J. Geomag. Geoelectr., 38, 573-597, 1986

Geomagnetic Temporal Change: 1903-1982,

A Spline Representation

R. A. LANGEL1, D. J. KERRIDGE2, D. R. BARRACLOUGH2, and S. R. C. MALIN3

1 Geophysics Branch, Goddard Space Flight Center, Greenbelt, Md. 20771, USA 2 British Geological Survey, Murchison House, West Mains Road, Edinburgh EH9 3LA, Scotland, UK 3 National Maritime Museum, Greenwich London SE10, UK, also Dept. of Physics and Astronomy,

University College, London

(Received September 24, 1985; Revised February 17, 1986)

The secular variation of the Earth's magnetic field is itself subject to temporal
variations. We investigate these with the aid of the coefficients of a series of spherical harmonic models of secular variation deduced from data for the interval 1903-1982
from the worldwide network of magnetic observatories. For some studies it is convenient to approximate the time variation of the spherical harmonic coefficients with a smooth, continuous, function; for this we have used a spline fitting. The phenomena that are investigated include periodicities, discontinuities and correlation
with the length of day. The numerical data we present will be of use for further investigations and for the synthesis of secular variation at any place and at any time
within the interval of the data-they are not appropriate for temporal extrapolations.

1. Introduction

It is well-known that the geomagnetic field changes with time. In the usual representation of the internal potential,

(1)

where ƒÁ, ƒÆ, ƒÓ are the usual spherical coordinates, p m are Schmidt normalized
Legendre functions, a is the mean radius of the earth taken to be 6371.2km, and the
gnm and hnm are called spherical harmonic or gauss coefficients. The magnetic field, B,
is then given by B=-‡™V. The temporal variation is usually included by expanding
the gmn and hnm in a Taylor series:

gn

m(t)=gnm(t0)+gnm(t0)[t-t0］+gnm (t0)[t-t0]2/2!十...... (2)

Most determinations have truncated (2) after the first derivative, or secular variation, terms.
It is now clear that the representation given by Eq. (2) is only adequate to represent the temporal variation over periods of a few years. This can be seen by

573

574 R.A. LANGEL et al.

considering Fig. 1 which shows computed values of g10(t) for 1903 to 1982. The computation is described more fully in a later paragraph; for now, observe that g10(t) has varied considerably with time. Some derivations of gmn(t) have included higher than first derivatives. The GSFC(12/66) model (CAIN et al., 1967) and the IGS75 model (BARRACLOUGH et al., 1975) included second derivative or secular acceleration terms. A plot of g10(t) from these models is included in Fig. 1. The GSFC (9/ 80) model (LANGEL et al., 1982), also shown in the figure, included the third time-derivative terms. It is clear from the figure that g10(t) is not well represented by a first derivative term alone, nor by the models with higher derivatives, except for limited time periods. A linear temporal variation, i.e. constant g10, would provide a good first order description only for 1916-1922, 1940-1950 and 1962-1980. For 1950 to 1965 and 1922 to 1940 the addition of a constant secular acceleration to give a linear variation in g10 would constitute a good model, as, for example, the IGS75 model shown on the figure. The GSFC (9/80) model, with its third time derivative, is adequate only for 1955 to 1980 and has rapidly increasing error the further one goes outside that period. The GSFC (12/66) model was based on data from 1900 to 1965 and included first and second time derivatives. It is a good "average" model but does not describe the

Fig. 1. Data points: g,° versus time from fits to observatory first differences. Solid lines: g10 from the indicated field models.

Geomagnetic Temporal Change 575

detailed variations. These examples illustrate clearly that an alternative representa-tion to (2) is required, one which is considerably more complex.
As part of attempting this more complex representation, a new set of secular variation models were computed. This was done year-by-year from available observatory data, as described in the next section. The resulting yearly values of the secular variation coefficients were then fitted with a cubic spline. The spline representation gives accuracy comparable to the best Taylor series models and has validity over a longer time interval. Selected plots are presented with discussion of some of the major features.
Examination of plots of the yearly secular variation coefficients indicates possible periodicities. A section of the paper is devoted to identifying any real periodicities by spectral analysis.
Finally, the spline representation is combined with a 1980 main field model based
on Magsat data to compute westward drift, which is compared to decade fluctuations in the rotation rate of the earth.

2. The Secular Variation Models

The data set used to derive the set of Secular Variation models was based on the

files of all day observatory annual means maintained by World Digital Data Center C1 (WDDC-C1) in Edinburgh (see, e.g., LEWIS, 1981). Only post 1900 data were used. These were corrected where necessary and checked for internal consistency. At many observatories, site changes or changes in standard have taken place since 1900. All such known changes were taken into account so that all data were referred to the most recent site or standard. For some observatories data for some years were incomplete and in these cases, and also in instances of a single missing value, the data were plotted and values interpolated from the plots. Isolated annual means with no immediately preceding or following values were deleted from the file. After these stages of processing, the file contained annual mean values from which first differences of the annual means could be calculated.

The first differences of the annual means for each observatory were found and plotted. The plots were inspected and a number of erroneous values were found. In cases of suspected error the actual measured elements were inspected. A number of corrections were made but in many cases no obvious source of error could be found. Where the values under suspicion were greatly outside the bounds of the overall level of noise they were deleted. The corrected data were then replotted and reinspected. At the end of this process a file of annual means free from obvious errors was available for analysis. The number of observatories from each hemisphere contributing data in each year from 1899.5 to 1982.5 is shown in Fig. 2.
To avoid non-linear equations, available data were transformed to the local rectangular coordinate system X, Y, Z, where X is north, Y is east and Z is down, assuming a spherical earth. The derivatives of these components, X, Y, and Z, can be computed from (1) and (2). If (2) is truncated after the first time derivative one has:

576 R.A. LANGEL et al.

Fig. 2. Number of observatories contributing to the secular variation models.

(3)

where n*, the truncation level, is chosen according to an estimate of the maximum degree at which meaningful coefficients can be determined from the available data. For each year the set of all available first differences was assembled and for each observation an equation of condition was formed. [This differs from the approach of HODDER (1981) who applied stringent data selection criteria nd smoothed the annual mean values prior to analysis in producing models at two year intervals.] Let

C (ρ; γ, θ, φ) represent any ofthe three derivatives, X, Yor Z, wherepis taken to be the vector of parameters, gnm and hnm, to be solved for. Ci, i=1,...., N are the

measurements. The equations of condition are then of the form:

Geomagnetic Temporal Change

Ci=Σk[∂C/∂pk］Pk+el,

577

(4)

where el is the, supposedly random, measurement error. In matrix form:

C=Ap+e, (5)

where A is the matrix of partial derivatives of the measurements with respect to the p. The equations of condition were solved by the method of least squares; the formation and solution of the normal equations followed the method of MALIN et al. (1982). In this procedure elements of the inverse of the normal equations matrix are generated which allows calculations of a lower bound on the condition number of the normal

matrix to be calculated. The condition number of a matrix, which is equal to the ratio of largest to smallest eigenvalues, is a measure of the relative uncertainty in the solution vector compared to the relative uncertainty of the data (FORSYTHE and MOLER, 1967). For example, if the data are accurate to one part in 105 and the condition number is 10, then the uncertainty in the solution parameters may be as large as one part in 104. Figure 3 shows the lower bound on the condition number for models with n*=6. The figure divides naturally into four time spans: 1900-1930,

Fig. 3. Lower bound on condition number as indicator of the stability of the inversion.

578 R.A. LANGEL et al.

1931-1958, 1959-1980 and 1981-1982. The condition number increases in 1981 and 1982 because data from relatively few observatories were available for the analysis. The form of the plot suggested that lower degree analysis should be attempted before 1931 and a higher degree analysis after 1958. Accordingly a set of n*=5 models for 1903-1930, a set of n*=6 models for 1931-1958 and 1981-1982, and a set of n*=8 models for 1959-1980 were calculated. This choice may be compared with that of HODDER (1981) who calculated n*=5 models before 1927 and n*=6 models from 1927 onwards. The analysis for the n*=5 models remained stable to 1903, but for 1900, 1901 and 1902, where fewer than 40 observatories contributed data, the analysis failed, producing spurious values of the coefficients. The resulting lower bounds of the condition number for the n*=5 models were generally 30 or less; for the n*=8 models this lower bound went up to about 40 and, in some cases, to near 80. No problems were encountered with the n*=8 models. Coefficients for the resulting models are available from WDDC-C1 upon request.

3. Spline Representation of Secular Variation Models

A representation of the secular variation models was achieved by fitting the values for each coefficient with cubic B-splines. B-splines are described by Cox

(1972), DE BOOR (1972) and HAVES (1974), and the algorithms implemented in the "Nag" software of the National Algorithms Group (Mayfield House, 256 Banbury

Road, Oxford OX2 7DE, United Kingdom). Following Cox (1972); if ti, i=1,..., mare a set of strictly increasing real numbers; and if s(t) is a polynomial of degree n-1 or less in each of the intervals

t≦t1; ti-1≦t≦ti, i=2,..., m; tm≦t

and if s(t) and its derivatives up to order n-2 are continuous; then s(t) is a spline function of order n or degree n-1 with knots at the {ti}. The B-spline of degree n-1, with knots at ti-n, ti-n+1,... ti, is defined by

where

(6)

(7)

W'ni=d/dt[Wni]=d/dt[(t-ti-n)(t-ti-n+1)….(t-ti),. (8)

Mni is zero for t•†t, and for t•…ti-n and is strictly positive for ti-n•…t•…ti. For cubic B-splines n is four. The resulting M4i are thus non-zero only over the range t i-4<t<ti, are positive over that range, and intersect the t-axis with zero first and second derivatives at t=ti-4 and t=ti. To fit a set of data over the range ti to th, say, it is

Geomagnetic Temporal Change 579

necessary to specify the desired knots, {t1, i=1,...n}, in the interval (ti, th) and to introduce eight additional knots, t-3, t-2, t-1, to, to+l, to+2, tn+3, tn+4, such that

t-3≦t-2≦t-1≦t0≦tl

th≦tn+1≦tn+2≦tn+3≦tn+4.

Then the general cubic B-spline over [ti, th] is given by

(9)

where the ƒÁi are coefficients determined by a least squares fit of s(t) to the data. If two
or more knots are chosen at the same t, the continuity of the spline is reduced at that point. The Nag software sets t-3=t-2=t-1=t-0=ti and tn+i=tn+2=tn+3=tn+4=th and uses a normalized B-spline defined by

Ni(t)=(ti-ti-4)M4i. (10)

The knots for each gauss coefficient were selected (subjectively) according to the complexity of its variation. As far as possible the same set of knots was used for each coefficient. For coefficients of degree seven and eight (for the interval 1959 to 1980) the fit is accomplished with a single "internal" knot at 1970 plus four knots added by the Nag software at each end point, 1959 and 1982. Similarly, only one internal knot is required for coefficients of degree five and six, this time chosen at 1940 and 1958, respectively. For coefficients of degree one to four the number of internal knots varies from one to six as indicated in Table 1. When one knot is chosen it is at 1940.0. When the number of internal knot is four or greater they consist of a standard set of four at 1920.0, 1935.0, 1950.0 and 1965.0 augmented by knots at 1970.0 (h22, g31, h31, g33), a double knot at 1970.0 (h12), a knot at 1912.0 (g10, g11, g22) or knots both at 1912.0 and 1970.0 (g20, g24). The addition of knots at 1912.0 and/or 1970.0 was required to represent particularly rapid changes in some coefficients at those epochs.
Table 1 gives the representation's standard deviation from the coefficients on
which it is based. These standard deviations should be regarded as a lower bound on the error of the corresponding coefficients when computed from the spline fit. The actual error is some complicated function of the errors in the original spherical harmonic coefficients which, in turn, depends on the data quality and distribution in time and space and on any effects of aliasing due to the finite value of n*.

The representation may be used to compute gnm` or hnm and their first, second and third derivatives (i.e., gnm, g, g); but note that the third derivatives are discontinuous
at the knots and the second derivatives are discontinuous at double knots. A Fortran subroutine, together with additionally required subroutines, which will carry out such a calculation is available from the authors.

580 R.A. LANGEL et al.

Table 1. Number of internal knots and standard deviation of fit to spline coefficients.

4. Results of the Spline Analysis

The resulting spline fits provide a continuous analytic representation of the spherical harmonic coefficients from 1903 through 1982 for degrees 1 through 5, from 1931 through 1982 for degree 6 and from 1959 through 1980 for degrees 7 and 8. Figure 4 gives plots of twelve of the secular variation coefficients. The points are the model from which the spline representation was derived and the solid line is computed from the representation. It can be seen, e.g., fromthat in some cases there were not enough knots in the representation to permit a fit of all the apparent variations.

Geomagnetic Temporal Change 581

However it is also clear that over most of the plots the original coefficients scatter about the spline function. This indicates that in general the error introduced by the spline fits is small relative to the error in the computed spherical harmonic coefficients. Then, to the extent that the scatter in the original spherical harmonic coefficients is an indication of their accuracy, the standard deviations of Table 1 should be a crude estimate of that accuracy. At present it is not clear that the original model accuracy warrants a closer fit; this is left for future study. The variations with time are clearly complex, in some cases wave-like, in other cases having slow trends and in still others exhibiting rapid near-reversals of trend. In view of this complexity it is not surprising that attempts to predict secular variation for more than a year or two

have not given satisfactory results and that the usual Taylor series representation for g and h (Eq. 2) is not accurate over long time periods, even where adequate data are available.

Figure 5 shows selected plots of secular acceleration coefficients (g, h) computed from the representation, together with previously published model secular accelera-tion estimates. The agreement between the model secular acceleration values and those of the spline representation is good considering the uncertainties in each and confirms the validity of the spline fit. As in the case of the secular variation models shown in Fig. 1, it is clear that constant or linear trending secular acceleration is a valid approximation only for limited time intervals.
A test of the accuracy of the spline fit comes from comparing X, Y and Z computed from the representation with first differences at magnetic observatories. For illustration, seven observatories were selected, widely spaced over the earth's surface. These include poor fits as well as good fits. Figure b shows plots of first differences of the annual means of the three components (the points) and of the

582 R.A. LANGEL et al.

(A) YEARS SINCE 1900

(B) YEARS SINCE 1900

(

C) YEARS SINCE 1900

(D) YEARS SINCE 1900

(E) YEARS SINCE 1900

(F) YEARS SINCE 1900

Fig. 4. Secular variation coefficients versus time. Points are from models; solid lines are from the spline representation. Units are nT/yr.

corresponding component derivative computed from the representation (the solid line). Although the spline representation for each coefficient is continuous, the value of n in Eq. (3) changes at 1930-1931 and at 1958-1959. This results in small discontinuities in the computed values of X, r and Z, i.e., in the solid lines on Fig. 6.

Geomagnetic Temporal Change 583

(G) YEARS SINCE 1900

(H) YEARS SINCE 1900.

(

I) YEARS SINCE 1900

(J) YEARS SINCE 1900

(K) YEARS SINCE 1900

(L) YEARS SINCE 1900

Fig. 4 (continued).

Overall, the agreement is quite good. The worst disagreement occurs in pre-1950 X and Yat Apia,)and Z at Coimbra and 2 at Tucson; in pre-1930 X and Yat Sitka and Tucson; and in pre-1920 X and Y at Pilar. Also, some wave-like variations, e.g., Y at Alibag and at Tucson, are too complicated to be followed in detail by the splines. In general the data show more internal scatter and are less well-described by the representation at earlier times, i.e., times of deteriorating quality of data distribution.

5. Periodicities in the Secular Variation

Some coefficients exhibit an apparent wave-like variation, e.g., g10 g21, and h22 in Fig. 4. Caution in interpretation is called for because the spline functions can appear

584 R.A. LANGEL et al.

(A) YEARS SINCE 1900

(B) YEARS SINCE 1900

(C) YEARS SINCE 1900

(D) YEARS SINCE 1900

(E) YEARS SINCE 1900

(F) YEARS SINCE 1900

Fig. 5. Secular acceleration coefficients from the spline representation. For comparison, secular acceleration is shown from the models of (a) CAIN et al, 1967; (b) MALIN, 1969; (c) BARRACLOUGH et al., 1975; (d) BARRACLOUGH and MALIN, 1979; (e) LANGEL and ESTES, 1982.

sinusoidal when the data are not. To determine actual periods three separate spectral analyses were performed: (1) a maximum entropy (MEM) analysis of the entire time period; (2) a Fast Fourier Transform (FFT) analysis of the first sixty four years; and, (3), a FFT analysis of the last sixty four years. These analyses were performed on the coefficient values themselves, as opposed to values computed from the spline fit. In the MEM, the percentage of prediction error coefficients compared to the number of data points is always less than 30%, which should reduce the problem of spurious peaks in the spectra. Table 2 summarizes the results. Underlined periods are considered to be present in all three analyses, allowing for the probable accuracy of the analyses.

Geomagnetic Temporal Change 585

For comparison we note the results of CURRIE (1976) [see also CURRIE, 1973; and COURTILLOT and LE MOUIL, 1976] who computed line spectra from the as magnetic index and noted harmonics of the sunspot cycle and double sunspot cycle, namely periods of 22.9, 11.5, 6.93, 5.58, 4.35, 3.75, 3.37, 3.08, 2.75, 2.40, 2.27 and 2.11 years. The periods of Table 2 recur from coefficient to coefficient, as indicated in Table 3. Some liberty has been taken in the compilation of Table 3 in that ranges of periods are grouped rather than specific periods. This we believe is warranted by the uncertainty of the results. In the case of the period range 9-13 years, associated with the single sunspot cycle, we have grouped three periods: 12.8, 10.7 and 9.1 years, but it is not inconceivable that three distinct periods are involved. The grouping is justified by noting, with HARWOOD and MARLIN (1977), that the sunspot cycle is not asimple sinusoid, but rather shows considerable variation in period and amplitude from one cycle to the next. Note that periods found in only two out of the three spectral analyses were not included in Table 3. These are, perhaps, absent or weak during part of the 1903-1982 time period but may well be significant during some other part of that interval. Many of the periods shown in Table 3 correspond closely enough to those found by CURRIE (1976) to consider them identical. In these cases Currie's results are included in the Table. As noted, CURRIE (1976) attributed all these to the effects of the sunspot cycle, presumably because they correspond to harmonics of his basic 11.5 year period. We cannot confirm or deny the correspondence. This study did not attempt separation of the field into internal and external parts, nor do we think that the data prior to 1959 will sustain such a separation. This means that periodicities associated with external fields can arise in the results both from aliasing caused by omission of the external terms in the models and from associated induced internal

fields. We would note, however, that the usual notion regarding near-earth fields associated with the solar cycle is that they are due mainly to the ring current with some contributions from magnetopause and magnetotail currents. These fields are expected to be almost completely described by the 1st degree gauss coefficients. Contrary to this expectation, Table 3 indicates such periodicities to degree and order three. The geometry of a causative external source is difficult to conceive. Prominent in Table 3, and on the plots, is a period near 23 years. ALLDREDGE (1977) noted such a period in data from magnetic observatories and attributed it to internal sources because it does not appear in all obsevatories (17 out of 49 in his study) and is not in phase at those observatories at which it is present. An external source of reasonable geometry would not result in such fields. We tend to agree that these variations are primarily internal.
Several authors (e.g., ORLOV, 1965; SLAUCITAJS and WINCH, 1965; CURRIE, 1973, 1976; PAPITASHVILLI and ROTANOVA, 1979) claim to have found internal variations with a period of about 60 years. On theoretical grounds BRAGINSKII (1970a, b) has proposed that such periods are expected from "torsional magneto-hydrodynamic vibrations" in the core. The morphology of such waves has been explored in several papers (e.g., GOLOVKOV and KOLOMIYTSEVA, 1971; PAPITASHVILLI et al., 1980; BENKOVA et al., 1982; KALININ and ROZANOVA, 1982; ROTANOVA etal., 1982a, b, 1983). We note that much of the variation in g10 can be

586 R.A. LANGEL et al.

(A)

(B)

(C)

(D)

(E)FG)(H)(

I)

(J)K)

(L)

Fig. 6. Comparison of derivatives computed from the spline representation with first differences from observatory data. Units are nT/ yr.

modeled with a wave of about 54 years period and suggest that this is the major source of the 60-year variations. Similar periods occur in g30, g31 (58.1yrs.), in g32 and g22 (70 yrs.), and in g33 (64yrs.).

Geomagnetic Temporal Change 587

(M)

(N)

(O)

(P)

(Q)

(R)

(S)

(T)

(U)

Fig. 6 (continued).

However, all identifications of the 60-year period are based on a data span of about 100 years or less which, as pointed out by COVRTILLOT et al. (1977), makes its identification as a periodic phenomenon tentative at best. This is particularly true

588 R.A. LANGEL et al.

Table 2. Summary of spectral analysis.
MEM: Results from maximum entropy analysis of entire interval.
FFT1: Results from FFT analysis of 1st 64 years.
FFT2: Results from FFT analysis of last 64 years.
Underline indicates approximate periods considered to be present in all three analyses.

Geomagnetic Temporal Change 589

Table 3. Groups of periodicities.

* c means present in CURRIE's (1976) results.

since MALIN and BOLLARD (1981) failed to find a 60-year cycle in the 400-year records
of declination and inclination at London.

6. Correlation with Length of Day

An apparent periodicity of nearly 60 years also occurs in the decade fluctuations
in the rate of rotation of the earth, called d(ƒ¢T)/dt (MORRISON, 1979, Fig. 4).
STOYKO (1951) was probably the first to notice a correlation in these rate-of-rotation
variations and the magnetic field intensity variations at Paris. Subsequently VESTINE
(1953) claimed to have found a correlation between d(ƒ¢T)/dt and the westward drift
of the eccentric dipole. He noted that conservation of the earth's total angular
momentum requires that fluctuations in the rotation rate of the outer layers, e.g., the
crust and mantle, be accompanied by compensating fluctuations in other layers, e.g.,

590 R.A. LANGEL et al.

the core, and hence, supposing the magnetic field to be frozen into the core fluid, in the
rate of westward drift. Such correlations have since been studied by, e.g., SMIRNOV
(1965), VESTINE and KAHLE (1968), KAHLE et al. (1969), MALIN (1969), YUKUTAKE
(1972, 1973), PRESS and BRIGGS (1975), JIN and THOMAS (1977), GOLOVKOV (1977),
COURTILLOT et al. (1978), LE MOUEL et al. (1981), LE MOUEL and COURTILLOT

(1981, 1982), BRAGINSKII (1982a) and COURTILLOT and LE MOUEL (1984). The work
of PRESS and BRIGGS (1975) considers not only the association of rate of earth
rotation with rate of change of westward drift, but also the association of those

phenomena with the Chandler Wobble. They note two types of time periods: (1) when
the Chandler Wobble is being excited and (2) when it is decaying. Using a pattern
recognition technique they propose the hypothesis:

"Seismic activity in particular belts is associated with dis
placements of lithospheric plates,
which are sufficient to excite the Chandler Wobble. The displacements for these particular
earthquakes include as a component large preseismic and postseismic deformation. In a
subsequent seismically inactive period the Chandler Wobble decays by transfer of angular
momentum to the earth's fluid core. The core motions produce compensatory changes in the
rotation of the earth, conserving angular momentum. The motions in the core also perturb the
geomagnetic field, which becomes evident on the surface after several years of diffusion through
the mantle."

This hypothesis, however, must be considered very speculative.
The most impressive correlation between the geomagnetic field and d(ƒ¢T)/dt
comes from consideration of the secular variation of declination, D, at Paris by LE
MOUEL et al. (1981), reproduced here in Fig. 7. These authors claim that D at Paris is
dominated by westward drift and hence accurately reflects the drift rate. They find
that the magnetic variations lead the A T variations by about 10 years.

Fig. 7. Plot of the secular variation of declination D (solid curve and right hand scale) and of the excess
length of day d(ƒ¢T)/dt (dashed curve and left-hand scale) for the period 1861-1978. (From LE MOUEL
et al.,1981; the sign is reversed for d(ƒ¢T)/dt.)

Geomagnetic Temporal Change 591

We find this picture to be highly oversimplified. D at a single location can hardly
be taken as a measure of global westward drift. A more representative computation of
westward drift can be obtained by using the spline representation of secular variation.
Westward drift, ƒÓ, can be estimated at radius ƒÁ from (WHITHAM, 1958; NAGATA,
1962; JAMES, 1968, 1970; RICHMOND, 1969):

(11)

For comparison with the results of LE MOUEL et al. (1981), ƒÁ is taken equal to a, the
mean radius of the earth. Results will depend somewhat upon the degree to which the
summation is taken. To apply (11) we have used the spline representation (i) to
compute the gnm and hnm and, (ii), to project the GSFC(12/83) main field model
(LANGEL and ESTES, 1985) backward in time to give gnm and hnm at any desired epoch.
The summation was made over degrees 2 through 5. Figure 8 shows 3 computed from
(11) compared to d(•¢T)/dt from STEPHENSON and MORRISON (1984). Some
correlation between d(•¢T)/dt and 3 may be present, but it is not so clear as that for D
in Fig. 7. The cross-correlation between d(•¢T)/dt and 3, for 1903 to 1982, as a
function of the time, ƒÁ, by which 3 leads d(•¢T)/ dt is shown in Fig. 9. The maximum
correlation, 0.562, occurs for a lead time, ƒÁ, of 10 years, as noted by LE MOUE L et al.
(1981). A negative correlation of -0.42 occurs when 3 lags d(•¢T)/dt by 8 years.
The cross correlation was also computed using the results of a spline fit to
d(•¢T)/dt, like that used to derive 3. The maximum correlation was 0.578 at ƒÑ=10
years and the maximum negative correlation was -0.433 at ƒÁ=-8 years. It is clear
that consideration of the global westward drift, ƒÓ, rather than the local D, renders the
supposed correlation much less certain.

Fig. 8. Excess length of day, d(•¢T)/dt, as in Fig. 7, and westward drift, 4, computed from Eq. (9).

592 R.A. LANGEL et al.

Fig.9. Cross-correlation between d(△T)/dt andφ.

The situation is further complicated when it is noted that there is also a
correlation between d(•¢T)/dt and g10. A cross-correlation calculation between g10

and d(•¢T)/dt for 1903 to 1982 gives a peak value of 0.751 for ƒÑ=-10 years, i.e, for

d(•¢T)/dt leading g10 by 10 years. A repeat of the calculation using g10 for 1903 to 1982
but d(•¢T)/dt for 1893 to 1972 shows a cross-correlation of 0.772 when d(•¢T)/dt leads g10 by 13 years. It has been suggested (YUKUTAKE, 1972; WATANABE and YUKUTAKE,
1975) that the variation of the geomagnetic dipole moment should result in variations
in the electromagnetic coupling between core and mantle and hence in variations in
the interchange of angular momentum. BRAGINSKII (1982a, b; 1984) modeled
d(•¢T)/dt with a temporal function consisting of a trend and some damped oscillatory
functions. He found the best fit for a period of 65 years and first and second
harmonics, i.e., 32.5 and 21.7 years. A period near 32.5 years is evident in h22 and h32 in
Table 3; periods close to 21.7 years are also present and have already been discussed.
Interchange of angular momentum between the core and mantle provides a
mechanism whereby the total angular momentum of the earth is conserved while its
outer layers vary in rotation rate. Further, there certainly appear to be correlations
between d(•¢T)/dt and geomagnetic phenomena. However, the physics of these
correlations is far from being firmly established and the subject is at present fraught with speculation.

7. Geomagnetic Jerks

The time variation of g20, h21, h31, and g30, shows a relatively sharp change of
slope at about 1970. This behavior is also exhibited in the corresponding secular
acceleration, which in each case changes sign from pre-to post 1970, and also in the

Geomagnetic Temporal Change 593

westward drift, ƒÓ (see Fig. 8). This change of slope has been discussed, e.g., by
COURTILLOT et al. (1978), DUCRUIX et al. (1980), LE MOULL et al. (1982), MALIN et al. (1983), GUBBINS (1984) and MCLEOD (1985). DUCRUIX et al. (1980) call it a
"secular variation impulse"
, LE MOUL et al. (1982) an "impulse of the secular
acceleration rate", while MALIN et al. (1983) have dubbed it "the geomagnetic jerk of
1970. "It appears to be well approximated by a spike in the third derivative of the field

(g, h) or, equivalently, a step function in the second derivative (g, h), although this is debated. DUCRUTX and LE MOUEL (1983) and GIRE et al. (1983) propose that secular variation can be well-modeled by a linear trend from 1913 (where an earlier impulse is suggested) to 1969 followed by a different linear trend after 1969. The time of the jerk is variously put at "late 1960's" (DUCRUIX et al., 1980), "1969" (LE MOUEL et al., 1982), and "1970" (MALIN et al.,1983).
The sharpness of the jerk as seen in the observatory data is not reproduced by the model representation, although the change in g20, h21, h31, g33 near 1970 looks somewhat like a smoothed step function. In the data plots (see Fig. 6) the 1970 jerk is very apparent in the Z component at Alibag, Apia and Tucson, the Y component at Chambon-la-Foret, Coimbra, Pilar and Sitka and the X component at Chambon-la-Foret, Sitka and Tucson. The spline representation is inherently smoother than the models from which it is derived and so cannot resolve the jerk. In some cases the representation follows the observatory variation very well, as the Y component at Chambon-la-Foret and Sitka, while in other cases it does not follow the jerk at all, as the Y component at Pilar.
DUCRUIX and LE MOUEL (1983) have concluded from examination of observa-tory data that a jerk also occurred at about 1912 or 1913. There is some evidence of this in Fig. 4. The models (but not the spline fit) of g11 show a sharp excursion near 1910; g20 g21 and h33 all show significant changes of slope near 1910, although not as sharp as those seen near 1970; and a less-well defined change occurs in g33 near 1910. None of these changes are as definitive as those seen near 1970, but the data available at the earlier epoch are quite sparse and poorly distributed and may not be sufficient to determine well the variation in the gauss coefficients.
The 1912/13 jerk is apparent in some of the observatory plots of Fig. 6, although
it is not completely clear that the jerk-like variations all occur simultaneously at 1912/13. Some such feature is evident in the Y and Z components at Alibag and in the Y component at Pilar. The representation seems to follow these variations in a reasonable fashion.

From Figs. 4 and 5 it is apparent that the proposed secular variation model of DUCRUIX and LE MOUEL (1983) and GIRE et al. (1983), i.e., linear trends up to 1913, from 1913 to 1969, and from 1969 on, with jerks at 1913 and 1969, is not adequate. The actual secular acceleration is not constant; considerable non-linear change occurs in most coefficients, albeit not as rapid as those near 1912 and 1970. A sharp change is also apparent near 1978 in g10, g21, h21, h22, and h31 (Fig. 4) and in the observatory data: Apia (Y component), Chambon-la-Foret (Y component), Pilar (Z component) and Tucson (X and Ycomponents) (Fig. 6). More post-1980 data from observatories will be required before this phenomenon can be studied in detail.

594 R.A. LANGEL et al.

8. Conclusions

The secular change of the geomagnetic field has undergone a number of significant changes over the interval 1903-1982. Such changes occur in nearly all of the harmonic components of the secular variation, but differ from one harmonic to another. However, spectral analysis reveals a number of periodic or quasi-periodic features that are common to many of the harmonics.
The most commonly occurring period is 2.6-2.8 years which was found in 11 harmonic coefficients. This is followed by 9-13 years (20 harmonics), presumably associated with the sunspot cycle. At the long period end, 40-80 years was found in 8 harmonics, but with a period so close to the length of the data interval this could well be due to a couple of chance turning points and there is little reason to suppose that this effect is truly cyclic. Numerous other periods were detected in fewer harmonics. One of the long-standing problems of geomagnetism is the reality or otherwise of the correlation between westward drift and length of day. Theoretical considerations suggest that there should be such a correlation, and this may have made scientists less critical than they should have been in examining the evidence in the observational data. Certainly there are some geomagnetic phenomena that appear to correlate with the length of day, particularly if one allows a lag or lead of one set of data over the other. However, here we are concerned specifically with westward drift (the global phenomenon) and length of day. Although our data are probably the best available for such a study, and the results are suggestive of such a correlation, we have to conclude that the case is still not proven.
A more convincing correlation exists between g10 and length of day with the former lagging behind the latter by 13 years. The explanation of this correlation, if it is real, is far from obvious.
Another phenomenon that has excited considerable interest over recent years is the geomagnetic jerk-an impulse in the third time derivative of the magnetic field. The most obvious example is clearly visible in annual mean data from individual observatories near 1970 (though another interpretation has been advanced), and others may have occurred near 1910 and 1978. Most previous studies have been concerned with magnetic elements at specific sites, but here we have looked for evidence of the jerk in the harmonic coefficients. Because of the intrinsic smoothing in the analysis, we would not expect and neither do we find as sharp a jerk as in the original data. However, it is found to be present in g20, h21, h31, and g30, for 1970, and in g20 , g21, h33, and (less convincingly) g33 in 1910.
The spline representation proved to be a useful and flexible means of representing the time variations of secular change, though it would be desirable to have an objective criterion for the selection of knots, rather than the subjective choice used here. For some of the above studies, where higher time resolution was required, it was necessary to use the un-splined coefficients. Where a smooth representation is required, the spline function has considerable advantages over, for example, polynomials in the efficiency and accuracy of fit.

Geomagnetic Temporal Change

595

We are grateful to Ron Estes for aid with some of the computations. Some of this work was done while RAL was on sabbatical leave at Bullard Laboratories, University of Cambridge. This paper is published with the permission of the Director, British Geological Survey (NERC).

REFERENCES

ALLDREDGE, L.R., Geomagnetic variations with periods from 13 to 30 years, Geomag. Geoelectr., 29, 123-135, 1977.
BARRACLOUGH, D.R. and S. R. C. MALIN, Geomagnetic secular acceleration, Geophys. J.R. Astron. Soc., 58, 785-793, 1979.
BARRACLOUGH, D.R., J.M. HARwooD, B.R. LEATON, and S.R.C. MALIN, A model of the geomagnetic field at epoch 1975, Geophys. J.R. Astron. Soc., 43, 645-649, 1975.
BENKOVA, N.P., V.P. GOLOVKOV, N.V. KULANIN, and T.N. CHEREVKO, Spatial and temporal distribution of the 60-yr. geomagnetic variations, Geomagn. & Aeron., 22, 523-526 (in English trans., 638-642 in Russian), 1982.
BRAGINSKII, S.I., Torsional magnetohydrodynamic vibrations in the earth's core and variations in day length, Geomagn. & Aeron., 10, 1-8 (In English trans., 3-12 in Russian), 1970a.
BRAGINSKII, S.I., Oscillation spectrum of the hydromagnetic dynamo of the earth, Geomagn. & Aeron., 10, 172-181 (In English trans., 221-233 in Russian), 1970b.
BRAGINSKII, S.I., Analytical description of the secular variation of the geomagnetic field and the speed of rotation of the earth, Geomagn. & Aeron., 22, 88-94 (In English trans., 115-122 in Russian), 1982. BRAGINSKII, S.I., Short-period geomagnetic secular variation, Geophys. Astrophys. Fluid Dyn., 30, 1-78, 1984.
CAIN, J.C., S.J. HENDRICKS, R.A. LANGEL, and W.V. HUDSON, A proposed model for the International Geomagnetic Reference Field-1965, J. Geomag. Geoelectr., 19, 335-355, 1967.
COURTILLOT, V. and J.L. LE MOUEL, On the long period variations of the earth's magnetic field from 2 months to 20 years, J. Geophys. Res., 81, 2941-2950, 1976.
COURTILLOT, V. and J.L. LE MOUEL, Geomagnetic secular variation impulses, Nature, 311, 709-716, 1984. COURTILLOT, V., J.L. LE MOUEL, and P.N. MAYAUD, Maximum entropy spectral analysis of the geomagnetic activity index as over a 107-year interval, J. Geophys. Res., 82, 2641-2649, 1977. COURTILLOT, V., J. DUCRUIX, and J. L. LE MOUEL, Sur une acceleration recente de la variation seculaire du champ magnetique terrestre, C.R. Acad. Sci. Paris, D287, 1095-1098, 1978.
Cox, M.G., The numerical evaluation of B-splines, J. Inst. Maths Applics., 10, 134-149, 1972. CURRIE, R.G., Geomagnetic line spectra-2 to 70 years, Astrophys. Space Sci., 21, 425-438, 1973. CURRIE, R.G., Long period magnetic activity-2 to 100 years, Astrophys. Space Sci., 39, 251-254, 1976. DE BOOR, C., On calculating with B-splines, Approximation Theory, 6, 50-62, 1972.DUCRUIX
, J. and J. L. LE MOUEL, Existence et caractere planetaire d'une secousse de la variation seculaire
du champ geomagnetique en 1912-1913, C.R. Hebd., Seanc. Acad. Sci., Paris, B296, 1419-1424, 1983.
DUCRUIX, J., V. COURTILLOT, and J.L. LE MOUEL
, magnetic variation and the electrical conductivity of the deep mantle, Geophys. J.R. Astron. Soc., 61, 73-94, 1980.
FORSYTHE, G.E. and C.B. MOLER, Computer Solution of Linear Algebraic Systems, 148 pp. Prentice Hall, Englewood Cliffs, N.J., 1967.
LIRE, C., J.L. LE MOUEL, and J. DUCRUIX, Evolution of the geomagnetic secular variation field from the beginning of the century, Nature, 307, 349-352, 1983. GOLOVKOV
, V.P., Relation between secular geomagnetic field variations and the daily rotation of the earth, Geomagn. & Aeron., 17, 488-491 (In English trans., 735-741 in Russian), 1977. GOLOVKOV
, V.P. and G.I. KOLOMIYTSEVA, Morphology of 60-year geomagnetic field variations in Europe, Geomagn. & Aeron.,11, 571-574 (In English trans., 674-678 in Russian), 1971.
GUBBINS, D., Geomagnetic field analysis-II. Secular variation consistent with a perfectly conducting core, Geophys. J.R. Astron. Soc., 77, 753-766, 1984.

596 R.A. LANGEL et al.

HARWOOD, J.M. and S.R.C. MALIN, Sunspot cycle influence on the geomagnetic field, Geophys. J.R. Astron. Soc., 50, 605-619, 1977.
HAYES, J.G., Numerical methods for curve and surface fitting, Bull. Inst. Math. Applications, 144-152, May/June, 1974.
HODDER, B.M., Geomagnetic secular variation since 1901, Geophys. J.R. Astron. Soc., 65, 763-776 and microfiche GJ65/1, 1981.
JAMES, R.W., An equation for estimating westward drift, J. Geomag. Geoelectr., 20, 429-431, 1968. JAMES, R.W., Decomposition of geomagnetic secular variation into drifting and non-drifting components, J. Geomag. Geoelectr., 22, 241-252, 1970.
JIN, R.S, and D.M. THOMAS, Spectral line similarity in the geomagnetic dipole field variations and length of day fluctuations, J. Geophys. Res., 82, 828-834, 1977.
KAHLE, A.B., R.H. BALL, and J.C. CAIN, Prediction of geomagnetic secular changes confirmed, Nature, 223, 165, 1969.
KALININ, Yu. D., and T.S. ROZANOVA, Internal and external components of the 60-yr, variations in the geomagnetic fields, Geomagn. & Aeron., 22, 395-396 (In English trans.), 1982.
LANGEL, R.A. and R.H. ESTES, The near-earth magnetic field at 1980 determined from Magsat data, J. Geophys. Res., 90, 2495-2509, 1985.
LANGEL, R.A., R.H. ESTES and G.D. MEAD, Some new methods in geomagnetic field modeling applied to the 1960-1980 epoch, J. Geomag. Geoelectr., 34, 327-349, 1982.
LE MOUEL, J.L. and V. COURTILLOT, Core motions, electromagnetic core-mantle coupling and variations in the earth's rotation: new constraints from geomagnetic secular variation impulses, Phys. Earth Planet. Int., 24, 236- 241, 1981.
LE MOUEL, J.L. and V. COURTILLOT, On the outer layers of the earth and geomagnetic secular variation, J. Geophys. Res., 87, 4103-4108, 1982.
LE MOUEL, J.L., T.R. MADDEN, J. DUCRUIX, and V. COURTILLOT, Decade fluctuations in geomagnetic westward drift and earth rotation, Nature, 290, 763-765, 1981.
LE MOUEL, J.L., J. DUCRUIX, and C, H, DUYEN, The worldwide character of the 1969-1970 impulse of the secular variation rate, Phys. Earth Planet. Int., 28, 337-350, 1982. LEWIS
, A.G., Annual mean values of geomagnetic elements 1978, Geomag. Bull. Inst. Geol. Sci., 10, 1-296, 1981.

MALIN, S.R., Geomagnetic secular variation and its changes, 1942.5 to 1962.5, Geophys. J.R. Astron. Soc., 17, 415-441, 1969.
MALIN, S.R.C. and E. BULLARD, The direction of the earth's magnetic field at London, 1950-1975, Phil Trans. R. Soc. Lond., 279A, 357-423, 1981.
MALIN, S.R.C., D.R. BARRACLOUGH and B.M. HODDER, A compact algorithm for the formation and solution of normal equations, Computers and Geosciences 8, No. 3-4, 355-358, 1982.
MALIN, S.R.C., B.M. HODDER, and D.R. BARRACLOUGH, Geomagnetic secular variation: A Jerk in 1970, Pub. Obs. L'bro, Memoria No. 14, 239-256, 1983.
MCLEOD, M.G., On the geomagnetic jerk of 1970, submitted to J. Geophys. Res., 1984.
MORRISON, L.V., Redetermination of the decade fluctuations in the rotation of the earth in the period 1861-1978, Geophys, J.R. Astron. Soc., 58, 349-360, 1979.
NAGATA, T., Two main aspects of geomagnetic secular variation-westward drift and non-drifting components, in Proc. Benedum Earth Magnetism Symp., pp. 39-55, 1962.
ORLOV , V.P., The leading trends of the secular variation investigation, J. Geomag. Geoelectr., 17, 277-286,
1965.
PAPITASHVILI, N. Ye. and N.M. ROTANOVA, Method of maximum entropy and its application to the analysis of geomagnetic time series, Geomagn. & Aeron., 19, 359-363 (In English trans., 543-550 in Russian), 1979.
PAPITASHVILI, N. Ye., N.M. ROYANOVA, and A.N. PUSHKOV, 60-year geomagnetic field variations in Europe, Geomagn. & Aeron., 20, 493-496, (In English trans., 711-717 in Russian), 1980.
PRESS, F, and P. BRIGGS, Chandler wobble, earthquakes, rotation and geomagnetic changes, Nature, 256, 270-273, 1975.

Geomagnetic Temporal Change 597

RICHMOND, A.D., Relation of the westward drift of the geomagnetic field to the rotation of the Earth's core, J. Geophys. Res., 74, 3013-3018, 1969.
ROTANOVA, N.M., N. Ye. PAPITASHVILI, and A.N. PUSHKOV, Space-time analysis of the 60-yr field variations according to data from a global network of observatories, Geomagn. & Aeron., 22, 692-695
(In English trans., 847-851 in Russian), 1982a.
ROTANOVA, N.M., N. Ye. PAPITASHVILI, and A.N. PUSHKOV, Use of natural orthogonal components to distinguish and analyse the 60-yr geomagnetic field variations, Geomagn. & Aeron., 22, 821-825 (In English trans., 1003-1009 in Russian), 1982b.
ROTANOVA, N.M., N.E. PAPITASHVILI, S.V. FILIPPOV, and T.A. CHERNOVA, Identification and analysis of 60-yr variations of the geomagnetic field from the time-series of spherical harmonic coefficients, Geomagn. & Aeron., 23, 673-679 (In English trans., 829-836 in Russian), 1983.
SLAUCITAJS, L. and D.E. WINCH, Some morphological aspects of geomagnetic secular variation, Planet. Space Sci., 13, 1097-1110, 1965.
SMIRNOV , N.P., Variations in the length of the day and geomagnetic field variations, Geomagn. & Aeron.,
5, 146-149 (In English trans., 193-197 in Russian), 1965.
STEPHENSON, F.R. and L.V. MORRISON, Long-term changes in the rotation rate of the Earth: 700 B.C. to A.D. 1980, Phil. Trans. R. Soc. Lond., A313, 47-70, 1984.
STOYKO, N., Sur les variations du champ magnetique et le rotation de la Terre, C. R. Hebd. Seanc. Acad. Sci. Paris, 233, 80-82, 1951.
VESTINE, E.H., On variation of the geomagnetic field, fluid motions, and the rate of the earth's rotation, J. Geophys. Res., 58, 127-145, 1953.
VESTINE, E.H. and A.B. KAHLE, The westward drift and geomagnetic secular change, Geophys. J.R. Astron. Soc., 15, 29-37, 1968.
WATANABE, H, and T. YUKUTAKE, Electromagnetic core-mantle coupling associated with changes in the geomagnetic dipole field, J. Geomag. Geoelectr., 26,153-173, 1975.
WHITHAM, K., The relationship between the secular change and the non-dipole fields, Can. J. Phys., 36, 1372-1396, 1958.
YUKUTAKE, T., The effect of change in the geomagnetic dipole moment on the rate of the earth's rotation, J. Geomag. Geoelectr., 24, 19-47, 1972.
YUKUTAKE, T., Fluctuations in the earth's rate of rotation related to changes in the geomagnetic dipole field, J. Geomag. Geoelectr., 25, 195-212, 1973.