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GEOMAGNETIC MODELS FROM SATELLITE SURVEYS

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1. INTRODUCTION

Following the first successful operation of artificial satellites during the International Geophysical Year, Chapman (1961), Vestine (1960, 1961), and others recommended that spacecraft carrying magnetometers aid in the World Magnetic Survey planned for the coming International Years of the Quiet Sun (1964-1965). As seen in Figure 1 (Cain, 1966), the total magnetic survey effort of the past two decades left large areas completely void of data and many others with only a single series of observations from which it would be impossible to accurately follow the secular change. It appeared that low altitude, polar orbiting spacecraft would be able to obtain a uniform worldwide net of observations in a period of only a few weeks. At that time more thought was given to the problem of acquiring the observations than to their use in defining a model of the geomagnetic field.

This review will concern itself with an evaluation of how the subsequent spacecraft experiments measuring magnetic field from a low altitude (below 1500 km) have contributed to this survey and how their data have been used to determine numerical models of the internal geomagnetic field. It will be shown that the evidence indicates that surface surveys are no longer needed for defining the broad features of the main field and following secular change. As expected, spacecraft observations are much more comprehensive and quickly obtained, and result in models with higher accuracy.

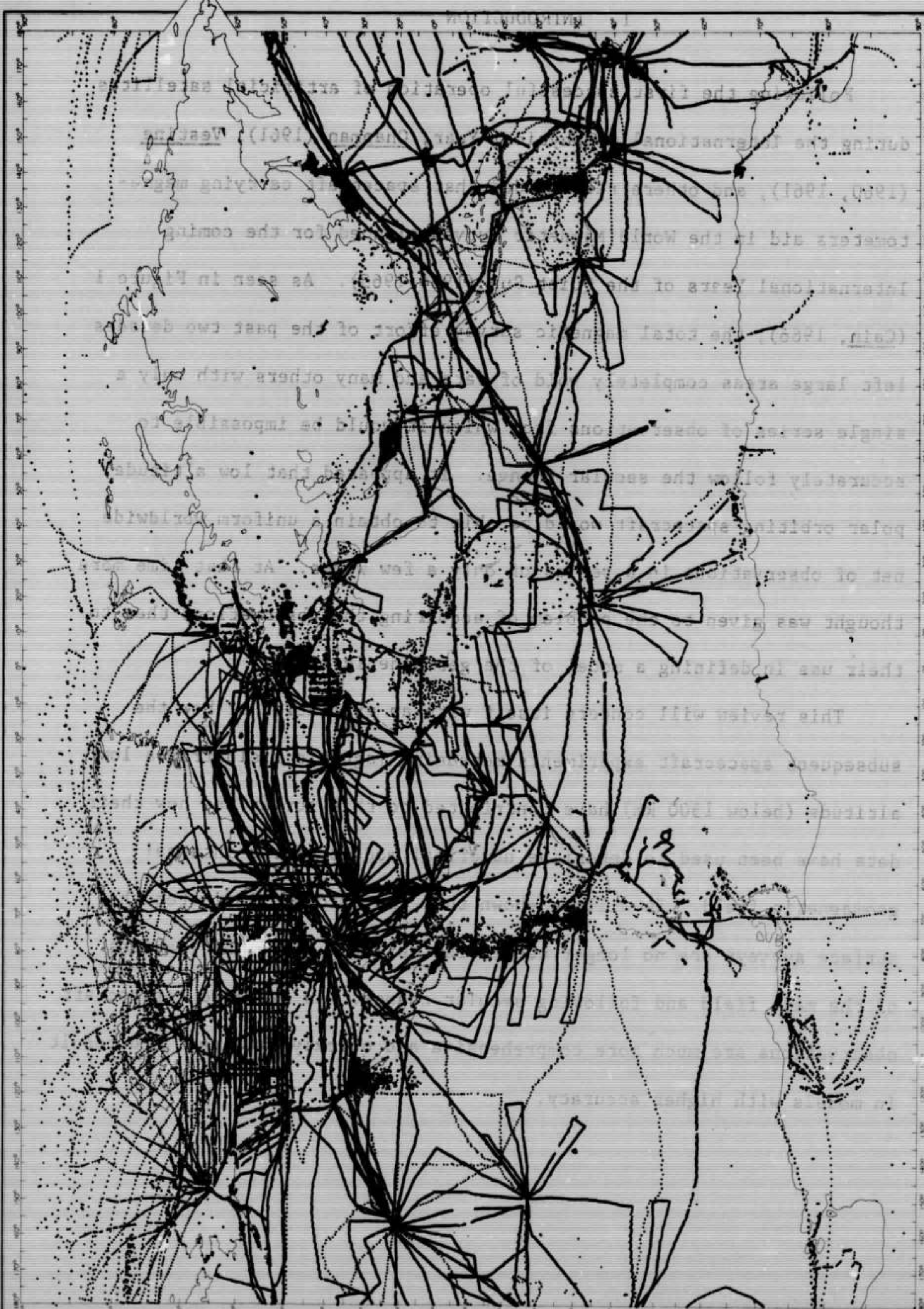

FIGURE 1

MAGNETIC SURVEY OBSERVATIONS 1945 – 1964

2. SATELLITE DATA

It was found at an early stage that vector measurements from spacecraft would be difficult to obtain since the attitude of the instrument would need to be known to a high accuracy. For altitudes under 1500 km where the field intensity is of the order of 0.3 Gauss (= 30,000γ), an uncertainty of only one minute of arc in direction corresponds to component errors of about 9γ. The first, and only, attempt to date to make measurements of the field direction and intensity was by Dolginov et al. (1962) with fluxgate magnetometers on Sputnik 3. Due to various problems, the direction was never determined accurately and only a cursory analysis was possible on the intensity.

As seen in Table 1, the subsequent magnetic surveys were performed with proton precession (Packard and Varian, 1954; Heppner et al., 1958) and alkali vapor (Heppner, 1963; Farthing and Folz, 1967, Dolginov et al., 1970) total field magnetometers. Owing to the lack of on-board recording devices, data from both Vanguard 3 (Cain et al., 1962) and satellite 1964-83C (Zmuda et al., 1968; Zmuda, 1970) could be obtained only when the spacecraft were in sight of receiving stations. All of the Cosmos (Dolginov et al., 1965, 1966; Benkova and Dolginov, 1970; Dolginov et al., 1970) and the POGO (Cain and Langel, 1968) satellites carried recorders and thus acquired data for the whole orbit. Each obtained a complete geographic coverage of the earth up to the latitude given by its inclination.

TABLE 1

Low Altitude Satellite Geomagnetic Measurements 1958-1970

Orbit		Interval	Magnetometer	Coverage	Estimated Accuracy (γ)		
Incl	Alt Range	Months/Year			Instr	Position	Total
65°	440-600 km	V-VI/58	Fluxgates	USSR	-	40	100
33	510-3750	IX-XII/59	Proton Precession	Near Ground Station	4	9	10
49	270-403	III/64	Proton Precession	Whole Orbit	(no published information)		
50	261-488	X-XI/64	Proton Precession	Whole Orbit	3	20	22
90	1040-1089	XII/64-VI/65	Rubidium Vapor	Near Ground Stations	20	3	22
87	413-1510	X/65-IX/67	Rubidium Vapor	Whole Orbit			
86	412-908	VII/67-I/69	Rubidium Vapor	Whole Orbit	3	9	10
82	397-1098	VI/69-VIII/70*	Rubidium Vapor	Whole Orbit			
71	270-403	I-III/70	Cesium Vapor	Whole Orbit	(no published information)		

nts of data for partial orbits are still being acquired

The accuracy of each experiment is estimated in the last column of Table 1. Except for Sputnik 3, the basic accuracy of the magnetometer sensing units was a few gammas. A higher figure is given for 1964-83C since the spacecraft used a strong magnet to orient itself approximately parallel to the geomagnetic field and the resulting calibration uncertainties were about $\pm 20\gamma$.

For the absolute magnetometers the uncertainty in position normally overshadows the errors due to the instrument and the effects of spacecraft produced fields. Positional uncertainty arises both from errors in knowing the absolute time of an observation and knowing the spacecraft coordinates at the assumed time. These factors are of consequence in spacecraft magnetic surveys since it is the difference between the (scalar) measurements and the field, predicted on the basis of some model and the position of the spacecraft, that is actually used in the analysis. Heppner et al. (1960) in a preliminary analysis of Vanguard 3 data noted that plots of ΔF (= $F_{measured}$ minus $F_{calculated}$) showed discontinuities on the days where the orbital arcs were adjusted. Cain et al. (1962) made comparisons between various precision orbits for Vanguard 3 and determined that the error was of the order of 1 km vertically and 4 km horizontally. They showed that gradients in the earth's field would produce errors of 9γ root-mean-square (rms) and up to 50γ maximum. For Sputnik 3 Dolginov et al. (1962) estimate the orbital error contribution to be 40γ, a figure smaller than the 100γ overall error. Part of the large position error noted for COSMOS 49 resulted from a timing uncertainty to ± 0.5 seconds.

3. ANALYSIS

The usual analytic representation of the internal geomagnetic field is a scalar potential function expressed in spherical harmonics (Chapman and Bartels, p. 639, 1940) as follows:

$$V = a \sum_{n=0}^{\infty} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} (g_n^m \cos m\,\varphi + h_n^m \sin m\,\varphi)\; P_n^m(\theta)$$

where

- r, θ, φ = spherical (geocentric) coordinates fixed to the earth. $\theta = \pi/2$ is the equatorial plane; $\varphi = 0$ is the Greenwich half plane.
- a = a scale factor usually taken to be the mean radius of the earth (6371 km)
- g, h = spherical harmonic coefficients
- P = Schmidt's quasi-normalized spherical functions

If there are no electric currents flowing within the volume of measurement, the internal field contribution is given by $\bar{F} = -\nabla V$. Cain et al. (1968) give computer programs useful for evaluating the field from a given set of spherical harmonic coefficients.

Since surface observations were normally of field components, it was easy to determine a finite set of g and h coefficients either from magnetic charts (Vestine et al., 1947; Jones et al., 1953; Fanselau and Kautzleben, 1956; Finch and Leaton, 1957; Jensen and Whitaker, 1960; Nagata and Oguti, 1962, Adam et al., 1962; Vestine et al., 1963; Leaton et al., 1965) or from data themselves (Hurwitz et al., 1966; Fougere, 1963a,b, 1964, 1965, 1966; Leaton, 1963; Winch, 1966). Observations in H (horizontal intensity), D (declination), and either Z (vertical intensity) or I (inclination) were converted to orthogonal components X (north), Y (east) and Z. The expansions were linear in the spherical harmonics and the solutions straightforward (see Chapman and Bartels, 1940, Chapter 20). These analyses were handicapped by the fact that the data covered only part of the earth and were taken so infrequently that it was difficult to account for secular change.

The first attempt to create a field model from only total field satellite observations was made by Cain et al. (1962) and Cain and Hendricks (1964) in analyzing the Vanguard 3 data. In an attempt to organize these satellite data to study time variations, it was noted by Heppner et al. (1960) that even the best recent models based on the analysis of magnetic charts (Finch and Leaton, 1957; Jensen and Whitaker, 1960) were inadequate. Using non-linear least square techniques, models were created which fit only the Vanguard 3 data to an rms deviation between the model and data of only 20γ. As Cain et al. (1962) noted and Leonard (1963) verified, such a model based on sparsely distributed data was unrealistic, particularly over areas of the earth remote from the observations.

Since no comprehensive satellite survey data were then available, the further work with direct analysis of observations was done using combinations of satellite and surface observations. The techniques used consisted essentially in making iterative corrections to an initial set of spherical harmonics so as to minimize the mean square deviations between the observations and the model. Observations of the angles D and I were combined with measurements of force components by weighting their residuals with computed values of H and F respectively. Cain et al. (1965) originally analyzed an assortment of data using weights based on instrumental accuracy estimates for different sources. However, Cain et al. (1967) reevaluated this position to consider the surface anomaly variation as part of the measurement inaccuracy of the surface data. This latter point of view becomes important when one considers that the appropriate weighting factors for a minimum least-squares residual are those inversely proportional to the square of the measurement error. Since the root-mean-square "noise" near the surface due to crustal anomalies is 200-400γ whereas the satellite data are good to at least 50γ including scatter caused by time variations, the satellite data are weighted at least $(200/50)^2=16$ times a surface measurement. Thus in a combination of data the surface observations have relatively low weight and may contribute little to the resulting model. Certainly, surface observations of total field contribute very little. However, there may be some stabilizing effect of surface component observations in spite of their small weight.

The problem of secular change was dealt with by expanding the g and h coefficients in a power series in time and simultaneously solving for the time derivatives. This technique cannot eliminate the problem of data gaps and irregular distribution. The neglect of large areas of the earth must affect these results as much as it does those derived by separately "updating" for secular change. As Cain (1966) has indicated, the larger the number of terms used in the analysis, the larger is the possible error in areas devoid of data.

3.1 Earth Oblateness

One refinement originally considered by Schmidt (Chapman and Bartels, 1940) revived by Jones et al. (1953), and used by Cain et al. (1965) was to include the earth's oblateness in the utilization of surface data. This sophistication was shown to be necessary for achieving high accuracies but has the disadvantage that a synthesis of the field at the surface requires using positions with varying radius with latitude, and rotating the geocentric vectors F_r and F_θ by a slight angle to obtain X and Z. Malin and Pocock (1969) argue that this consideration is unnecessary. However, Kahle et al. (1964, 1966) have agreed with the value of this procedure and pointed out the possible corrections to the older coefficients sets (those based on surface data where the earth is considered spherical) to allow comparison with the newly derived models.

4. RESULTS

4.1 Accuracy of Satellite Models

Before abandoning surface magnetic surveys for the study of the internal field, one should verify that models based only on spacecraft total field data accurately fit surface vector data taken at the same epoch. At this date no such comparisons have been made exactly in this way since there are not yet available sufficient surface measurements later than 1966. Dawson (1970) has compared 1945-1966 Canadian surface vector data, extrapolated to 1970, with various models and concluded that a model based only on POGO data (Cain and Cain, 1968) gave the best agreement.

One should question first whether an analysis in which a least square fit is made to a set of only total-field data will result in a unique vector field model. Cain and Langel (1968) have described a numerical experiment in which a grid of total-field observations is generated from a specific set of spherical harmonic coefficients. This set of synthetic data is then analyzed by the iterative technique described by Cain et al. (1967) with the result that, no matter what are the initial input coefficients, the coefficients that generated the data are retrieved within the limits of computer round-off error. Although such numerical exercises are not rigorous mathematical proof, they do provide some confidence that the technique gives unique results.

However, as applied to a set of real data, it appears that more variation is possible in the components of the resulting field model than in the scalar magnitude. In comparing two models based on coefficients, Cain and Langel (1968) showed that, in the volume of space occupied by the observations, the two models agreed to 5γ (rms) in magnitude, but only 20, 40, and 50γ in the components X,Y,Z respectively. The maximum differences, and the differences extrapolated to the earth's surface or projected ahead in time, were larger.

A comparison of the coefficients indicates that the sectoral harmonic terms ($n = m$) show more variance between analyses than the zonals or tesserals. Since the sectoral harmonics are those with zeroes along the geographic meridians, one would assume that they would be the most responsive to changes in average field from orbit-to-orbit. At this writing it is unclear as to the reasons for this variations or whether it is related to the larger differences noted in the resulting components compared with the total field.

4.2 Effect of External Sources

A major but deliberate omission in the generation of models of the internal field is the neglect of sources arising from electric currents in the ionosphere (Chapman and Bartels, 1940; Matsushita and Campbell, 1967) and plasma pressures in the magnetosphere. It is now well known that the magnetosphere contributes effects of the order of 30γ during quiet intervals (Hess and Mead, 1968; Williams and Mead, 1969; Sugiura, 1970) and more during magnetic disturbance (Akasofu, 1968; Zmuda, 1967; Langel and Cain, 1968). Since such models as described by Cain and Sweeney (1970) were based on data taken during very quiet intervals, we may assume that the systematic errors would be of the order of a few tens of gammas. The one recent attempt to simultaneously include an external source in an analysis of predominantly surface data (Cain, 1966) did not produce a plausible direction though the magnitude was a reasonable 30γ.

In the analysis of samples of satellites data taken over a limited time span, one should also allow for the fact that although the geographic coverage can be complete and uniform, the orbit plane will undergo only a limited variation in local time. Magnetospheric and ionospheric effects that are local time-dependent would then be averaged into the model.

It would thus appear that these systematic effects could be at least partly responsible for the previously mentioned discrepancies in the models based on spacecraft data. However, it is not clear how systematic time variations would give greater variation to the sectoral harmonics. One would think that systematic errors would be hidden in the zonal terms (m = 0) since they are the only ones constant in geographic longitude. If these systematic effects could be determined and included in the field analysis, the resulting models could be improved so that the errors would be only a few gammas instead of the present estimates of the order of 50γ.

4.3 Secular Change from Satellite Models

Dipole Decrease

If spacecraft total-field data accurately model the field at a given epoch, they should also give an equally accurate model of its secular change. Of course, the available spacecraft data span only a few years so it would not be surprising that the details of the secular change determined from spacecraft data might be inaccurate over areas where the secular change is slow. In an effort to see whether the low-order terms from spacecraft-derived models are plausible, we first collect recent estimates for the secular change of the dipole terms based on conventional surface data. Given in Table 2 are the rates of change of the dipole moment estimated from three different analyses using magnetic observatory and repeat station data. The moment is here expressed in terms of H_0, the equatorial value of the eccentric dipole best approximating the earth's field. The expression for its rate of change in terms of the spherical harmonic components is

$$\dot{H}_0 = (g_1^0 \dot{g}_1^0 + g_1^1 \dot{g}_1^1 + h_1^1 \dot{h}_1^1)/[(g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2]^{\frac{1}{2}}$$

An inherent deficiency in the results shown in Table 2 is the fact that no data for the oceanic areas of the earth were used. All three of these analyses indicate a decrease in H_0 of 17 or 18γ/year in 1960 increasing in 1965 by almost a third. For comparison, Table 3 lists values of $\dot{H}_0$ based on direct fits to surface and satellite data and to satellite data alone.

TABLE 2

Secular Change of Centered Dipole From Analysis Of Magnetic Observatory Annual Means

	Hurwitz and Fabiano (1969)	Leaton and Malin (1967)	Orlov et al. (1970)
Epoch		H_0 (γ/yr)	
1940	-15		
1945	-12	-7	
1950	-10	-7	
1955	-12	-13	
1960	-18	-17	-17
1965	-24		-22

TABLE 3

Secular Change of Centered Dipole From Analysis Of Survey (including observatory) and/or Satellite Data

Data Range	Mean Epoch T_M	H_0 (γ/yr) at T_M	Model	Reference
1900-1965.7	1946	-16	GSFC(12/66)	Cain et al. (1967); Cain and Hendricks (1968)
1940-1963	1957	-19	GSFC(4/64)	Cain et al. (1965)
1945-1964	1959	-21	GSFC(9/65)	Hendricks and Cain (1966); Cain (1966)
1965.8-1967.7	1967	-27	POGO(3/68)	Cain and Cain (1968)
1965.8-1967.9	1967	-26	POGO(10/68)	Cain and Langel (1968)
1965.8-1968.4	1967	-27	POGO(8/69)	Cain and Sweeney (1970)

Cain and Hendricks (1968) evaluated, for the GSFC(12/66) model, values of $\dot{H}_0$ which appeared to show only slight changes since 1900. The values for 1910, 1930, and 1950 were -20, -18, and -16γ/year respectively. These results are discordant with those shown for the GSFC(4/64) and (9/65) models which gave substantially higher values. At this point we must conclude that the GSFC(12/66) results represent an average rate over the interval 1900-1960 and did not contain enough terms to adequately follow the details of the variation of only a decade in period.

Comparing the results from the POGO models given in Table 3 with an extrapolation from Table 2 shows good agreement. Both of these techniques appear to confirm that at least for the last decade there has been a significant increase in the collapse rate of the main dipole. Whether this is merely a modulation hitherto undetected whose period is of the order of about two decades or whether it represents an indication of an imminent reversal (Cox and Doell, 1964; Cox et al., 1967; Cox, 1968) is yet to be seen. Although it is known from such compilations as Vestine et al. (1963) that the decrease of dipole moment since 1830 has averaged 0.05% per year corresponding approximately to the 16γ/year average given by the GSFC(12/66) model, a modulation which could double or halve this rate over a period of one or two decades would have previously gone undetected.

Cox (1968) argues that if field polarity reversals are related to the relative strengths of the geomagnetic dipole and non-dipole component, paleomagnetic evidence indicates that the present field configuration allows only a 5% probability that the current decrease will continue.

However, so little is known of the causes and mechanisms of this process that such predictions cannot be taken too seriously. Indeed, in the same discussion he notes that the distribution of polarity intervals above 0.05 million years in length gives an increasing frequency towards shorter intervals, and predicts that an increasing number of durations less than 50,000 years will be discovered within recent epochs.

Westward Drift

A second general aspect of the core field is that most major features drift westward at about 0.2° per year. This characteristic has been analyzed by Carlheim-Gyllensköld (1897), Bullard et al. (1950), Nagata (1962), and others, and is of great theoretical interest in forming theories of the origin of the field (Elsasser, 1956; Hide and Roberts, 1961; Roberts, 1970; Rikitake, 1970). It also appears that there is good evidence that the rate of westward drift is related to variations in the earth's rate of rotation caused by small alterations in its angular momentum (Vestine, 1953; Cogniard, 1960; Rochester, 1960; Hide, 1966, 1967; Vestine and Kahle, 1968). Since the rate of earth rotation has decreased since 1961, Ball et al. (1968) predicted that the drift of the eccentric dipole position (as defined by the lowest six terms of spherical harmonic expansion) would also soon decrease from its approximate 0.3°/year westward movement. The POGO(3/68) model (Cain and Cain, 1968) did show a value of 0.11°/year for westward drift at epoch 1967 (Kahle et al., 1969). This change is also confirmed by Hurwitz and Fabiano (1969) for epoch 1965 in their analysis of observatory annual means.

The lowest order terms of the satellite-derived model do thus appear consistent with other estimates and give the latest picture of secular change trends.

4.4 Crustal Anomalies

In the preceding discussions we have evaluated the analysis of the earth's internal field and the secular change of the major contribution from its core. It is well known from surface magnetic surveys that there are spatially small (1-50 km) and sometimes intense (.005-2 G.) magnetic features due to magnetic material in the cooler upper 20 km of the earth's crust. Since these sources are localized in this relatively shallow layer, it has been assumed that their effect would disappear by satellite altitude. Rocket measurements (Davis et al., 1965) at one location verify the hypothesis that a strong anomaly observed near the surface begins to disappear at altitudes of the order of 100 km. Sample spectral studies of the surface magnetic field distribution by Alldredge et al. (1963) and Serson and Hannaford (1957) have shown that the amplitude of the spatial anomaly spectrum is highest for wavelengths below a few kilometers and decreases to equivalent amplitudes of only a few gammas beyond 200 km. Above a few thousand kilometers the long wavelength component from the core appears.

The irregular coverage and low accuracy of surface surveys have made the study of possible anomalies in the 200-2000 kilometer range very difficult. Indeed, maps depicting anomalies have generally been constructed over small areas by taking out an assumed background field by passing smooth curves through the data. With such techniques it was generally found that adjacent maps would have discontinuities at their borders.

Only recently have attempts been made to use satellite data to study these longer wavelength anomalies. Although the overall accuracy of the POGO observations are 5-10γ compared with the COSMOS-49 errors of 20-30γ, the latter spacecraft has accrued data at a much lower altitude. For anomaly structures of an extent small compared with the altitude of measurement, the intensity should follow the inverse cube law for a dipole. It is thus not surprising that the first interpretation of a crustal signature in satellite data was done using the COSMOS-49 observations (Zietz et al., 1970). The result was that after fitting the data with a model using the techniques developed by Cain et al. (1967), the residuals were averaged and showed features a few hundred kilomenters across and a few tens of gammas in amplitude. Although there was some tentativeness about the averages due to the large scatter of the data, plausible correlations were noted with tectonics and heat-flow data.

5. CONCLUSIONS AND FUTURE WORK

It should be possible to construct very accurate models of the main geomagnetic field and its secular change using only low-altitude total-field observations obtained with satellites. The present models have been shown to be very precise in total field, but somewhat less so in components. The correction of the data for time variations should reduce these uncertainties by an order of magnitude. There is now available a large body of data from several spacecraft, which should allow more accurate models to be derived from more sophisticated analysis.

Before discontinuing surface magnetic surveys (except for studies of crustal anomalies with wavelengths under 200 km) it would be appropriate to compare surface-vector observations with these definitive models based only on spacecraft data.

Even at the present time it appears that the satellite results give a more up-to-date picture of significant changes in the field than is available from surface sources. The recent increase in the rate at which the main dipole is weakening needs to be followed to determine the future trend. Such analyses as those by Walker and O'Dea (1952), Currie (1968), and Hurwitz et al. (1966) show that secular change cannot be accurately interpolated over intervals longer than four or five years. Thus only spacecraft would be able to perform surveys quickly enough to maintain accurate models of the field.

The benefits of these continued surveys should also be seen in the new information to be learned about the earth's interior. Hide and Malin (1970) have shown that there may be important relations between secular change and the earth's gravitational field. Also, in the course of the more sophisticated analyses necessary to remove time variations, one will need to obtain a more detailed picture of the conductivity of the upper mantle than was determined by Lahiri and Price (1939).

Although the specific form and volume distribution of the sources of magnetic distortion in the magnetosphere may not be of direct interest in deriving models of the core field, it may be easier to incorporate data from satellites measuring the ambient field at higher altitude than to include the external sources as an added unknown in the analyses. The possible effects of substantial electric currents flowing along the field lines in auroral regions (Alfvén, 1950; Bostrom, 1964; Cummings and Dessler, 1967) need to be carefully considered. Since the analyses of potential assume zero current density, any currents within the volume of measurement would have some effect. The choice of data taken only at times when the field is quiet and known auroral electrojet currents are absent does not completely avoid the problem.

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