Geomagnetic Pulsations and the Earth's Outer Atmosphere

T. Obayashi* and J. A. Jacobs†

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Summary

Hydromagnetic oscillations of the Earth's ionized outer atmosphere along the geomagnetic lines of force are considered. The observational evidence of world-wide geomagnetic pulsations yields the distribution of ionic density in the outer atmosphere extending beyond the ionosphere. It is found that the ion density is about 10^3 per cm^3 at a distance of a few earth radii decreasing exponentially to a value of the order of 5 per cm^3 in interplanetary space. A theoretical consideration of the temperature of the outer ionosphere is also discussed.

1. Introduction

Little was known about the nature of the atmosphere beyond the Earth's ionosphere until quite recently. Exploration by radio waves and the study of the emission spectra of night air glow and aurorae give considerable information about the ionosphere, but yield little on the outer ionosphere at greater heights. However, some evidence indicating the existence of an extending outer atmosphere has been known for a long time.

Twilight flashes of the red oxygen lines can be traced up to a height of 1300 km (Elvey 1948). Auroral streamers, particularly those in the sunlit atmosphere, have also been observed up to heights as great as 1000 km (Störmer 1937). Astrophysicists, on the other hand, have been concerned with interstellar gases. Zodiacal light indicated evidence of the scattering of solar rays by extensive clouds of particles in interplanetary space, and Siedentopf, Behr & Elsasser (1953) showed that the density of interplanetary gas is as much as 600 hydrogen atoms per cm^3. Recent developments in radio astronomy, especially the observation of the 21 cm line of hydrogen, provides much information about hydrogen clouds. It is well established that solar corpuscular streams ejected from the Sun are responsible for aurorae, magnetic storms and ionospheric disturbances. Furthermore, certain cosmic ray intensity variations are now generally regarded as being due to the movements of interplanetary clouds (Simpson 1956, Forbush 1956).

Considerable developments have taken place in the last few years. Storey (1953) has shown that "whistling atmospherics" consist of waves travelling along the Earth's magnetic lines of force through the ionized outer atmosphere, and

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* Radio Research Laboratory, Kokubunji, Tokyo, Japan.
† Department of Physics, University of British Columbia, Vancouver 8, Canada.

further investigation should reveal important information on the nature of the Earth's outer atmosphere. Thus theattention of many scientific disciplines has been concentrated in this field. Further insight into the problem has been given by Dungey (1954) who proposed the theory that geomagnetic pulsations might be caused by hydromagnetic oscillations of the Earth's ionized outer atmosphere.

Geomagnetic pulsations are small regular fluctuations of the geomagnetic field, the oscillations lasting for an hour or more. The period varies from a few seconds to about three minutes and the amplitude is usually several gammas. They occur not only during storms but also on otherwise quiet days. Although the history of the study of geomagnetic pulsations is very old, observational knowledge is still inconsistent and sparse. Kato & Watanabe (1957) gave an extensive survey of the observational knowledge including recent interesting results of their own comprehensive researches. In their paper, they emphasize the necessity of a systematic world-wide morphology of pulsations, stressing the value of the International Geophysical year.

Since this problem seems to offer great promise for a clearer insight into the physical nature of the outer atmosphere, some preliminary investigations have been attempted. In the present paper, a theoretical consideration is given of the hydromagnetic oscillations of the ionized outer atmosphere as a cause of geomagnetic pulsations. The latitude variation of the period of world-wide pulsations is also obtained. Applying this observational result to the theoretical model, the distribution of ionic density in the atmosphere is determined. Some discussion of the physical nature of the outer atmosphere is also given.

2. Hydromagnetic oscillations of the outer atmosphere

An ionized gas is capable of a wide variety of oscillatory motions. In general these oscillations may be classified into three particular types of waves, electromagnetic waves, hydromagnetic waves and electrostatic waves (plasma oscillations). For the investigation of geomagnetic pulsations, hydromagnetic waves are the most important since the frequency range corresponding to geomagnetic pulsations is far below the gyro-frequency of positive ions. In a hydromagnetic wave, the positive ions provide the inertia of the oscillation, while the restoring forces are largely magnetic. Thus the oscillation may be regarded as waves in the magnetic line of force, which behaves as a stretched string allowing the propagation of transverse waves.

The possibility of the existence of such waves was first suggested by Alfvén (1942). Waves, known as Alfvén waves, are propagated along the magnetic lines of force with a velocity whose square is equal to the magnetic stress density divided by the ionic density ρ_i,

$$V = \frac{H}{(4\pi\rho_i)^{\frac{1}{2}}}. \tag{1}$$

As the magnetic energy density of the gas becomes large compared with the kinetic energy density, all hydromagnetic motions reduce to hydromagnetic waves. Thus in the outer terrestrial atmosphere, a small disturbance, which may possibly be due to the invasion of ionized clouds into the Earth's magnetic field, will excite hydromagnetic perturbations and propagate as Alfvén waves along the geomagnetic lines of force. This possibility has been considered by Akasofu (1956) and the general solution of outer atmospheric oscillations has been obtained under particular conditions.

In the present study, the oscillation of hydromagnetic waves induced along the geomagnetic lines of force is treated by an analogy to stretched strings; the period along a certain geomagnetic line of force corresponding to co-latitude θ_0 is given by

$$T_p = \int_{\theta_0} \frac{2ds}{V}. \tag{2}$$

As illustrated in Figure 1, the integration is along the line of force, which cuts the Earth's surface at θ_0. The geomagnetic field is assumed to be a centred dipole. The equation of the line of force is then given by

$$\nu = \frac{r}{a} = \frac{\sin^2\theta}{\sin^2\theta_0} \tag{3}$$

and

$$H = \frac{C_0}{\nu^3}(1+3\cos^2\theta)^{\frac{1}{2}} \tag{4}$$

where ν is the distance from the centre of the Earth measured in units of earth radii, and C_0 is 0·3 Γ. Then equation (2) becomes

$$T_p = \frac{8\pi^{\frac{1}{2}}}{C_0\sin^8\theta_0}\int_{\theta_0}^{\frac{1}{2}\pi} \rho_i^{\frac{1}{2}}\sin^7\theta\, d\theta. \tag{5}$$

Since Tp can be observed as the period of geomagnetic pulsations in different latitudes (θ_0), the problem is to solve this integral equation in order to determine the unknown function ρ_i. However, a formal mathematical solution is complicated. Therefore, assuming a particular mathematical form for the function ρ_i, Tp is calculated by numerical integration.

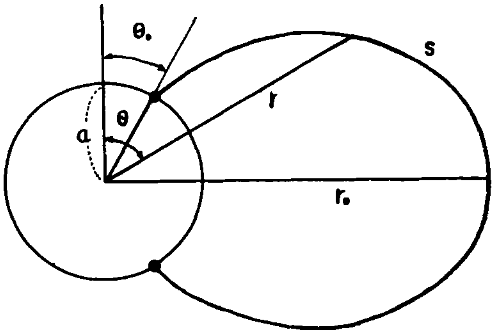

Fig. 1.—*Geometry of the geomagnetic lines of force.*

Following Dungey (1955) the outer atmosphere is assumed to consist chiefly of ionized hydrogen in thermal and hydrostatic equilibrium under gravity. Then

$$\rho_i = m_H N \tag{6}$$

and

$$N = N_s \exp\left(\frac{a}{H_0\nu}\right), \tag{7}$$

where N_s is the ionic density in interplanetary space far from the Earth, and H_0

is the scale height of the ionization at the given temperature and at the value of gravity which is found at the Earth's surface. Putting $\alpha = a/H_0$,

$$T_p = \frac{8a(\pi m_H)^{\frac{1}{2}}}{C_0 \sin^8 \theta_0} N_s^{\frac{1}{2}} \int_{\theta_0}^{\frac{1}{2}\pi} \exp\left(\frac{\alpha \sin^2 \theta_0}{2 \sin^2 \theta}\right) \sin^7 \theta \, d\theta. \tag{8}$$

This equation can also be written in the form

$$\ln T_p = \ln N_s^{\frac{1}{2}} + \ln F(\theta_0, \alpha) \tag{9}$$

and

$$\frac{d}{d\theta_0} \ln T_p = \frac{d}{d\theta_0} \ln F(\theta_0, \alpha). \tag{10}$$

From the observed values of T_p, which are functions of θ_0, α and N_s can be determined by solving the simultaneous equations (9) and (10).

A graph of the calculated curves of $T_p/N_s^{\frac{1}{2}}$ for different values of α is shown in Figure 2. The curves are plotted against $(\cos \Phi)^{-2}$ since, with this abscissa, the relationship is nearly linear for small values of α. $(\cos \Phi)^{-2}$ is proportional to the maximum distance from the Earth's surface of the geomagnetic line of force through geomagnetic latitude Φ.

3. Observational results of geomagnetic pulsations

For the systematic study of world-wide geomagnetic pulsations it is necessary to examine the simultaneous records from many magnetic stations and over a sufficiently long period of time. Though it was not possible to fulfil such requirements in the present study, a considerable number of good photostat copies of magnetograms from several stations were fortunately available. These had originally been collected for the purpose of studying the sudden commencements of magnetic storms. Thus about 80 days of records from 1949 to 1953 were used to investigate simultaneous geomagnetic pulsations.

Magnetic stations whose data are used in the present analysis are listed in Table 1.

Table 1

List of magnetic stations

Station	Abbreviation	Φ	Λ	Station	Abbreviation	Φ	Λ
Tromso	Tr	67·1	116·7	Toyohara*	Ty	36·9	203·5
College	Co	64·5	255·4	Onagawa*	On	28·2	206·0
Sitka	Si	60·0	275·4	Toolangi	Tl	−46·7	220·8
Niemegk	Ni	52·2	96·5	Amberley	Am	−47·7	252·5
Cheltenham	Ch	50·1	350·5	Macquarie Is.	Mq	−61·1	243·1
Tucson	Tu	40·4	312·2				

* For these stations, statistical results from other authors and not actual magnetograms were used.

The magnetograms used here are the ordinary records of the H, D and Z components which run 20 mm per hour. Therefore the minimum resolution of the

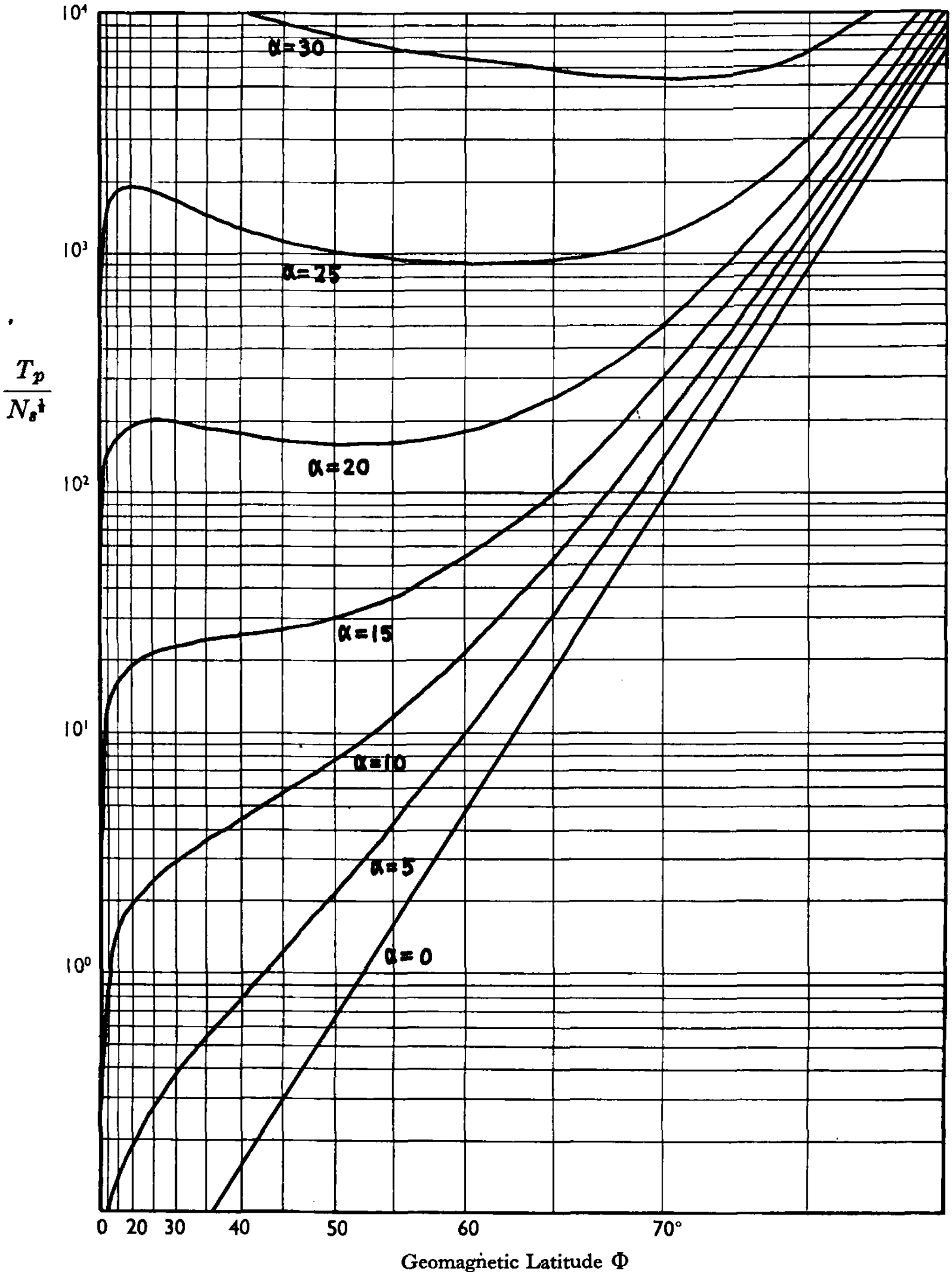

FIG. 2.—*Computed curves of* $T_p/N_s^{\frac{1}{2}}$.

period is of the order of one minute. To find geomagnetic pulsations, special attention was paid to the data from College, Sitka, Cheltenham and Amberley. In particular from the latter two stations, very beautiful pulsations of about one minute period appearing as serrations on the magnetogram were often observed, though for stations in the auroral zone it was rather difficult to identify them owing to additional disturbances. In the present analysis, only those pulsations which were observed simultaneously at more than two of the above mentioned stations were accepted.

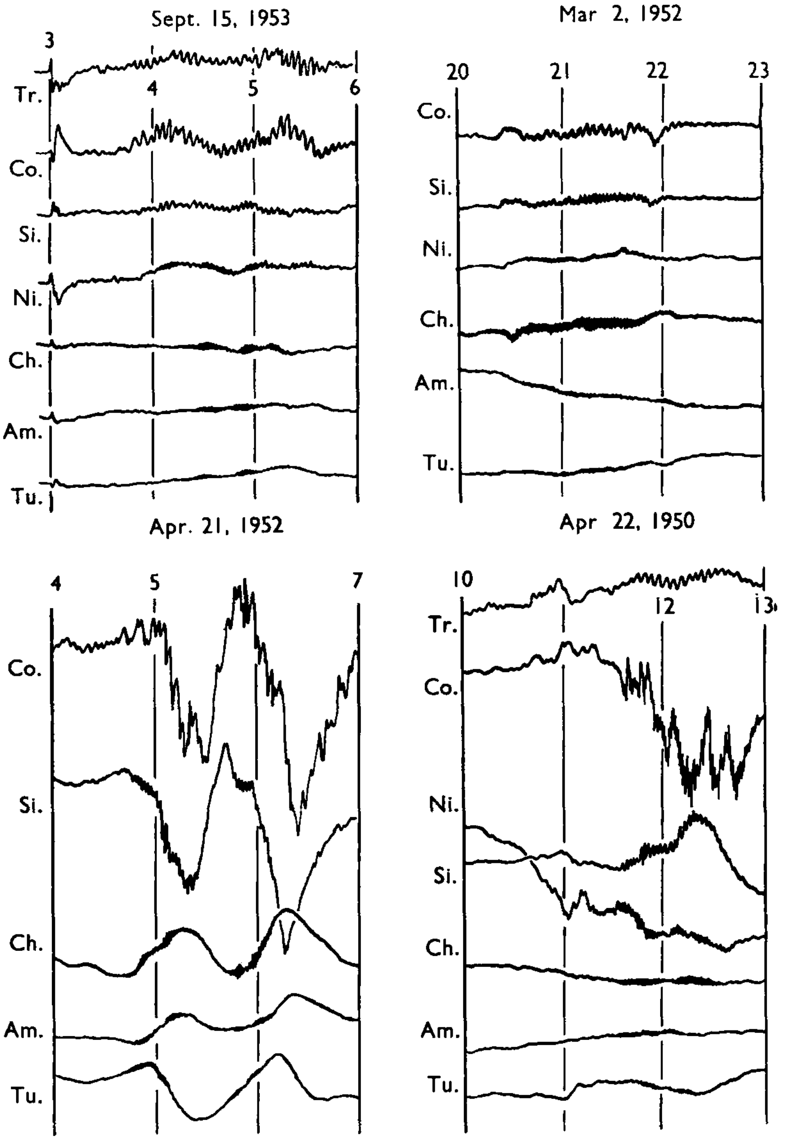

Fig. 3.—*Typical examples of simultaneous world-wide geomagnetic pulsations.*

Typical examples of pulsations are reproduced in Figure 3. Some of them occurred during (magnetically) rather quiet conditions, while others were accompanied by disturbances particularly at the beginning of "bay disturbances". An examination of the data showed that in general the period becomes longer and the amplitude larger with increasing geomagnetic latitude, although during severe disturbances very rapid oscillations were observed in the auroral zone on several occasions.

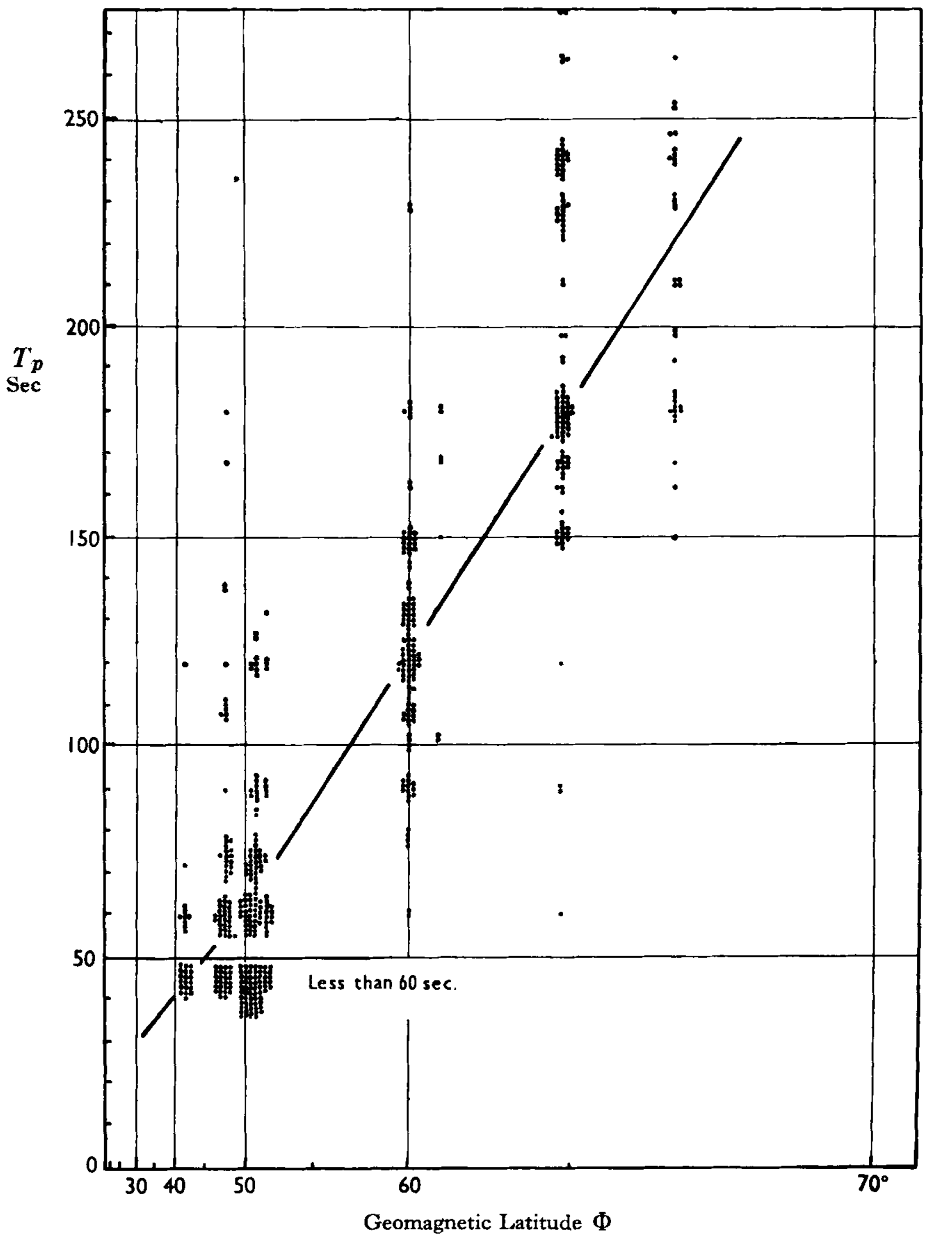

Fig. 4—*Plot of observed periods of geomagnetic pulsations.*

A statistical scatter diagram of the period of pulsation with respect to geomagnetic latitude is shown in Figure 4. Although there is considerable scatter for individual pulsations, the period becomes longer with increasing latitude and, as has been mentioned, an almost linear relationship exists between the period and $(\cos \Phi)^{-2}$.

4. The Earth's outer atmosphere

Applying the observational relationship obtained above, the distribution of the ionic density in the outer atmosphere has been determined. As shown in Figure 5, the best agreement with observational results is obtained when N_s is 3·75 and α is 15. In Figure 5, observational results obtained in Japan by Kato, Ossaka, Watanabe, Okuda & Tamao (1955), Kato & Watanabe (1957) and Hatakeyama (1938) are also added.

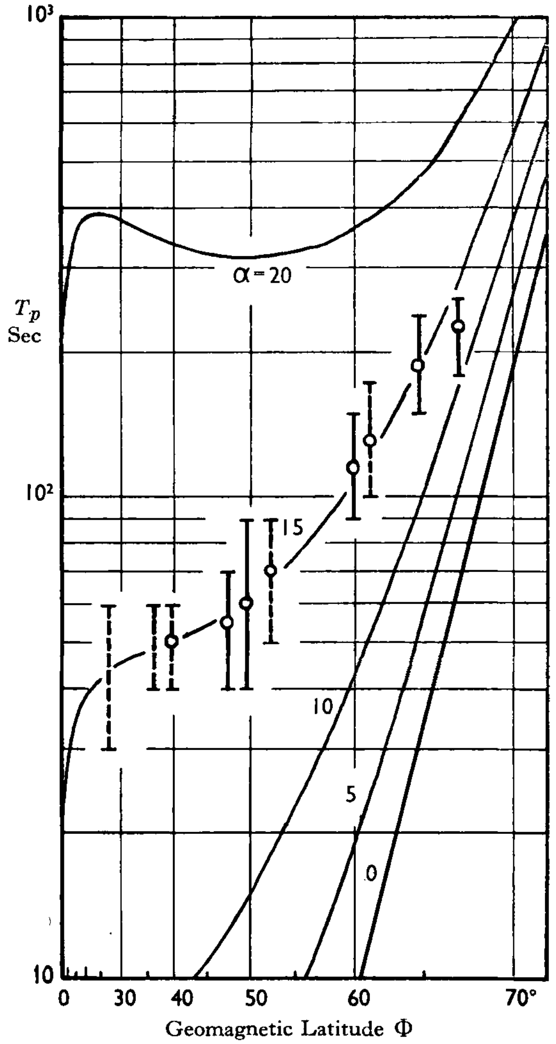

Fig. 5.—*Comparison between observed and theoretical values T_p for $N_s = 3{\cdot}75/cm^3$.*

These values of N_s and α then yield the distribution of the ionic density, which is illustrated in Figure 6. The observational data are obtained mainly from stations whose geomagnetic latitude lies in the range 40° to 65°, the estimated ion distribution from this range corresponding to values of ν between 2 and 5. This portion is indicated by a solid line in the figure.

The ion density is about 10^7 per cm^3 at the Earth's surface (i.e. at the top of the ionosphere), and 10^3 per cm^3 at a distance of a few earth radii. These values are in striking agreement with the completely independent results of ionospheric observations, which give a maximum electron density in the F2 layer of about 10^6 per cm^3. "Whistling atmospherics" indicate 1000–2000 per cm^3 at $\nu = 2 \sim 3$ (Storey 1957). Furthermore, the density in interplanetary space obtained here is of the same order as the value in interstellar space, viz. $N = 1$ atom per cm^3 (in spiral arms) as estimated by Spitzer (1954).

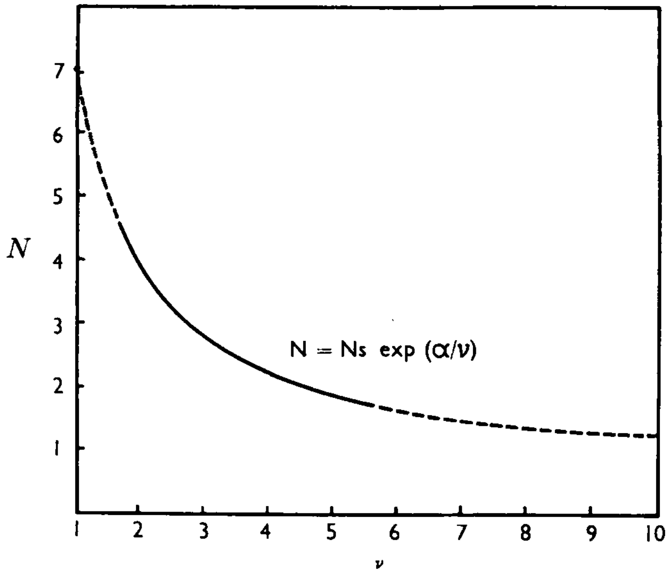

FIG. 6.—*Distribution of ionic density in the outer atmosphere for* $\alpha = 15$, $N_s = 3{\cdot}75/cm^3$.

The temperature of the atmosphere can also be estimated from the value of α. Assuming isothermal hydrostatic equilibrium conditions under gravity, equation (7) can be written in the form

$$\alpha = \frac{a}{H_0} = a \cdot \frac{\mu m_H g_0}{kT} = \frac{a\mu}{0{\cdot}85T}, \qquad (11)$$

where μ is the mean molecular weight and g_0 the value of gravity at the Earth's surface. Taking the value of α equal to 15 gives the scale height $H_0 = 425$ km. Therefore T/μ is equal to 500°K. If the hydrogen gas is fully ionized i.e. $\mu = \frac{1}{2}$, then it gives a temperature of about 250°K.

However, this value of the temperature is certainly too low for ionized hydrogen gas. There is considerable evidence that the temperature of the uppermost regions of the ionosphere is about 1000°–2000°K (Spitzer 1949). The temperature estimated from the scale height does not agree with this. However, there is considerable doubt that the outer atmosphere is in hydrostatic equilibrium governed only by gravity and temperature. The mean free path of the particles becomes extremely large in the outer atmosphere, and also the motion and distribution of the particles, if ionized, will be profoundly influenced by the Earth's magnetic field. Therefore the temperature estimated from the condition of hydrostatic equilibrium is certainly not reliable and it is more likely that the temperature in interplanetary space is well above a few thousand degrees.

In the present study, the cause of the geomagnetic pulsations is attributed to certain ionized clouds which invade the Earth's magnetic field. Since there is some evidence that geomagnetic pulsations show appreciable 27-day recurrence tendency (Kato & Watanabe 1957), it is almost certain that these clouds are of solar origin. If the density of ionized clouds is assumed to be of the order of 10 per cm^3 and the velocity of the order of 500 km/s, the kinetic energy of the clouds is balanced by the stress energy of the geomagnetic field at the distance of ν_0, namely

$$\tfrac{1}{2}\rho V^2 = \frac{H^2}{8\pi}, \tag{12}$$

giving $\nu_0 = 7$. Hence clouds are stopped where the corresponding geomagnetic line of force cuts the Earth's surface where $\Phi \simeq 67{\cdot}5°$. This explains why the amplitude of geomagnetic pulsations near the auroral zone is large, while decreasing gradually towards low latitudes.

Although the observational material used here is not sufficient to reach a firm conclusion, characteristics of geomagnetic pulsations can be explained fairly consistently. The distribution of ionic density in the outer atmosphere obtained from this analysis also shows good agreement with other independent investigations.

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University of Toronto
Toronto, Canada

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