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RESEARCH NOTE

Geomagnetic effects of the Earth's ellipticity

Andrew Jackson,[1] Denis Winch[2] and Vincent Lesur[1]

[1] *Department of Earth Sciences, Leeds University, Leeds,* LS2 9JT, *UK. E-mail: a.jackson@earth.leeds.ac.uk*

[2] *Department of Applied Maths, University of Sydney, Sydney,* NSW 2006, *Australia*

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SUMMARY

We analyse the external field generated by a uniform distribution of magnetic susceptibility contained in an oblate spheroidal shell when it is magnetized by an internal magnetic field of arbitrary complexity. The situation is more relevant to the Earth than that of a spherical shell considered by Runcorn (1975a) (in the context of lunar magnetism), because of the larger flattening of the Earth than that of the Moon. We find that, to first order in the susceptibility, each internal harmonic in a spheroidal harmonic expansion of the magnetic potential generates just one non-vanishing external field coefficient, unlike in the spherical case when all harmonics vanish identically. The field generated is proportional to the susceptibility, thickness of the shell and square of the Earth's eccentricity, and hence it appears that this field amplification mechanism will be very ineffective for the Earth.

Key words: Earth's ellipticity, geomagnetism.

1 INTRODUCTION

A great deal of effort continues to be devoted to the calculation of the crustal magnetic field in the waveband where it dominates the magnetic field due to the core (e.g. Arkani-Hamed *et al.* 1994; Whaler 1994; Cohen & Achache 1990, 1994; Ravat *et al.* 1995; Langel & Whaler 1996; Purucker *et al.* 1997, 1998). Fig. 1 shows that in a plot of the power spectrum this is characterized by roughly spherical harmonic degrees of 13 and greater. Below this spherical harmonic degree the measured field is primarily of core origin with a small admixture of crustal field. As a result of Gauss' theorem, the contribution of the crustal field in this region is essentially unknown, although attempts have been made to estimate it in one form or another (e.g. Langel *et al.* 1989; Jackson 1990, 1994, 1996; Cohen & Achache 1990, 1994). It is clear that the source of crustal magnetization is both remanent and induced magnetization within the crust and lithosphere (referred to in this paper as the crust) in the region above the Curie isotherm, which lies at some tens of kilometres below the surface. Given this fundamental inability to separate the two sources of magnetic field on anything other than spectral arguments, it has been of interest to attempt to build geologically reasonable models of source distributions in order to test whether they could predict the magnetic fields which are actually measured by high-resolution surveys such as that performed by the satellite Magsat, which flew in 1980. Many, but not all (e.g. Arkani-Hamed 1988, 1989; LaBrecque & Raymond 1985) workers have

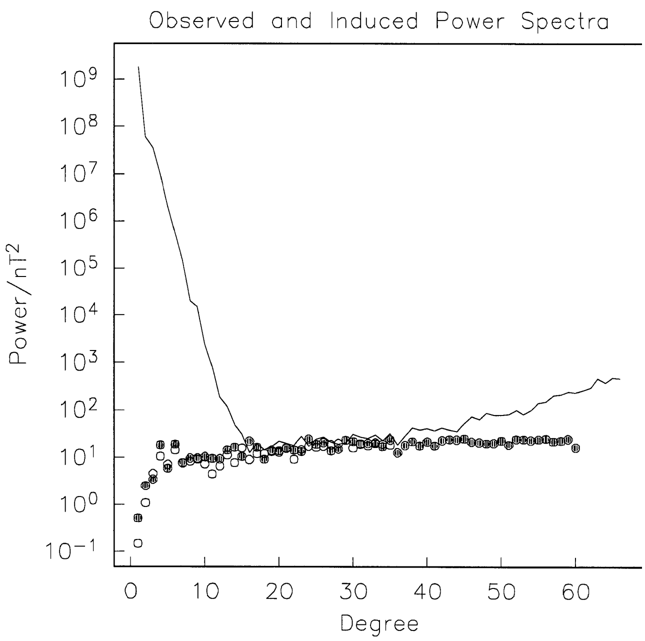

Figure 1. Power spectrum of the magnetic field as a function of spherical harmonic degree from model M07AV6 of Cain *et al.* (1989) (solid line). Also plotted are the induced power spectrum generated by the continent function of Counil *et al.* (1991) and an internal source (filled circles) when $\chi_0 d = 1.4$ km, and the spectrum for the model of the induced field CRST-70-F-22-22- of Hahn *et al.* (1984) (open circles).

treated the magnetization **M** as being induced magnetization due to the material having susceptibility χ in the presence of an internal magnetic field $\mathbf{B}_i$. The magnetization is related to these quantities by

$$\chi \mathbf{B}_i = \mu_0 \mathbf{M}\,, \tag{1}$$

where μ_0 is the permeability of free space, $4\pi \times 10^{-7}$ H m^{-1}. To first order in χ, the external potential ϕ_m that is produced at position $\mathbf{r}_j$ is given by

$$\phi_m(\mathbf{r}_j) = \frac{\mu_0}{4\pi} \int_V \nabla \frac{1}{|\mathbf{r}_j - \mathbf{s}|} \cdot \mathbf{M}(\mathbf{s})\, d^3\mathbf{s}\,, \tag{2}$$

where V is the volume of the crust and the associated external magnetic field is $\mathbf{B}_e = -\nabla\phi_m$. Values for susceptibilities encountered in typical crustal rocks can be found in e.g. Vacquier (1972), Harrison (1987) or Gee *et al.* (1989). Suffice it to say that there is a great variability in this property, but a value in the range 10^{-2}–10^{-1} is not atypical. Early work (Meyer *et al.* 1983, 1985; Hahn *et al.* 1984) built a model of the material characteristics of the crust and used the internal magnetic field to induce magnetization within it. It became clear that it is possible to generate an essentially white magnetic field spectrum of about the right amplitude in the $l > 13$ range where the crustal field dominates the core field (see Fig. 1). A simple order of magnitude calculation shows why this is so. From (2) we find $\mathbf{B}_e \sim (d/a)\mu_0\mathbf{M} \sim (d/a)\chi\mathbf{B}_i$, where d is the thickness of the source shell with outer radius a, so for an internal field of 50 000 nT and a typical value of χd of 1 km (e.g. Cohen & Achache 1990, 1994; Jackson 1990) we have $\mathbf{B}_e \sim 10$ nT, which is in reasonable accord with the observations. This clearly gives only an order of magnitude estimate, but it shows the possibility of generating the observed spectrum beyond $l = 13$. In a somewhat simpler model, Counil *et al.* (1991) attempted to explain the spectral characteristics by a difference in the depth-integrated susceptibility between the continents and oceans. Certainly some margins show evidence of this contrast, but the applicability of the model on a global scale is not clear (see e.g. Jackson 1990). This model, again, is capable of reproducing the high-degree power spectrum (Fig. 1). It is useful for workers interested in attempting to extrapolate the low-degree ($l < 13$) part of the core field down to the core–mantle boundary (CMB) to have estimates of the contamination in this waveband due to the crust, in order either to account for it statistically (Langel *et al.* 1989; Jackson 1990) or even to subtract its effect. These forward models therefore play a vital role in this process. Even extremely simple models which nevertheless acknowledge the presence of crustal magnetization can be useful as a first approximation to the Earth, and we consider such models below.

A very powerful result was obtained by Runcorn (1975a,b) for the case of a perfectly spherical shell of uniform magnetic susceptibility. He showed that when a shell is magnetized by a purely internal source (of arbitrary complexity), the magnetic field produced by this magnetization external to the body is exactly zero, to first order in the susceptibility. He also showed that the result could be generalized to the case where susceptibility varies purely with radius. This result was used to great effect in explaining the magnetization and magnetic field of the moon. Measurements of remanent magnetization of rock samples returned from the moon showed them to be significantly magnetized, whereas the magnetic field currently measured outside the moon is negligible (the dipole field is less than 1 nT at the equator). The answer to this conundrum (Runcorn 1975a,b) seems to be that the moon once possessed a dynamo operating within its core, which was responsible for magnetizing rocks near the surface when they cooled through their Curie temperature. If either the rocks were magnetized simultaneously or the core field did not vary significantly over the time that the rocks were emplaced, they will approximate magnetized spherical shells and the mathematical result, now almost universally known as Runcorn's theorem, will apply. It is interesting to note that the moon obeys rather well the conditions of the theorem, since its flattening is very small (roughly one part in 1000, corresponding to a difference of less than 2 km between the maximum and minimum radii), whereas the depth to the Curie temperature may be as large as 100 km (Runcorn 1975b). This means that despite the great thickness of the shell, it is highly spherical.

Conditions on the Earth are rather different. A flattening of approximately 1 part in 300 corresponds to a difference of 22 km between the polar and equatorial radii. This is large, and indeed is comparable to the depth to the Curie isotherm. We will therefore consider the idealized case of an oblate spheroidal shell of uniform susceptibility which is magnetized by an internal magnetic field. The result can be generalized to the case of susceptibility which is depth-dependent but which is constant on surfaces of constant flattening. In view of our poor knowledge of the depth dependence of susceptibility, we will not devote attention to this scenario. Although it has been suggested that there are significant differences between the depth-integrated susceptibilities of the oceans and continents, it is very difficult to see this in satellite data, and the constant-susceptibility question is nevertheless relevant because such ocean/continent differences can be considered by adding them to the result for a magnetized shell. The situation we consider is illustrated in Fig. 2. Fig. 2(a) shows the susceptibility in an oblate spheroidal shell. Since the focal radius is the same for both inner and outer surfaces, the shell thickness is not quite constant; however, for a very thin shell the deviation is tiny. Also shown in Fig. 2(b) are inner and outer spheres with radii equal to the length of the major axis of the inner spheroidal surface, and equal to the length of the minor axis of the outer spheroidal shell. Fig. 2(b) shows that, by Runcorn's theorem, only the susceptibility distribution in crescent-shaped regions is relevant to actually generating an external magnetic field. Given the fact that the distribution is large scale, and given the estimate for $\mathbf{B}_e$ given above, it is of

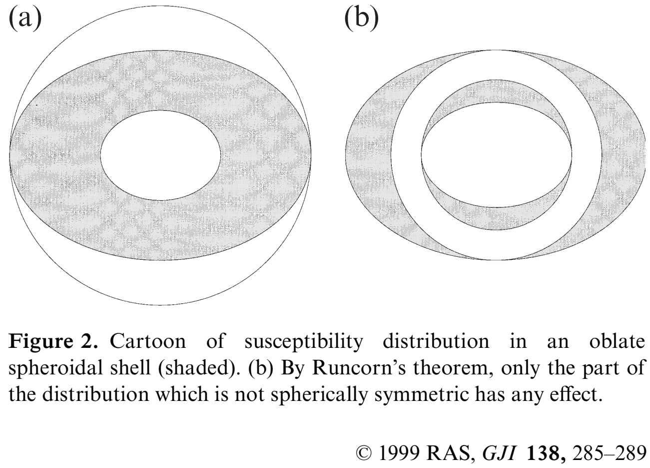

Figure 2. Cartoon of susceptibility distribution in an oblate spheroidal shell (shaded). (b) By Runcorn's theorem, only the part of the distribution which is not spherically symmetric has any effect.

interest to examine the generalization of Runcorn's theorem to the case of an oblate spheroidal shell appropriate to the Earth. The problem can be solved exactly using confocal spheroidal coordinates, since Laplace's equation separates in this coordinate system.

2 A MAGNETIZED OBLATE SPHEROIDAL SHELL

Here we consider the external magnetic field generated by an oblate spheroidal shell of uniform susceptibility which is magnetized by an internal source. We begin by briefly reviewing the result which applies to a spherical shell under the same conditions, the result which has become known as Runcorn's theorem.

We represent the internal magnetic field in spherical polar coordinates (r, θ, ϕ) in the form

$$\mathbf{B}_{\rm i} = -\nabla V_{\rm i}\,, \tag{3}$$

and $V_{\rm i}$ is expanded in terms of fully normalized spherical harmonics with coefficients β_l^m, analogous to the conventional Gauss coefficients $\{g_l^m; h_l^m\}$:

$$V_{\rm i} = a \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \left(\frac{a}{r}\right)^{l+1} \beta_l^m Y_l^m(\theta, \phi)\,, \tag{4}$$

where $a = 6371.2$ km is the mean Earth radius. The spherical harmonics satisfy

$$\frac{1}{4\pi} \int_{\Omega} Y_l^m Y_p^q \, d\Omega = \delta_{lp}\delta_{mq}\,. \tag{5}$$

For constant susceptibility, (2) can be written

$$\phi_{\rm m}(\mathbf{r}_j) = -\chi \frac{1}{4\pi} \int_S \hat{\mathbf{n}} \cdot \nabla \frac{1}{|\mathbf{r}_j - \mathbf{s}|} V_{\rm i}(\mathbf{s})\, d^2\mathbf{s} \tag{6}$$

by an application of Gauss' theorem, where $\hat{\mathbf{n}}$ is a unit normal and S represents both the inner and the outer surfaces of the shell. We use the expansion of the reciprocal distance in spherical harmonics for $|\mathbf{r}_j| > |\mathbf{s}|$,

$$\frac{1}{|\mathbf{r}_j - \mathbf{s}|} = \frac{1}{r_j} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} \left(\frac{r}{r_j}\right)^l Y_l^m(\theta, \phi) Y_l^m(\theta_j, \phi_j)\,, \tag{7}$$

to obtain

$$\phi_{\rm m}(\mathbf{r}_j) = -\chi \frac{1}{4\pi} \int_S \sum_{l,m} \frac{l}{2l+1} \left(\frac{r}{r_j}\right)^{l+1} Y_l^m(\theta, \phi) Y_l^m(\theta_j, \phi_j) \times a \sum_{p,q} \left(\frac{a}{r}\right)^{p+1} \beta_p^q Y_p^q(\theta, \phi)\, d\Omega\,. \tag{8}$$

The orthogonality of the Y_l^m couples l and p in the sums, and leads to the result that each surface integral gives a contribution of the same absolute value but of opposite sign, and therefore the external potential generated is zero. This is Runcorn's theorem (1975a,b).

To treat the spheroidal case, we must introduce a confocal spheroidal coordinate system. Discussions of the solution of Laplace's equation in this coordinate system can be found in classical books such as Hobson (1931), Kellogg (1929), MacMillan (1958) or Ramsey (1959). Our treatment follows closely that of MacRobert (1927).

The confocal ellipsoidal coordinate system (r', θ, ϕ) that we use is related to the Cartesian coordinates (x, y, z) by

$$x = \sqrt{r'^2 + c^2} \sin\theta \cos\phi\,, \tag{9}$$

$$y = \sqrt{r'^2 + c^2} \sin\theta \sin\phi\,, \tag{10}$$

$$z = r' \cos\theta\,. \tag{11}$$

In terms of a standard ellipse with semi-major axis a, semi-minor axis b, eccentricity e and focal distance c, we have

$$c = ae; \quad a = \sqrt{b^2 + c^2}\,. \tag{12}$$

For the Earth $b = 6356$ km and $c = 522$ km. We take the shell to have outer and inner boundaries given by $r' = b$ and $r' = b - d$. In this coordinate system the internal (inducing) potential is described by

$$V_{\rm i} = b \sum_{l,m} \kappa_l^m Y_l^m(\theta, \phi) Q_l^m(ir'/c) \tag{13}$$

(e.g. Winch 1967), where the coefficients κ_l^m are real and Q_l^m is an associated Legendre function of the second kind. We need to calculate (6) in this coordinate system. We will need the expansion of reciprocal distance in oblate spheroidal harmonics, namely

$$\frac{1}{|\mathbf{r} - \mathbf{r}_j|} = \frac{i(-1)^m}{c} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \times \left[\left(\frac{(n-m)!}{(n+m)!}\right) Y_n^m(\theta, \phi) Y_n^m(\theta_j, \phi_j) Q_n^m\left(\frac{ir_j'}{c}\right) P_n^m\left(\frac{ir'}{c}\right)\right] \tag{14}$$

[MacRobert (1927), p. 199, amended for the phases given in (15) and (16)]. In this formula the $Y_l^m(\theta, \phi)$ have the same normalization as in (5). The associated Legendre functions of the first and second kinds (P_n^m and Q_n^m) have been left in unnormalized form; they are defined by the explicit forms for large argument ($|x| > 1$):

$$P_n^m(x) = (x^2 - 1)^{m/2} \left(\frac{d}{dx}\right)^m P_n(x)\,, \tag{15}$$

$$Q_n^m(x) = (-1)^m (x^2 - 1)^{m/2} \left(\frac{d}{dx}\right)^m Q_n(x)\,. \tag{16}$$

Care must be taken to account for the metrics in the definitions of surface area and gradients in this coordinate system; the element of surface area, dS, is

$$dS = \sqrt{(r'^2 + c^2)(r'^2 + c^2 \cos^2\theta)} \sin\theta \, d\theta \, d\phi\,, \tag{17}$$

whilst the $\hat{\mathbf{r}}'$-component of the gradient has the form

$$\hat{\mathbf{r}}' \cdot \nabla f = \sqrt{\frac{r'^2 + c^2}{r'^2 + c^2 \cos^2\theta}} \frac{df}{dr'}\,. \tag{18}$$

The integral (6) becomes

$$\phi_m(\mathbf{r}_j)=(-1)^{m+1}\frac{\chi i}{4\pi c}\int_S (r'^2+c^2)b\sum_{p,q}\kappa_p^q Y_p^q(\theta,\phi)Q_p^q(ir'/c)$$
$$\times\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[\left(\frac{(n-m)!}{(n+m)!}\right)Y_n^m(\theta,\phi)Y_n^m(\theta_j,\phi_j)\right.$$
$$\left.\times Q_n^m\left(\frac{ir'_j}{c}\right)\frac{d}{dr'}P_n^m\left(\frac{ir'}{c}\right)\right]d\Omega\,. \quad (19)$$

Performing the angular integration leads to

$$\phi_m(\mathbf{r}_j)=\frac{\chi b}{c^2}\sum_{n,m}(-1)^m\left(\frac{(n-m)!}{(n+m)!}\right)\kappa_l^m Y_n^m(\theta_j,\phi_j)Q_n^m(ir'_j/c)$$
$$\times\left[(c^2+r'^2)Q_n^m\left(\frac{ir'}{c}\right)\frac{d}{d\left(\frac{ir'}{c}\right)}P_n^m\left(\frac{ir'}{c}\right)\right]_{r'=b-d}^{r'=b}. \quad (20)$$

This is the final result: a single harmonic of the internal (inducing) field generates one single harmonic of the magnetic field due to the crust, where the latter is equal to the former multiplied by an 'amplification factor' α_n^m, where

$$\alpha_n^m=\frac{\chi}{c^2}\left(\frac{(n-m)!}{(n+m)!}\right)\left[(c^2+r'^2)Q_n^m\left(\frac{ir'}{c}\right)\frac{d}{d\left(\frac{ir'}{c}\right)}P_n^m\left(\frac{ir'}{c}\right)\right]_{r'=b-d}^{r'=b}. \quad (21)$$

For illustrative purposes, consider the field induced by the dipole component of the core field κ_1^0. In this case we find that the amplification factor α_1^0 is

$$\alpha_1^0=\frac{\chi}{c^2}\left[(r'^2+c^2)\left(\frac{ir'}{2c}\log\left(\frac{ir'/c+1}{ir'/c-1}\right)-1\right)\right]_{b-d}^{b}$$
$$=\frac{\chi}{c^2}\left[-(r'^2+c^2)\left(\frac{r'}{2c}\tan^{-1}\left(\frac{-2c/r'}{1-c^2/r'^2}\right)+1\right)\right]_{b-d}^{b}. \quad (22)$$

The argument of the $\tan^{-1}$ function is small and the function can be approximated by its Taylor series as $-2c/r'+2c^3/3r'^3-2c^5/5r'^5$; we find for a very thin shell, as is the case for the Earth, the approximate result

$$\alpha_1^0\approx\frac{4}{15}\frac{c^2}{b^2}\chi\frac{d}{b}\,. \quad (23)$$

3 RESULTS AND DISCUSSION

We compute typical results for an assumed homogeneous shell covering the Earth, taking the values to be those appropriate for old oceanic crust. Although it is accepted that the primary source of remanent magnetism in the oceans is layer 2A, we must consider the susceptibility of the much thicker underlying gabbros, down to the Moho. An upper value for the susceptibility of gabbro is probably 0.1 (with many gabbros falling below this; Vacquier 1972), and we take the thickness of the crust to be 10 km. These values give a product χd of 1 km, which is not at all unreasonable.

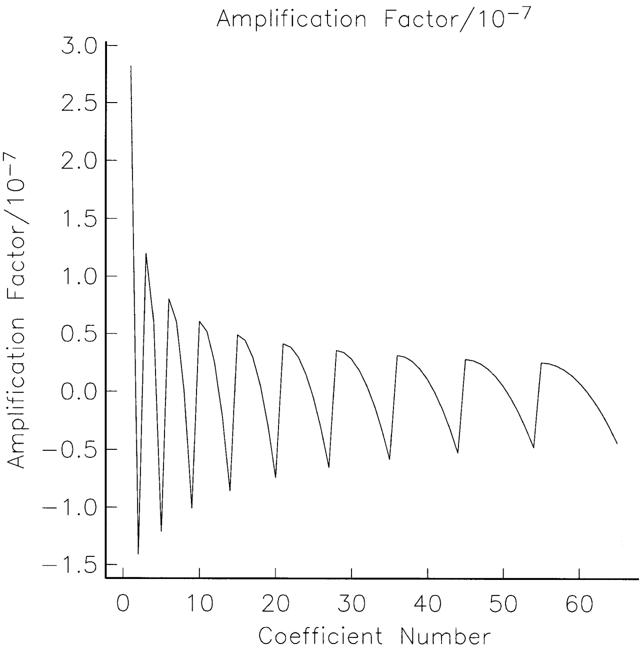

Figure 3. The amplification factor α_l^m which multiplies each spheroidal harmonic of the inducing field is plotted for each coefficient, with degree l and order m running from degree 1 up to degree 10. The coefficients are ordered with l increasing and m increasing within each degree l; the sawtooth pattern derives from the fact that within each degree l, α_l^m decreases from positive to negative values as m ranges from $m=0$ to $m=l$.

Fig. 3 displays the results for all α_l^m up to degree 10. As could be predicted from eq. (23), the amplification factors are tiny. Contrary to expectation, some of the amplification factors are negative, representing diminution of a particular inducing field harmonic; recall that we have treated only isotropic (inherently positive) susceptibilities, and this therefore represents a geometrical effect. Clearly the flattening of the Earth is insufficient to circumvent the effects of Runcorn's theorem to a large degree.

We should note that the treatment given here is inherently linear, and the effects of self-interactions are ignored. When the boundary value problem is solved in its entirety, non-linear effects are introduced which circumvent Runcorn's theorem, even in the spherical case. Results of these calculations are reported in Lesur & Jackson (1999).

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