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Geometric analysis and optimization of a parabolic concentrator for production at both thermal receivers at its focal point

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Abstract

The exploitation of solar energy for the production of electricity through a thermodynamic cycle is done according to several methods. The parabolic concentration method remains the best because of its high thermal efficiency (about 68%) but the least used because of difficulties related to the storage of its energy. Our work concerns a parabolic reflector with two receivers at its focal point. These two receivers are a boiler and a Stirling engine. The boiler and the Stirling engine receive the energy concentrated by the reflector. In the presence of the Sun it is the Stirling engine that produces the electricity. During the periods when the solar energy is insufficient or absent it is the thermal tank that will send the heat to the fluid that will turn the blades of a turbine to produce electricity. To meet the thermal needs of these two receivers, the efficiency of the reflector's heat production must be improved. To this end, we have studied the possibilities of optimizing the geometric parameters of the parabolic concentrator. For a surface of 12.56 m2 of the reflector, the use of standard geometric parameters allowed us to obtain a daily energy of 824.474 kWh when the solar power is 1104.938 kWh. For the same surface, using the optimized parameters, the production increased to 975.937kWh; a gain of 13.7%. This study allowed us to discover that the geometric and optical parameters strongly influence the thermal production of a reflector.

Keywords: Parabolic Concentrator; Dual Receiver; Optimization; Geometrical Parameters

1. Introduction

A solar concentrator refers to any technique that transforms the sun's radiation into heat and then into electricity through a thermodynamic cycle. Among all solar concentration technologies, the concentration technique using a parabolic reflector remains the best because of its high thermal efficiency (about 68%) [1] and is adaptable to isolated locations [2][3]. Unfortunately, it is the least mature because of the difficulties related to the storage of its energy [4][5]. This study is devoted to the development of a parabolic reflector with dual receivers at the focal point. These receivers consist of a boiler and a Stirling engine. However, in order to meet the thermal needs of these two receivers, it is important to make the thermal production of the concentrator as efficient as possible.

The general objective of this study is to investigate the possibility of optimizing the heat production of a parabolic concentrator with two receivers.

The specific objectives are as follows: 1) study the geometry of the parabolic reflector; 2) evaluate the production by acting on these standard geometrical parameters; and then 3) optimize and evaluate the production using the optimal geometrical parameters.

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2. Materials and method

The study focuses on the geometric and optical parameters of the reflecting surface of the parabolic concentrator, then the optimization of the production for two receivers: The boiler and Stirling engine.

The simulation was developed in python language in the Jupiter notebook environment. It allowed us to obtain the results presented in section III.

Direct radiation measurements were performed on the roof of the Physics Department of the Faculty of Science and Technology using two pyranometers. Figure 1 presents the direct radiation measurement station. The first pyranometer is mounted on a solar tracker in order to measure global radiation with solar tracking. The second one is equipped with a cover in order to measure the diffuse radiation. Thus the direct radiation is calculated by making the difference between the global radiation measured with sun tracking and the diffuse radiation for the same day[8].

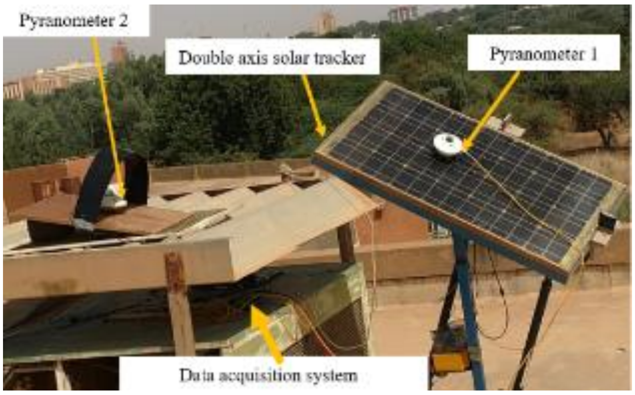

Figure 1 Direct radiation measurement station

For the calculation of energy, we have considered that the system starts to produce from 8am to 6pm, which corresponds to a production time of 10 hours per day.

2.1. Geometric and optical studies of a parabolic reflector

A parabolic reflector is constructed on the basis of its mathematical equation in a double entry frame. It is characterized by the aperture diameter D_{cp}, the focal length f and the depth p. These quantities are related by the relation (1).

$$P^2 = 4fD_{cp} \qquad (1)$$

Figure (2) shows the geometric parameters of a parabolic reflector. In this figure φ_j represents the rim angle of the reflector, r_j is the rim radius, D_{rec} is the diameter of the focal point of the reflector, and θ_i is the incident angle of the solar ray arriving on the reflector surface. La The sum of these angles along the half of the opening of the parabola makes an angle of 90 °[9].

$$\theta_{i1} + \theta_{i2} + \cdots \theta_{in} = 90° \qquad (2)$$

This angle is small and depends on the position of the solar ray on the reflecting surface and is less than 30°[9].

In the simulation we have chosen this angle considering that 900 direct rays of the Sun arrive on the reflective surface of one square meter of half angle of 90°. Thus each ray will have an incident angle of 10°.

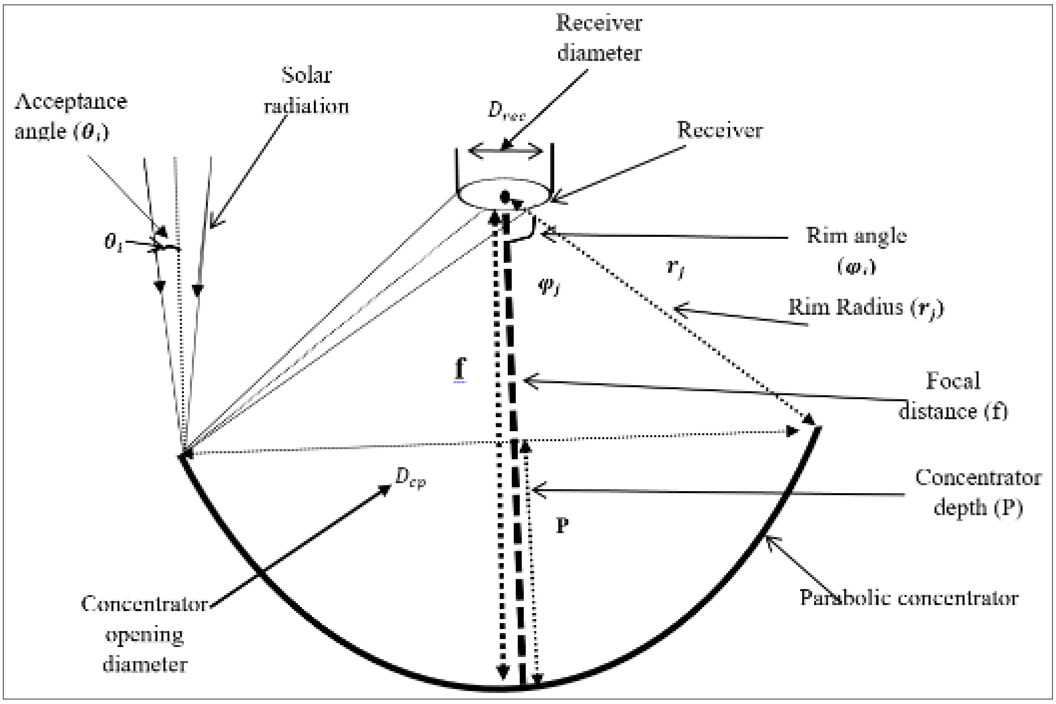

Figure 2 Identification of the geometrical parameters of a parabolic concentrator

The efficiency of a parabolic reflector generally depends on certain optical and geometric parameters.

The theoretical reflector concentration can be evaluated from expression (3). This factor is defined as the ratio of the aperture area of the concentrator S_{cp} to the area of the receiver S_{rec} [10][11].

$$C = \frac{S_{cp}}{S_{rec}} \qquad (3)$$

The parameter that most influences the production of a parabolic concentrator is its surface. In our research we found several expressions to calculate the surface of a parabolic concentrator. Among these expressions, we have chosen the one that is the most precise and the most used. This expression given in (4) allows us to calculate the surface as a function of the diameter D_{cp} of the parabola opening [12][13] [10][14].

$$S_{cp} = \frac{\pi}{4} {D_{cp}}^2 \qquad (4)$$

The aperture diameter of a concentrator D is a parameter that plays a determining role in the production of a parabolic concentrator. It is expressed by the expression (5) as a function of the focal length and the angle.

$$D_{cp} = \frac{4f\sin(\varphi_j)}{1+\cos(\varphi_j)} \qquad (5)$$

The focal area of a parabolic concentrator is fundamental for a better reception of the concentrated heat. It is defined by (6) as a function of the rim radius, the incident angle of the solar radiation arriving on the surface of the concentrator and the rim angle of the concentrator[10].

$$S_{rec} = \frac{\pi r_j^2 \theta_i^2}{{\cos(\varphi_j)}^2} \qquad (6)$$

The diameter of the receiver required to intercept the solar image concentrated at the focus of a reflector is also expressed by the relation (7). It depends on the focal length f, the angle of incidence θ_i and the rim angle φ_j [12][10].

$$D_{rec} = \frac{f\theta_i}{\cos(\varphi_j)(1+\cos(\varphi_j))} \qquad (7)$$

The focal length is expressed according to the formula (8) as a function of the aperture diameter and the rim angle [10].

$$f = \frac{D_{cp}}{4\tan\left(\frac{\varphi j}{2}\right)} \qquad (8)$$

The depth of the parabolic reflector is given by the relationship (9) as a function of the aperture diameter and the focal length of the reflector [15] [16][17][18].

$$P = \frac{{D_{cp}}^2}{16.f} \qquad (9)$$

The rim radius r_j is a very important parameter that should not be neglected during the sizing of a parabolic concentrator system. It plays a role in the production of a reflector. It is calculated from the expression (10). It depends on the focal length and the rim angle [19][14].

$$r_j = \frac{2f}{1+\cos(\varphi_j)} \qquad (10)$$

The power Q_{rec} expressed as a function of the various optical parameters, the geometry of the concentrator, and the material of the reception is given by the equation (11).

$$Q_{rec} = Q_{cp}(\rho\gamma\alpha\tau) \qquad (11)$$

ρ : reflectivity of the reflector; γ : interception proportion (fraction of energy leaving the reflector and not entering the receiver); α : absorption coefficient of the receiver ; τ : air transmittance;

Q_{cp} expressed in (12) is the power of solar radiation received by the opening of the concentrator [20] [21] [22] :

$$Q_{cp} = I_{bn} \times S_{cp} \qquad (12)$$

S_{cp} is the aperture area of the concentrator and I_{bn} is the direct radiation from the Sun.

For our system where we have a double receiver (boiler and Stirling engine) we must integrate in the expression of Q_{rec} the thermal absorption coefficient of these two receivers the expression (12) becomes (13) :

$$Q_{rec} = I_{bn} \times S_{cp}\left(\rho\gamma\alpha_{(s-r)t}\tau\right) \qquad (13)$$

$\alpha_{(s-r)t}$ represents the total absorption coefficient of the receiver which is defined as a function of the absorption coefficient of the Stirling engine by $\alpha_{(s-r)t}$ and the absorption coefficient of the boiler expressed in (14).

$$\alpha_{(s\_r)t} = \alpha_{se} + \alpha_{rc} \qquad (14)$$

The solar energy received by the opening of a parabolic concentrator for a given surface at a defined production time is calculated by (15) :

$$E_{cp} = \left(\sum_{I_{bn}i}^{I_{bnf}} S_{cp} I_{bn}\right) \times \int_{t_i}^{t_f} dt \qquad (15)$$

The thermal energy produced at the focus of the concentrator for a given surface and time is expressed by the formula (16):

$$E_{rec} = \left(\sum_{I_{bn}i}^{I_{bnf}} S_{cp} I_{bn}\, \rho\gamma\alpha_{(s\_r)t}\tau\right) \times \int_{t_i}^{t_f} dt \qquad (16)$$

3. Results and discussion

This part presents the results of our work based on a reflector with an opening diameter of 4 meters (m).

Figure (3) shows the variation of the focal length as a function of the rim angle for an opening diameter fixed at 4 m equation (8).

From this figure we can see that the focal length decreases as the rim angle increases. A rim angle corresponds to a precise value of the focal length in a parabolic reflector.

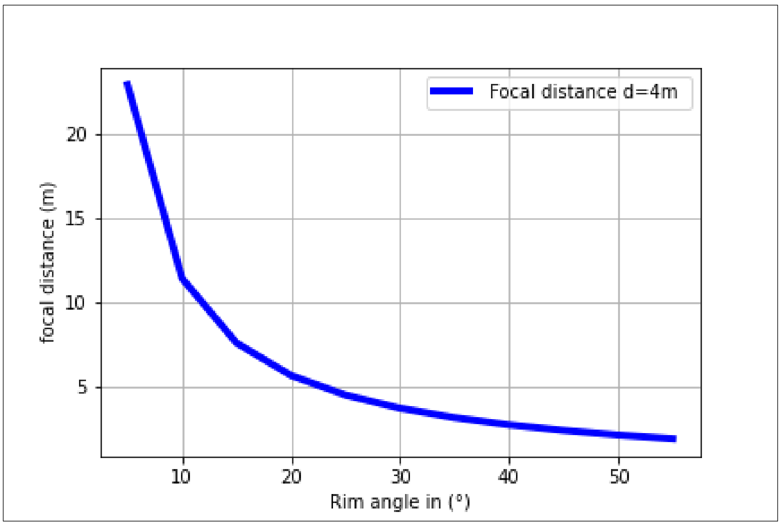

Figure 3 Variation of the focal length as a function of the rim angle for an opening diameter fixed at 400 cm

Figure (4) shows the variation of the concentrator depth as a function of the focal length.

From the curve in this figure, we can conclude that the depth of a parabolic reflector decreases when the focal length increases.

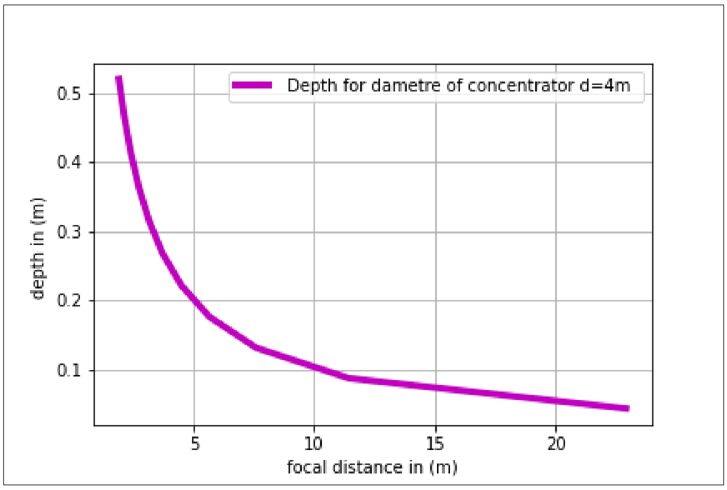

Figure 4 Variation of the depth of the concentrator for an opening diameter of 4 mm

Figures (3) and (4) show that for any given aperture diameter, there can be several depths and focal lengths depending on the rim angle.

The three geometrical parameters presented above (rim angle, focal distance and depth) are parameters to be treated with care because they have an important influence on the heat production of a parabolic reflector.

Figure (5) shows an example of the variation of direct radiation measured by using two pyranometers, the first of which is mounted on a sun-tracking system to measure global radiation with sun-tracking, and the second of which is equipped with a cover to measure diffuse radiation.

This figure shows that the measurement of the direct radiation of the Sun with a solar tracking system leads to a radiation of 200W at about 8 am. The maximum value of 1050W is reached from 13:00, the radiation starts to decrease and reaches 600W around 16:00 for the day of 02_11_21.

La figure (5) montre la variation de la radiation directe mesurée sur le site d'étude avec un système de suivi solaire à 02_11_21.

This figure shows that the direct radiation for this day varies from 200W around 8am to 800W at 9am o clock and reaches a peak with a value of 1050W around 13pm and decreases to a value of 600W around 5pm.

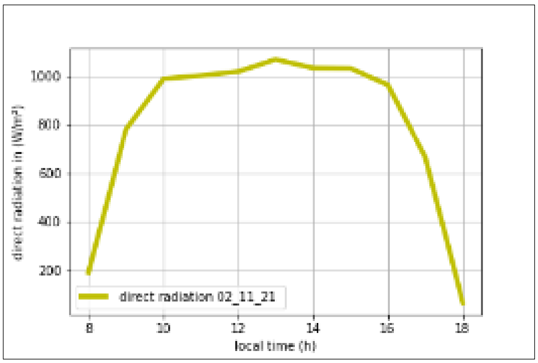

Figure 5 Direct radiation measured for the day of 02_11_21

Figure (6) shows the variation of the solar power received by the reflecting surface (relation (13)) and the thermal power concentrated (relation (14)) by this same surface with an opening diameter of 4 m which corresponds to a surface of 12.56 m^2 expression (4), according to the direct radiation received.

The analysis of this figure indicates that the power of the solar radiation received and concentrated varies according to the intensity of the solar radiation that this surface receives. We also notice that the thermal power varies from 2000W for a radiation of 200W to 9950 for a direct radiation of 1000W while the solar power received by the surface varies from 2050 for a radiation of 200W to 1050W for a radiation of 1000W.

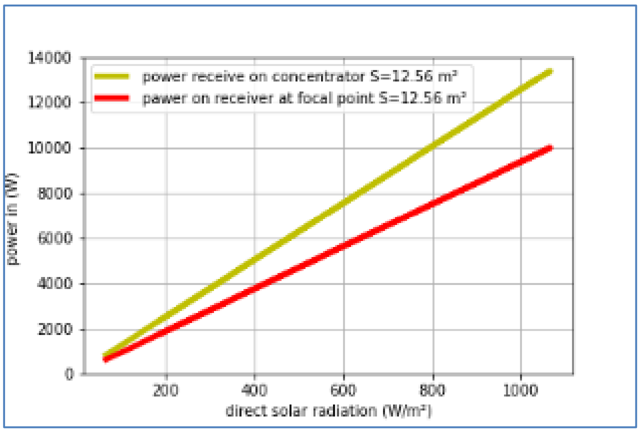

Figure 6 Solar power and thermal power according to the direct radiation received for an opening diameter of 4m

Figure (7) shows the variation of the received solar power and the concentrated thermal power for the opening area of 12.56m^2 as a function of time.

The analysis of this figure shows that the solar power received by the surface of the reflector is greater than the thermal power produced by the reflector at its focus. This is due to the heat loss at the reflector.

We notice that the thermal production for a radiation measured with tracking of the Sun varies from 8000 around 9 am to 10000W around 1 pm then goes down slightly to 8000W at 3 pm. Thereafter, it decreases until 9000W at 16h and 6000W towards 17h.

These curves illustrate the influence of the intensity of direct radiation on the production of a concentrator and that they have the same appearance.

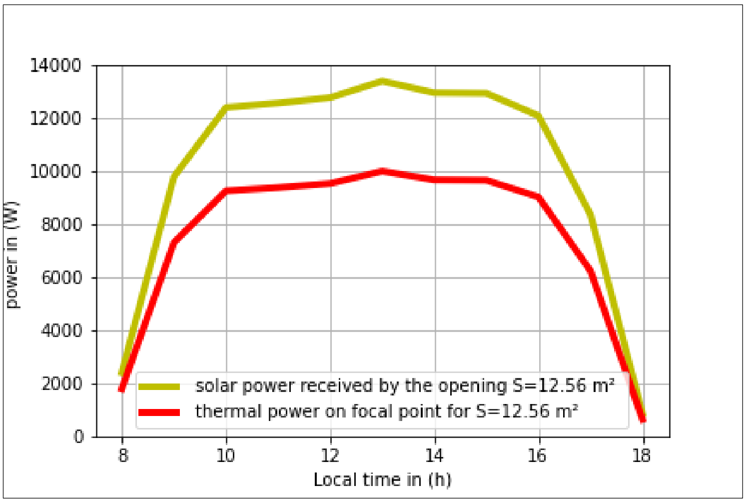

Figure 7 Power received and concentrated at the focus of the reflector according to the time of day

By applying the expression (33) the solar energy received by the opening of this parabolic concentrator for the surface of S = 12.56 m^2 at a time of production of 8h-18h or 10h is :

$$E_{cp} = 1104{,}938\ kWh$$

By applying the expression (34) the thermal energy produced at the focus of this parabolic reflector for this same surface and for the same time is:

$$E_{rec} = 824{,}474\ kWh$$

That is a daily thermal efficiency of 74,6%.

The energy lost due to geometric and optical influences is then :

$$E_p = E_{cp} - E_{rec} = 280{,}464 kWh$$

These losses are enormous, hence the need to reduce them by acting on some parameters related to geometrical influences.

3.1. Optimization of the concentrator production by acting on the geometrical parameters.

To optimize the production, we first expressed the thermal power concentrated at the focus as a function of the geometrical parameters.

- From the relation (4), (5) and (13) we deduce the relation (17) which allows us to express the thermal power as a function of the focal length and the rim angle, the other parameters being considered constant:

$$Q_{rec}(f,\varphi_j) = I_{bn} \times 4\pi f^2 \frac{sin^2(\varphi_j)}{(1+\cos(\varphi_j))^2} \rho\gamma\alpha_{(s\_r)t}\tau \qquad (17)$$

- From the relation (4), (5) and (9) we deduce the relation (18) it represents the thermal power as a function of the depth and the rim angle of the reflector, the other parameters being considered constant :

$$Q_{rec}(P,\varphi_j) = I_{bn} \times 4\pi \frac{P^2.(1+\cos(\varphi_j))^2}{(\sin(\varphi_j))^2} \rho\gamma\alpha_{(s\_r)t}\tau \qquad (18)$$

The opening diameter is fixed at 4 m and the direct radiation at 900W/m^2.

The focal length is calculated from the relation (8), the depth from the relation (9). The value of the aperture diameter obtained by calculation will be compared with the initial experimental value of 4 m.

Reading the data in this table, we notice that when the rim angle exceeds 25° the concentrator opening overflows.

This table allows you to locate the choice of focal length, depth and rim angle while avoiding distortion of the concentrator opening.

Table 1 Calculation of geometric parameters for optimization

rim angle in (°)	Focal length in (m)	Calculated depth in (m)	Diameter calculated in (m)
5	22,9038	0.04366094290851205,	3.9999999999999996,
10	11,4301	0.087488663525924,	3.9999999999999996,
15	7,59575	0.1316524975873959,	4.000000000000001,
20	5,67128	0.17632698070846495,	3.9999999999999996,
25	4,51071	0.2216946626429399,	4.0,
26	3.84	0.26114119243112264	4.0,
30	3,73205	0.26794919243112264,	3.9999999999999996,
35	3,17159	0.31529878887898344,	3.9999999999999996,
40	2,74748	0.36397023426620234,	4.0,
45	2,41421	0.4142135623730951,	4.0,
50	2,14451	0.4663076581549986,	4.0,
55	1,92098	0.5205670505517465	4.000000000000001

Figure (8) shows the production of the reflector with an opening diameter of 4 m, rim angle fixed at 26° and variable depth.

This figure shows that the production of a reflector increases with increasing depth. But it is necessary to identify the optimal depth that does not distort the diameter of the opening table (1).

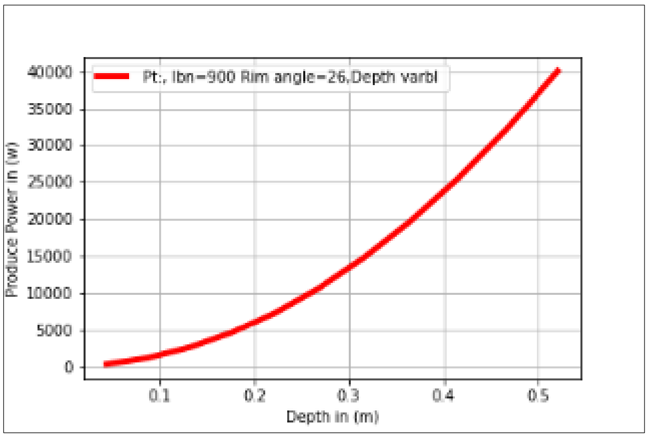

Figure 8 Variation of the reflector output by fixing the rim angle, direct radiation and depth being variable, for a diameter of 4 m

Figure (9) shows the production of the reflector with an opening diameter of 4 m, the depth of the reflector being fixed at 0.26 m.

We see that the production of the reflector decreases with increasing rim angle. It is very important then to identify the rim angle for an optimal production and which does not deform also the shape of the reflector.

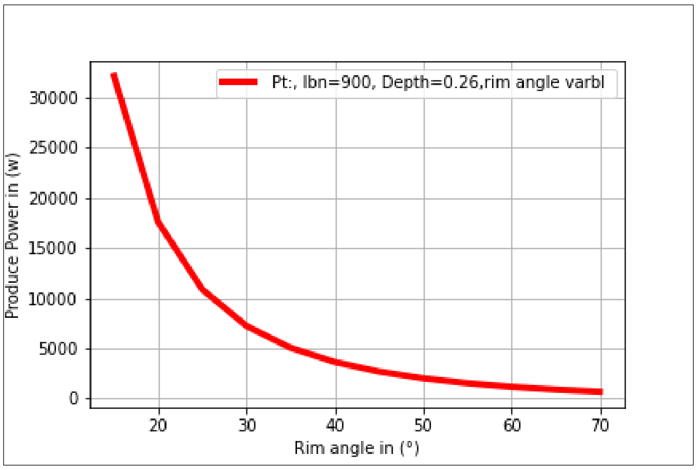

Figure 9 Variation of the reflector output by fixing the depth, the direct radiation and the rim angle being variable for a diameter of 4m

Figure (10) shows the comparison of the production for standard reflector surface and the production with optimized geometrical parameters for an opening diameter of 4 m.

The reading of this figure identifies that with the optimal parameters the production is more important than with the standard parameters.

We can see from this curve that the heat production at the focus of the receiver is only higher than the value obtained with the standard surface if the rim angle is equal to 26°C, with a depth.

Thus, around 9 am the thermal power produced with the standard parameters is 9950W while with the optimized parameters (rim angle 26°, depth 0.26 m) the production is higher than 11.000W from 9 am to 4 pm. At 1pm the production with the standard parameters is 10.000W while the optimized production is 12.000W. At around 4 pm the standard production is 9000W. For the optimized production at 11.000W at the same time.

The two receivers at the focus of the concentrator require an instantaneous thermal power of 11000W. The analysis of the figure (10) shows that with the production using the optimized parameters curved in red, we can provide the thermal need of these two receivers from 9 am to 4 pm.

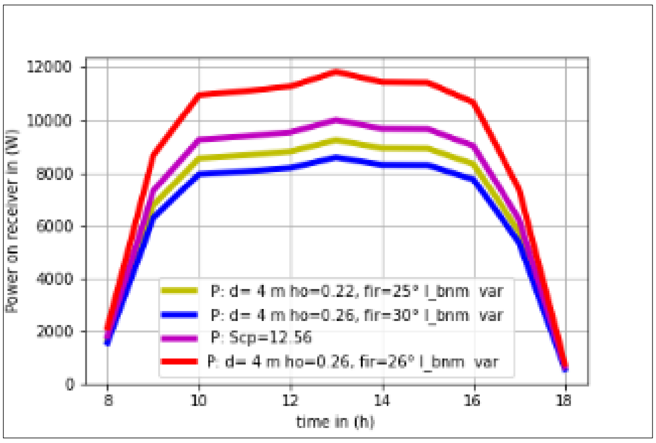

Figure 10 Standard production and optimized for a surface of 4m

The daily energy produced with the optimized geometrical angles of 8h-18h or 10h is:

$$E_{rec} = 975{,}937 kWh$$

This production corresponds to a daily thermal output of 88%.

Compared to the energy obtained with the use of standard geometrical parameters we have a gain of 13.7% or:

$$E_{rec} = 129\ kWh$$

The comparison of these data shows that the geometric parameters have a very strong influence on the output of a parabolic solar reflector.

Table 3 presents the geometric parameters of the reflector for a diameter of 4 meters. These are the optimized values of the geometric parameters. The rim radius and the diameter of the receiver are obtained respectively from the relations (7) and (10).

Table 2 Geometric characteristics of the reflector

Parameters	Values for 4 m diameter
Rim angle	26°C
Rim radius	4.04 m
Focal length	3.84 m
Depth	0.26 m
Receiver diameter	0.39 m

4. Conclusion

We first studied the geometrical parameters of a parabolic concentrator and modeled the heat transfers to the two receivers placed at the focus of a parabolic reflector.

This study proved that the focal length and depth of a parabolic reflector varies with the rim angle of the reflector.

The heat production of a reflector depends on the geometrical parameters, and the direct radiation received. Thus, with a surface of 12.56 m^2 the solar energy received by the surface of the reflector from 8am to 6pm or 10 hours and 1104.938Wh. At the receiver, the thermal energy received corresponding to 10 hours of operation is 824.474 kWh, a loss of 280.464 kWh. These losses are related to the influences of geometric parameters and the quality of the receiving material.

The optimization of the rim angle, the focal distance and the depth allowed us to increase the production. Thus, for the same direct radiation received, the production went from a daily thermal efficiency of 74.6% to 88% with the optimal geometrical parameters, i.e. a gain of 13.4%.

This study proves that it is possible to optimize the thermal production of a parabolic concentrator through the parameters.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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