

**GEOMETRIC PARTIAL
DIFFERENTIAL
EQUATIONS AND IMAGE
ANALYSIS**

GUILLERMO SAPIRO

University of Minnesota



**CAMBRIDGE
UNIVERSITY PRESS**

Contents

<i>List of figures</i>	<i>page</i> xi
<i>Preface</i>	xv
<i>Acknowledgments</i>	xvii
<i>Introduction</i>	xxi
1 Basic Mathematical Background	1
1.1. Planar Differential Geometry	2
1.2. Affine Differential Geometry	7
1.3. Cartan Moving Frames	12
1.4. Space Curves	15
1.5. Three-Dimensional Differential Geometry	17
1.6. Discrete Differential Geometry	20
1.7. Differential Invariants and Lie Group Theory	22
1.8. Basic Concepts of Partial Differential Equations	45
1.9. Calculus of Variations and Gradient Descent Flows	57
1.10. Numerical Analysis	61
Exercises	69
2 Geometric Curve and Surface Evolution	71
2.1. Basic Concepts	71
2.2. Level Sets and Implicit Representations	74
2.3. Variational Level Sets	91
2.4. Continuous Mathematical Morphology	92
2.5. Euclidean and Affine Curve Evolution and Shape Analysis	99
2.6. Euclidean and Affine Surface Evolution	129
2.7. Area- and Volume-Preserving 3D Flows	131
2.8. Classification of Invariant Geometric Flows	134
Exercises	142

3	Geodesic Curves and Minimal Surfaces	143
	3.1. Basic Two-Dimensional Derivation	143
	3.2. Three-Dimensional Derivation	165
	3.3. Geodesics in Vector-Valued Images	182
	3.4. Finding the Minimal Geodesic	191
	3.5. Affine Invariant Active Contours	197
	3.6. Additional Extensions and Modifications	205
	3.7. Tracking and Morphing Active Contours	207
	3.8. Stereo	215
	Appendix A	217
	Appendix B	218
	Exercises	220
4	Geometric Diffusion of Scalar Images	221
	4.1. Gaussian Filtering and Linear Scale Spaces	221
	4.2. Edge-Stopping Diffusion	223
	4.3. Directional Diffusion	241
	4.4. Introducing Prior Knowledge	248
	4.5. Some Order in the PDE Jungle	260
	Exercises	265
5	Geometric Diffusion of Vector-Valued Images	267
	5.1. Directional Diffusion of Multivalued Images	267
	5.2. Vectorial Median Filter	269
	5.3. Color Self-Snakes	281
	Exercises	283
6	Diffusion on Nonflat Manifolds	284
	6.1. The General Problem	287
	6.2. Isotropic Diffusion	290
	6.3. Anisotropic Diffusion	292
	6.4. Examples	293
	6.5. Vector Probability Diffusion	298
	Appendix	304
	Exercises	305
7	Contrast Enhancement	307
	7.1. Global PDE-Based Approach	310
	7.2. Shape-Preserving Contrast Enhancement	325
	Exercises	337
8	Additional Theories and Applications	338
	8.1. Interpolation	338

8.2. Image Repair: Inpainting	343
8.3. Shape from Shading	355
8.4. Blind Deconvolution	357
Exercises	358
<i>Bibliography</i>	359
<i>Index</i>	381