\frac{e' q_0'}{q_0}\right)$$

$$\gamma_p = \frac{GM}{a^2}\left(1 + \frac{m}{3}\,\frac{e' q_0'}{q_0}\right)$$

with

$$q_0' = 3\left(1 + \frac{1}{e'^2}\right)\left(1 - \frac{1}{e'}\,\text{arctan}\, e'\right) - 1$$

and

$$m = \frac{\omega^2 a^2 b}{GM}$$

(p. 69, eqs. (2-73) and (2-74); p.68, eq. (2-67)).

The constant :

$$f^* = \frac{\gamma_p - \gamma_e}{\gamma_e} \qquad \text{(gravity flattening)}$$

is also needed.

A check is provided by the closed form of **Clairaut**'s theorem for the equipotential ellipsoid :

$$f + f^* = \frac{\omega^2 b}{\gamma_e}\left(1 + \frac{e' q'_0}{2 q_0}\right)$$

(p. 69, eq. (2-75)).

The Gravity Formula

Somiglinana's closed formula for normal gravity is

$$\gamma = \frac{a\gamma_e \cos^2\phi + b\gamma_p \sin^2\phi}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

For numerical computations, the form

$$\gamma = \gamma_e \frac{1 + k\sin^2\phi}{\sqrt{1 - e^2\sin^2\phi}}$$

with

$$k = \frac{b\gamma_p}{a\gamma_e} - 1$$

is more convenient.

The conventional abbreviated series expansion is :

$$\gamma = \gamma_e\,(1 + f^* \sin^2\phi - \frac{1}{4} f_4 \sin^2 2\phi)$$

with

$$f_4 = \frac{1}{2} f^2 + \frac{5}{2} fm$$

(p.77, eqs. (2-115) and 2-116)).

More generally, the above closed formula for normal gravity may be expanded into the series

$$\gamma = \gamma_e \left[1 + \sum_{n=1}^{4} a_{2n} \sin^{2n}\phi\right]$$

where

$$a_2 = \frac{1}{2} e^2 + k, \qquad a_6 = \frac{5}{16} e^6 + \frac{3}{8} e^4 k,$$

$$a_4 = \frac{3}{8} e^4 + \frac{1}{2} e^2 k, \qquad a_8 = \frac{35}{128} e^8 + \frac{5}{16} e^6 k,$$

The average value of gravity over the ellipsoid is

$$\bar{\gamma} = \int_0^{\pi/2} \frac{\gamma\cos\phi\, d\phi}{(1 - e^2\sin^2\phi)^2} : \int_0^{\pi/2} \frac{\cos\phi\, d\phi}{(1 - e^2\sin^2\phi)^2}$$

$$= 1 + \frac{1}{6} e^2 + \frac{1}{3} k + \frac{59}{360} e^4 + \frac{5}{18} e^2 k +$$

$$+ \frac{2371}{15120} e^6 + \frac{259}{1080} e^4 k + \frac{270229}{1814400} e^8 + \frac{9623}{45360} e^6 k.$$

4- *Numerical values*

The following derived constants are accurate to the number of decimal places given. In case of doubt or in those cases where a higher accuracy is required, these quantities are to be computed from the defining constants by means of the closed formulas given in the preceding section.

Defining Constants (exact)

a	= 6378 137 m	semimajor axis
GM	= 3 986 005 x 10^8 m^3 s^{-2}	geocentric gravitational constant
J_2	= 108 263 x 10^{-8}	dynamic form factor
ω	= 7 292 115 x 10^{-11} rad s^{-1}	angular velocity

Derived Geometric Constants

b	= 6 356 752.3141 m	semiminor axis
E	= 521 854.0097 m	linear excentricity
c	= 6 399 593.6259 m	polar radius of curvature
e^2	= 0.006 694 380 022 90	first excentricity (e)
e'^2	= 0.006 739 496 775 48	secondexcentricity (e')
f	= 0.003 352 810 681 18	flattening
f^{-1}	= 298.257 222 101	reciprocal flattening
Q	= 10 001 965.7293 m	meridian quadrant
R_1	= 6 371 008.7714 m	mean radius R_1=(2a+b)/3
R_2	= 6 371 007.1810 m	radius of sphere of same surface
R_3	= 6 371 000.7900 m	radius of sphere of same volume

Derived Physical Constants

U_0	$= 6\ 263\ 686.0850 \times 10\ m^2\ s^{-2}$	normal potential at ellipsoid
J_4	$= -0.000\ 002\ 370\ 912\ 22$	
J_6	$= 0.000\ 000\ 006\ 083\ 47$	spherical-harmonic
J_8	$= -0.000\ 000\ 000\ 014\ 27$	coefficents
m	$= 0.003\ 449\ 786\ 003\ 08$	$m = \omega^2 a^2 b/GM$
γ_e	$= 9.780\ 326\ 7715\ ms^{-2}$	normal gravity at equator
γ_p	$= 9.832\ 186\ 3685\ ms^{-2}$	normal gravity at pole
f^*	$= 0.005\ 302\ 440\ 112$	$f^* = \dfrac{(\gamma_p - \gamma_e)}{\gamma_e}$
k	$= 0.001\ 931\ 851\ 353$	$k = \dfrac{(b\gamma_p - a\gamma_e)}{a\gamma_e}$

Gravity Formula 1980

Normal gravity may be computed by means of the closed formula :

$$\gamma = \gamma_e \frac{1 + k \sin^2 \phi}{\sqrt{1 - e^2 \sin^2 \phi}} ,$$

with the values of γ_e, **k**, and $\mathbf{e^2}$ shown above.

The series expansion, given at the end of sec. 3, becomes :

$$\begin{aligned}\gamma = \gamma_e\ (1 &+ 0.005\ 279\ 0414 \sin^2 \phi \\ &+ 0.000\ 023\ 2718 \sin^4 \phi \\ &+ 0.000\ 000\ 1262 \sin^6 \phi \\ &+ 0.000\ 000\ 0007 \sin^8 \phi) ;\end{aligned}$$

it has a relative error of 10^{-10}, corresponding to $10^{-3}\ \mu m\ s^{-2} = 10^{-4}$ mgal.

The conventional series

$$\gamma = \gamma_e\ (1 + f^* \sin^2 \phi - \frac{1}{4} f_4 \sin^2 2\phi)$$

$$= 9.780\ 327\ (1 + 0.005\ 3024 \sin^2 \phi - 0.000\ 0058 \sin^2 2\phi)\ m\ s^{-2}$$

has only an accuracy of $\mathbf{1\ \mu m\ s^{-2} = 0.1}$ **mgal**. It can, however, be used for converting gravity anomalies from the International Gravity Formula (1930) to the Gravity Formula 1980 :

$$\gamma_{1980} - \gamma_{1930} = (-\ 16.3 + 13.7 \sin^2 \phi)\ \text{mgal},$$

where the main part comes from a change of the Postdam reference value by - 14 mgal; see also (IAG, 1971, Publ. Spéc. n° 3, p.74).

For the conversion from the Gravity Formula 1967 to the Gravity Formula 1980, a more accurate formula, corresponding to the precise expansion given above, is :

$$\gamma_{1980} - \gamma_{1967} = (0.8316 + 0.0782 \sin^2 \phi - 0.0007 \sin^4 \phi)\ \text{mgal},$$

Since former gravity values are expressed in the units **"gal"** and **"mgal"**, we have, in the conversion formulas, used the unit $\mathbf{1\ mgal = 10^{-5}\ m\ s^{-2}}$.

Mean values of normal gravity are :

$\bar{\gamma}$ = 9.797 644 656 m s^{-2} average over ellipsoid,

γ_{45} = 9.806 199 203 m s^{-2} at latitude ϕ = 45°.

The numerical values given in this section have been computed independently by **Mr. Chung-Yung Chen**, using series developments up to f^5, and by **Dr. Hans Sünkel**, using the formulas presented in sec. 4.

5- *Atmospheric Effects*

The table given here is reproduced from (IAG, 1971, Publ. Spéc. n° 3, p.72). It shows atmospheric gravity correction **δg** as a function of elevation **h** above sea level. The values **δg** are to be added to measured gravity. The effect of this reduction is to remove, by computation, the atmosphere outside the Earth by shifting it vertically into the interior of the geoid.

Atmospheric Gravity Corrections δg
(to be added to measured gravity)

h [km]	δg [mgal]	h [km]	δg [mgal]
0	0.87	10	0.23
0.5	0.82	11	0.20
1.0	0.77	12	0.17
1.5	0.73	13	0.14
2.0	0.68	14	0.12
2.5	0.64	15	0.10
3.0	0.60	16	0.09
3.5	0.57	17	0.08
4.0	0.53	18	0.06
4.5	0.50	19	0.05
5.0	0.47	20	0.05
5.5	0.44	22	0.03
6.0	0.41	24	0.02
6.5	0.38	26	0.02
7.0	0.36	28	0.01
7.5	0.33	30	0.01
8.0	0.31	32	0.01
8.5	0.29	34	0.00
9.0	0.27	37	0.00
9.5	0.25	40	0.00

6- *Origin and Orientation of the Reference System*

IUGG Resolution n° 7, quoted at the begining of this paper, specifies that the Geodetic Reference System 1980 be geocentric, that is, that its origin be the center of mass of the earth. Thus, the center of the eliipsoid coincides with the geocenter.

The orientation of the system is specified in the following way. The rotation axis of the reference ellipsoid is to have the direction of the Conventional International Origin for the Polar Motion **(CIO)**, and the zero meridian as defined by the Bureau International de l'Heure **(BIH)** is used.

To this definition there corresponds a rectangular coordinate system **XYZ** whose origin is the geocenter, whose **Z**-axis is the rotation axis of the reference ellipsoid, defined by the direction of **CIO**, and whose **X**-axis passes through the zero meridian according to the **BIH**.

References

W.A. HEISKANEN, and H. MORITZ (1967) : Physical Geodesy. W.H. Freeman, San Francisco.

International Association of Geodesy (1971) : Geodetic Reference System 1967. Publi. Spéc. n° 3 du Bulletin Géodésique, Paris.

H. MORITZ (1979) : Report of Special Study Group N° 39 of I.A.G., Fundamental Geodetic Constants, presented at XVII General Assembly og I.U.G.G., Canberra.

Editor's Note

Additional useful constants can be obtained from :
"United States Naval Observatory, Circular N° 167, December 27, 1983, Project MERIT Standards", with updates of December 1985.