



دانشگاه کیمبریج

Geometric
Spanner Networks

Geometric Spanner Networks

Spring 2014

Course Outline

Textbook

Introduction

Algorithms Review

Greedy Algorithm (Org. and Imp.)

Apx. Greedy Algorithm

(Ordered) Θ -Graph
Algorithm (Sink and
Skip-list spanner)

Sink Spanner

WSPD-based Algorithm

Theoretical
bounds

Applications

Designing approximation
algorithms with spanners

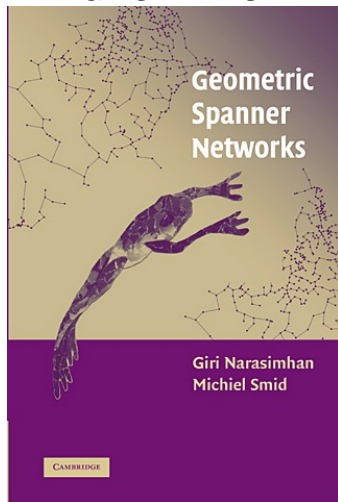
Metric space searching

Protein Visualization

Research Topics

Textbook:

Giri Narasimhan, Michiel Smid, **Geometric Spanner Networks**, CAMBRIDGE UNIVERSITY PRESS, 2007.



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London Underground Network

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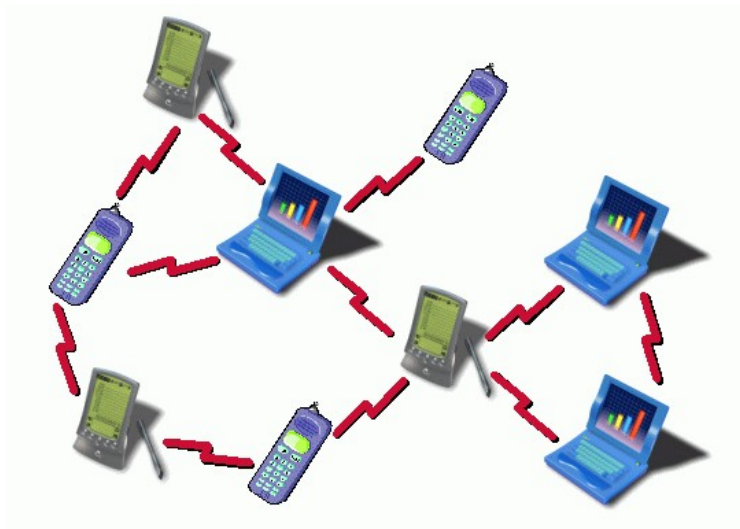
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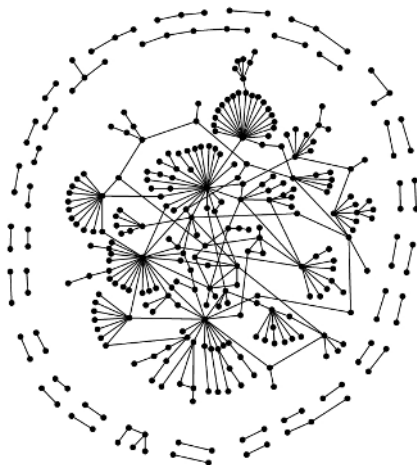
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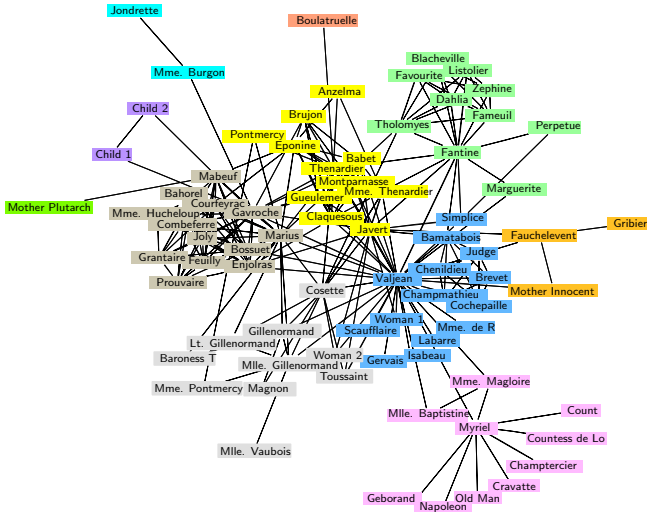
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A Social Network (Les Miserables characters)

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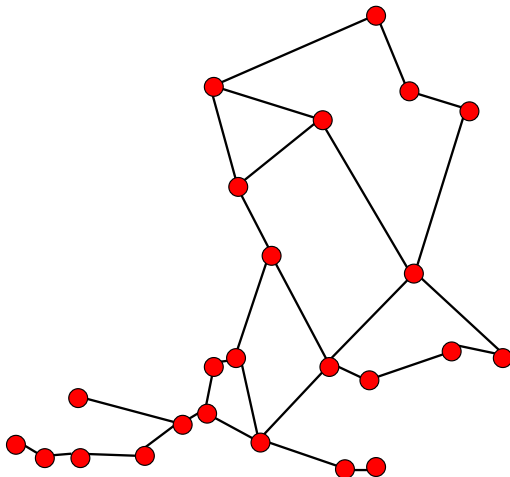
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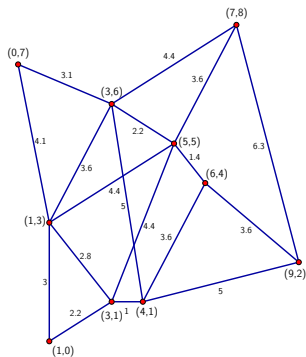
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Weighted undirected graph

$G(V, E)$ s.t.

- $V \subset \mathbb{R}^d$.
- $\forall e = (u, v) \in E, wt(e) = |uv|$.

Network Quality



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- Driving distance: 256 km. Actual distance: 198 km.
- $\frac{\text{Driving distance}}{\text{Actual distance}} = 1.27.$

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• Driving distance: 180 km. Actual distance: 136 km.

• $\frac{\text{Driving distance}}{\text{Actual distance}} = 1.32.$

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- Driving distance: 143 km. Actual distance: 100 km.
- $\frac{\text{Driving distance}}{\text{Actual distance}} = 1.43$.



Dilation (stretch factor)

• between a pair of vertices=

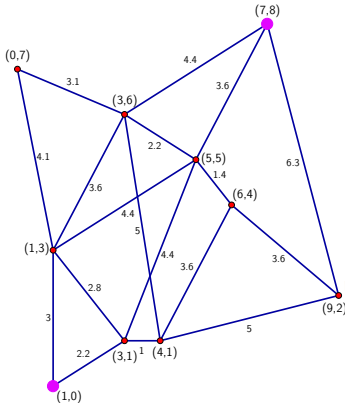
$$\frac{\text{Distance in the graph}}{\text{Euclidean distance}}$$

• of a network= maximum
dilation between all pairs.

t -spanner

A network with dilation at
most t , or
 $\forall u, v \in V$, there is a path
between u and v of length
 $\leq t \times |uv|$.

(t -path)



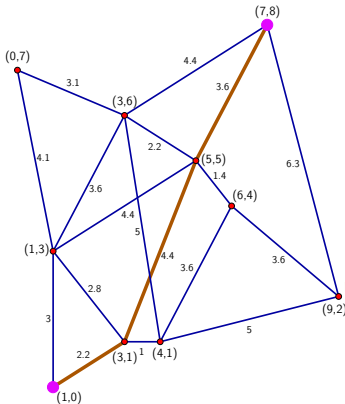


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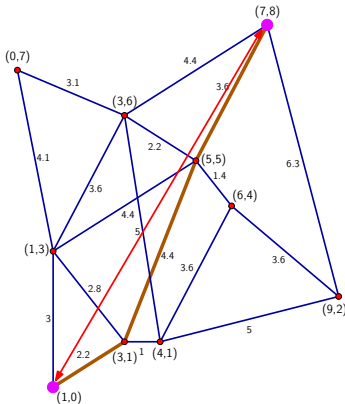
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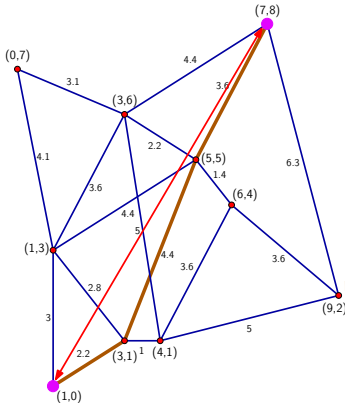


Dilation (stretch factor)

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Dilation (stretch factor)

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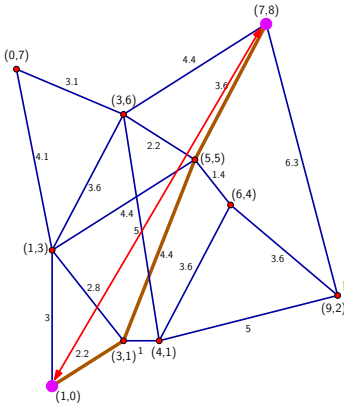
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t -spanner

A network with dilation at most t , or

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(t -path)



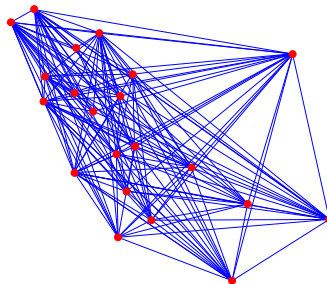
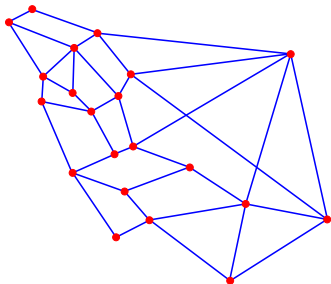
Network Quality



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$(1 + \varepsilon)$ -Spanners **approximate** the complete graphs with error ε .



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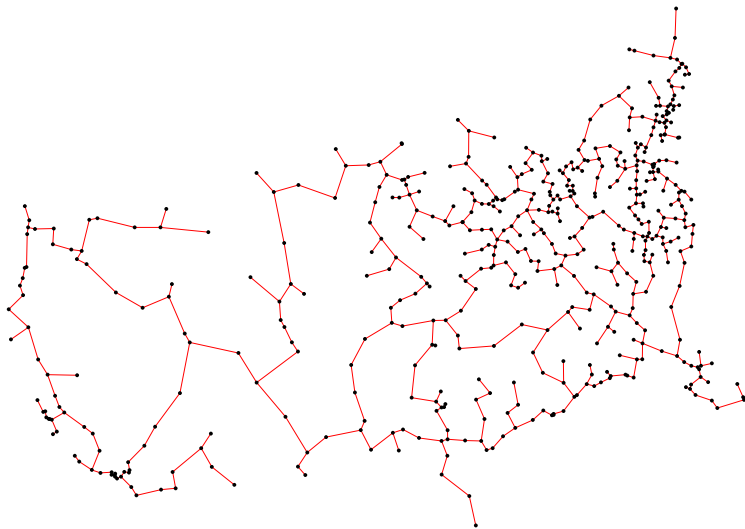
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10-spanner for 532 US-cities



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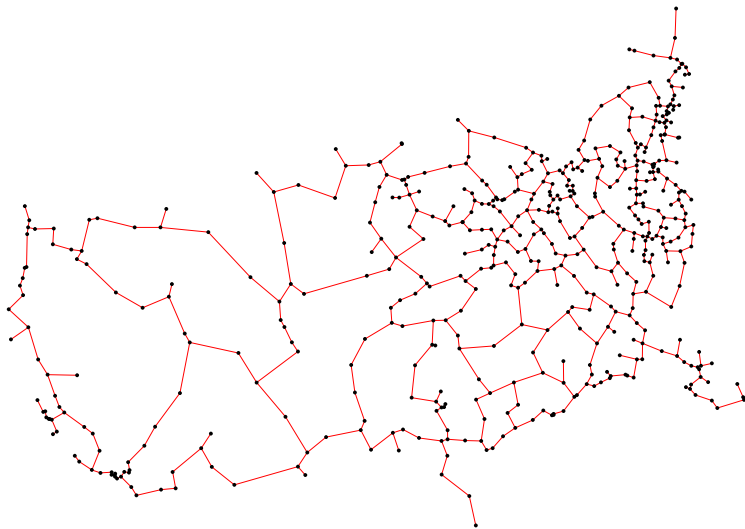
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5-spanner for 532 US-cities



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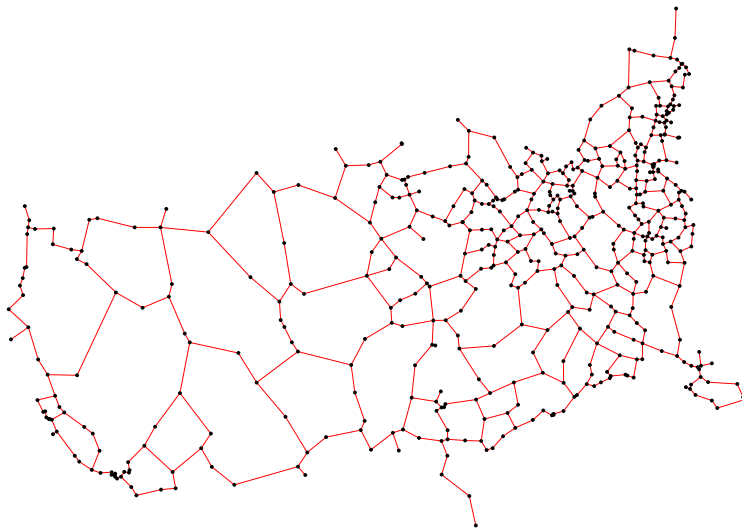
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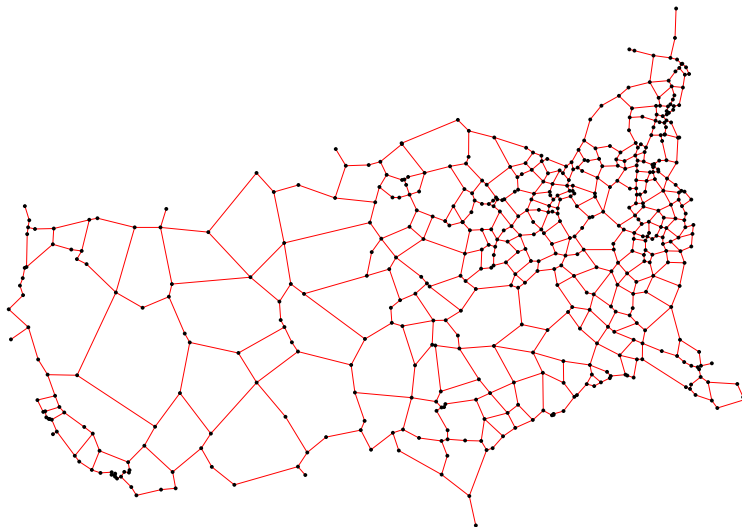
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2-spanner for 532 US-cities



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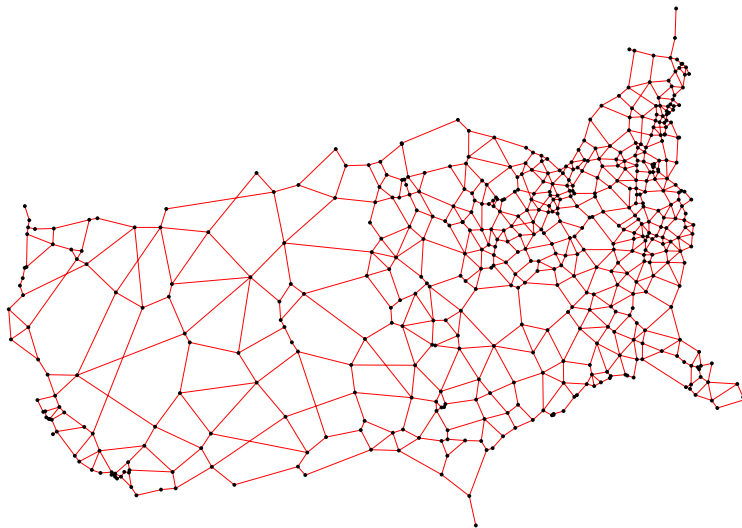
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1.5-spanner for 532 US-cities



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How to compute a good spanner?



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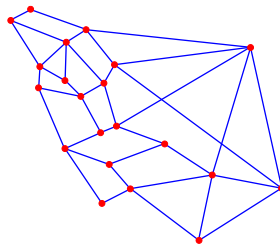
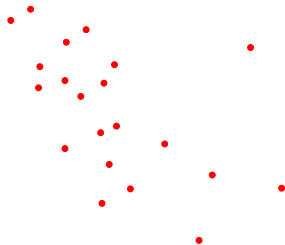
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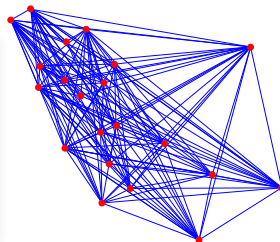
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Sparse t -Spanner



Given a set V and $t > 1$

Quality measurement:

- Number of edges (size)
- Weight (compared with MST)
- Maximum degree
- Diameter

How to compute a good spanner?



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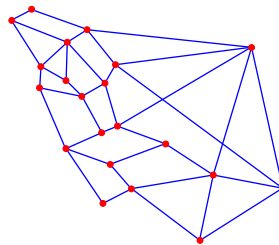
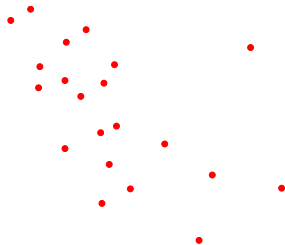
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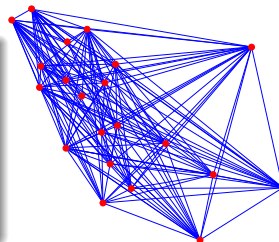
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Given a set V and $t > 1$

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How to compute a good spanner?



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Constructing sparse t -spanners:

- Greedy (Bern (1989) and Althöfer et al. (1993)).
- Θ -graph (Clarkson (1987) and Keil (1988)).
- Ordered Θ -graph (Bose et. al. (2004)).
- Well-Separated Pair Decomposition (Arya et. al. (1995)).

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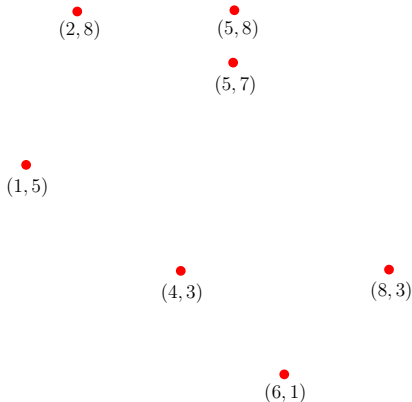
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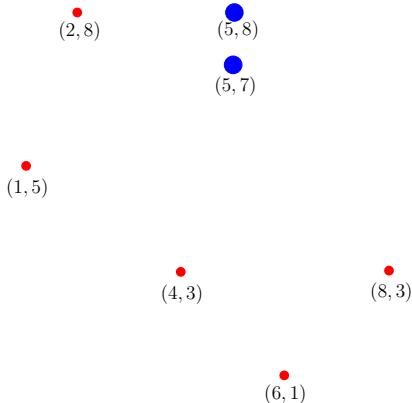
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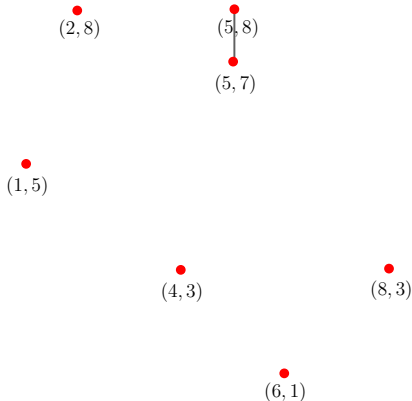
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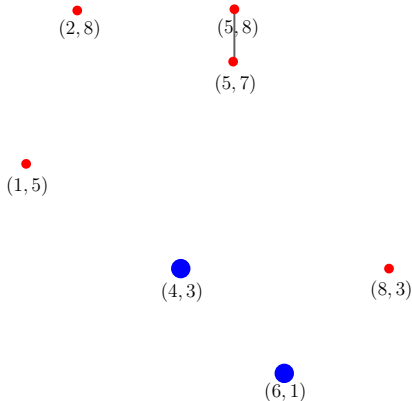
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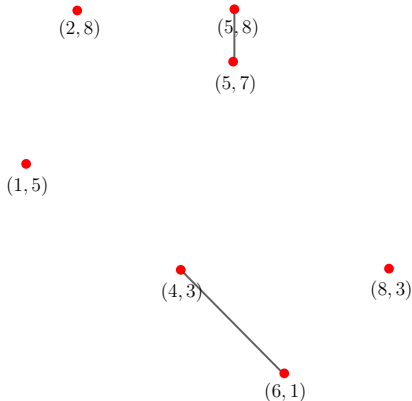
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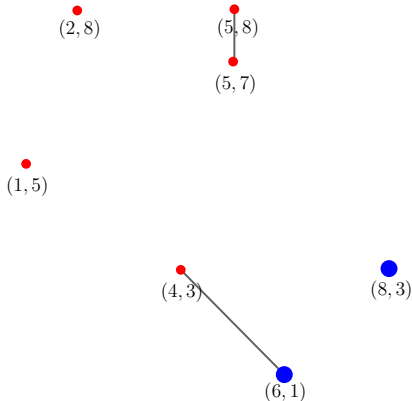
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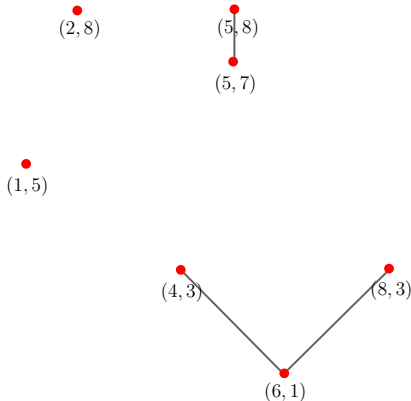
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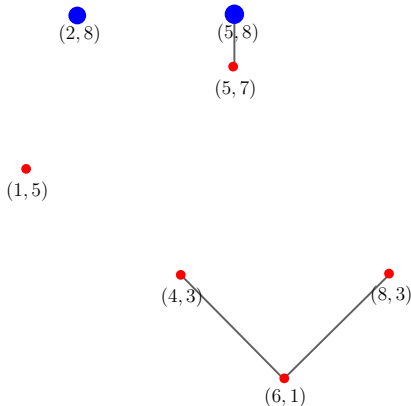
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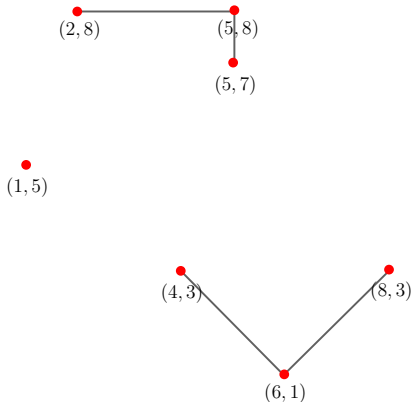
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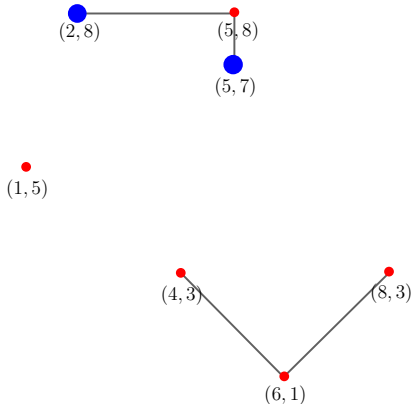
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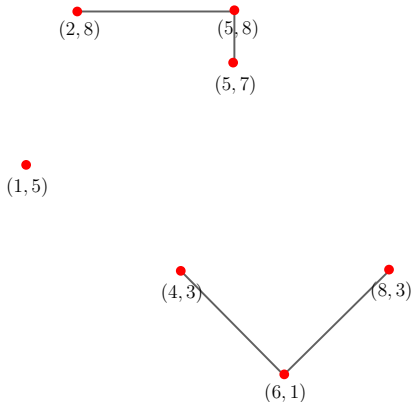
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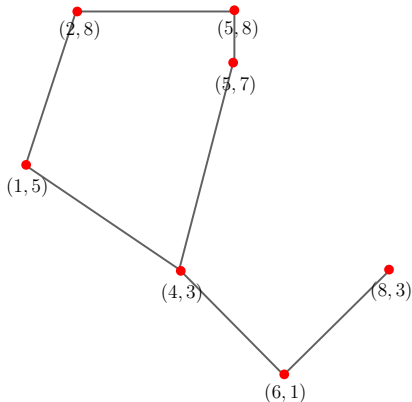


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ORG. GREEDY

Input: V and $t > 1$

Output: t -spanner $G(V, E)$

Sort pairs of points by non-decreasing order of distance;

$E := \emptyset$;

$G := (V, E)$;

for each pair (u, v) of points (in sorted order) **do**

if SHORTESTPATH(G, u, v) $> t \cdot |uv|$ **then**

 Add (u, v) to E ;

end

end

return $G(V, E)$;

Time Complexity: $\mathcal{O}(n^3 \log n)$.

Storage Complexity: $\mathcal{O}(n^2)$.

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Input: V and $t > 1$

Output: t -spanner $G(V, E)$

Sort pairs of points by non-decreasing order of distance;

$E := \emptyset; G := (V, E);$

for each pair (u, v) of points (in sorted order) do

if SHORTESTPATH(G, u, v) $> t \cdot |uv|$ **then**

 Add (u, v) to E ;

end

end

return $G(V, E)$;

Number of shortest path queries: $\Theta(n^2)$.



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for each pair (u, v) of points (in sorted order) do

if SHORTESTPATH(G, u, v) $> t \cdot |uv|$ **then**

 Add (u, v) to E ;

end

end

return $G(V, E)$;

Number of shortest path queries: $\Theta(n^2)$.

Observations:

- We only want to know if there is a t -path between u and v .
- The graph is only updated $\mathcal{O}(n)$ times.



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IMP. GREEDY

Input: V and $t > 1$

Output: t -spanner $G(V, E)$

for each pair $(u, v) \in V^2$ **do** Set $Weight(u, v) := \infty$;

Sort pairs of points by non-decreasing order of distance;

$E := \emptyset$; $G := (V, E)$;

for each pair (u, v) of points (in sorted order) **do**

if $Weight(u, v) \leq t \cdot |uv|$ **then**

 Skip (u, v) ;

else

 Compute single source shortest path with source u ;

for each w **do** update $Weight(u, w)$ and $Weight(w, u)$;

if $Weight(u, v) \leq t \cdot |uv|$ **then** Skip (u, v) ;

else Add (u, v) to E ;

end

end

return $G(V, E)$;

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Conjecture:

The running time of IMP. GREEDY is $\mathcal{O}(n^2 \log n)$.

Bose, Carmi, Farshi, Maheshvari and Smid (2008)

- The conjecture is wrong!
- They presented an algorithm which computes the greedy spanner in $\mathcal{O}(n^2 \log n)$ time (even for points from some metric spaces).

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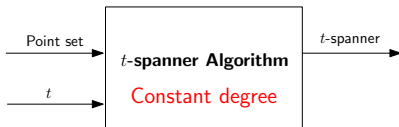
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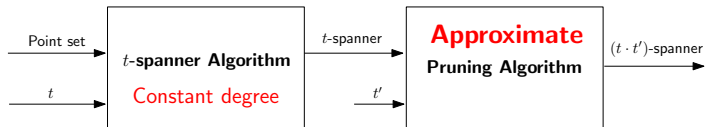
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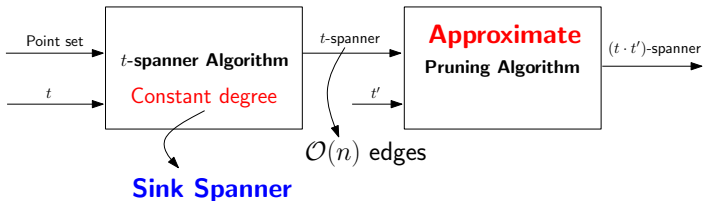
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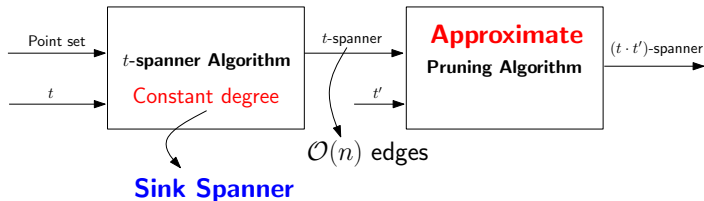
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Time Complexity: $O(n \log^2 n)$

Storage Complexity: $O(n)$.

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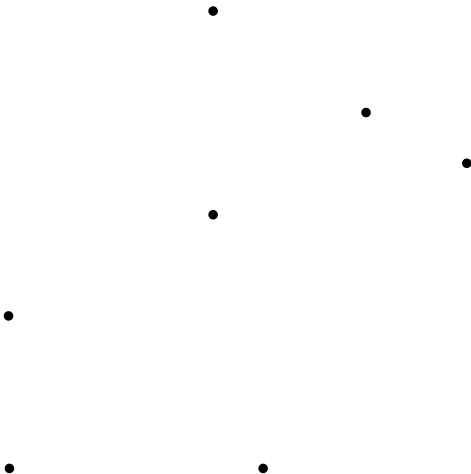
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$$t = 3, \Theta = \pi/6$$



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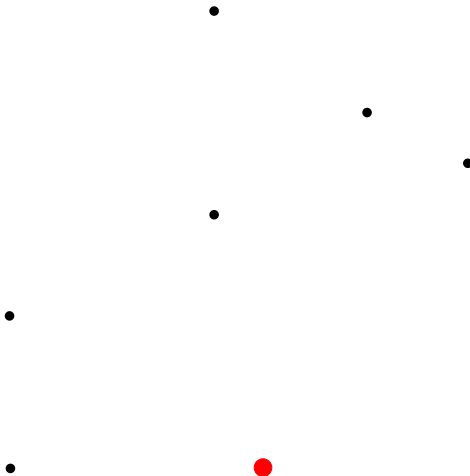
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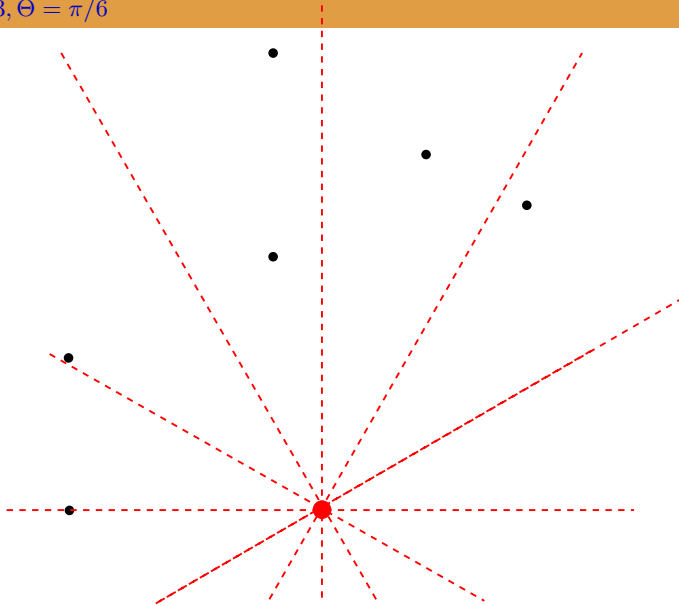
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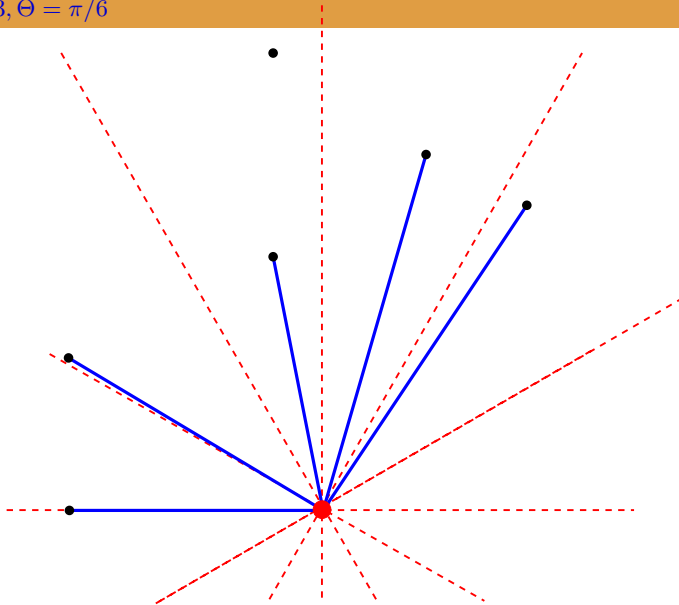
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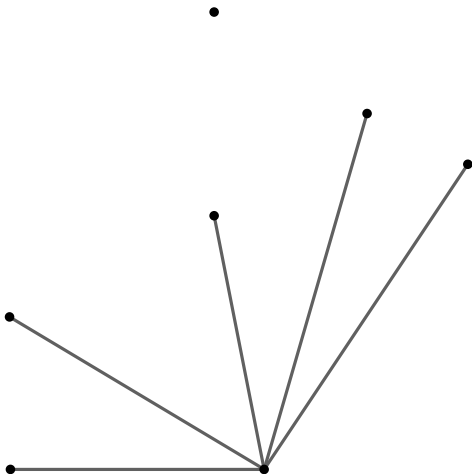
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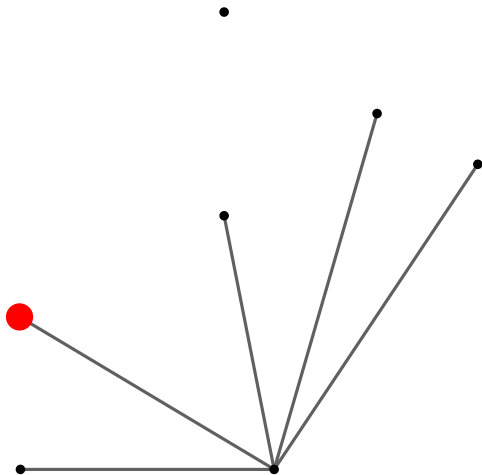
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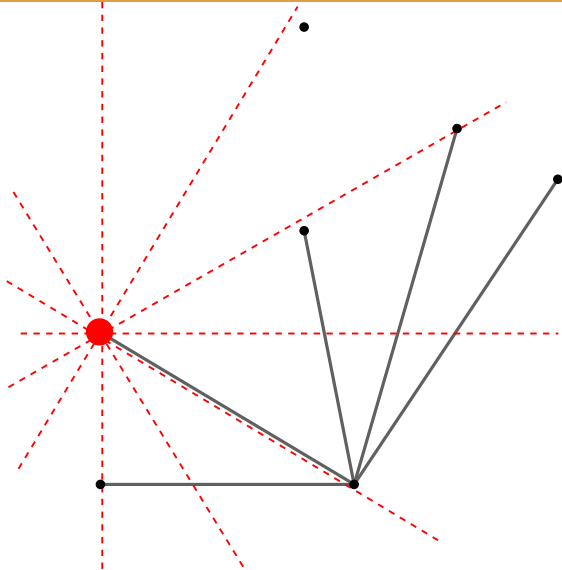
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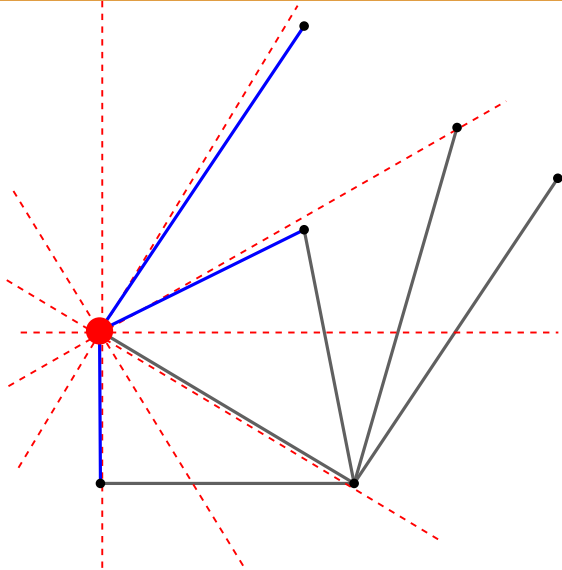
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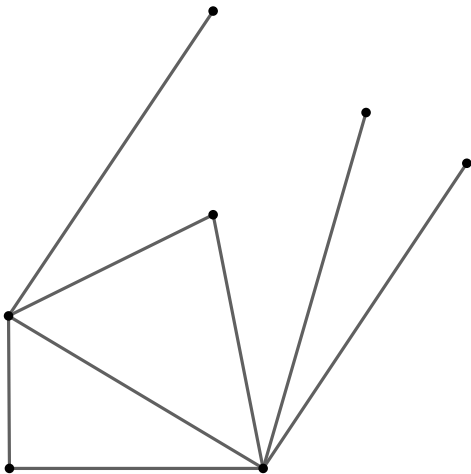
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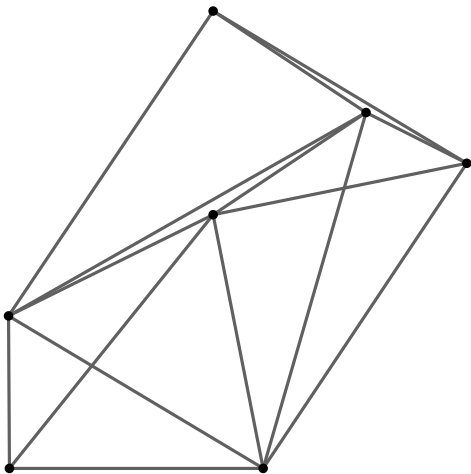
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Θ -Graph Algorithm

Θ -GRAPH

Input: V and $t > 1$

Output: t -spanner $G(V, E)$

Set $k :=$ the smallest integer such that $t = \frac{1}{\cos \theta - \sin \theta}$ for
 $\theta = 2\pi/k$;

$E := \emptyset$;

for each point $u \in V$ do

$C_1, \dots, C_k :=$ non-overlapping cones with angle θ
 and with apex at u ;

for each cone C_i do

 | Connect u to the closest point in C_i ;

end

end

return $G(V, E)$;

Time Complexity: $\mathcal{O}(n \log n)$.

Storage Complexity: $\mathcal{O}(n)$.



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Variants of Θ -Graph Algorithm

Ordered Θ -Graph– $\mathcal{O}(\log n)$ maximum degree

Same as the Θ -graph algorithm, except we add points one by one in a special order.

Random Ordered Θ -Graph– $\mathcal{O}(\log n)$ spanner diameter

We add points one by one in a random order.

Sink Spanner– bounded degree

Decrease the degree of nodes by replacing some edges by paths within other nodes.

Skip-List Spanner– $\mathcal{O}(\log n)$ spanner diameter

Decrease the diameter of Θ -graph by adding some extra edges.



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Sink Spanner

A variant of Θ -graph with bounded degree

Input: V and $t > 1$

Output: t -spanner $G(V, E)$

Construct a directed \sqrt{t} -spanner \vec{G} with bounded out-degree;

for each point $q \in V$ do

 Replace the “star” pointing to q by a \sqrt{t} - q -sink spanner

end

return $G(V, E)$;

Time Complexity: $\mathcal{O}(n \log n)$ Storage Complexity: $\mathcal{O}(n)$.



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Sink Spanner

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Input: V and $t > 1$

Output: t -spanner $G(V, E)$

Construct a directed \sqrt{t} -spanner \vec{G} with bounded out-degree;

for each point $q \in V$ **do**

 Replace the “star” pointing to q by a \sqrt{t} - q -sink spanner

end

return $G(V, E)$;

Time Complexity: $\mathcal{O}(n \log n)$ **Storage Complexity:** $\mathcal{O}(n)$.



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Skip-List Spanner

A variant of Θ -graph with $\mathcal{O}(\log n)$ spanner diameter

```
Input:  $V$  and  $t > 1$   
Output:  $t$ -spanner  $G(V, E)$   
Set  $V_0 := V; i := 1;$   
while  $V_{i-1} \neq \emptyset$  do  
|  $V_i$  contains each points of  $V_{i-1}$  with probability  $1/2;$   
end  
for each  $i$  do  
| Construct a  $t$ -spanner  $G_i(V_i, E_i)$  using the  $\Theta$ -graph  
| algorithm;  
end  
 $E = \cup_i E_i;$   
return  $G(V, E);$ 
```

Time Complexity: $\mathcal{O}(n \log n)$ Storage Complexity: $\mathcal{O}(n)$.



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Well Separated Pair:

$A, B \subset \mathbb{R}^d$ are s -well separated ($s > 0$), if \exists disjoint balls, D_A and D_B such that

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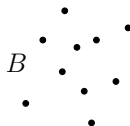
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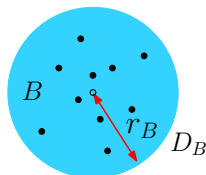
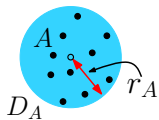
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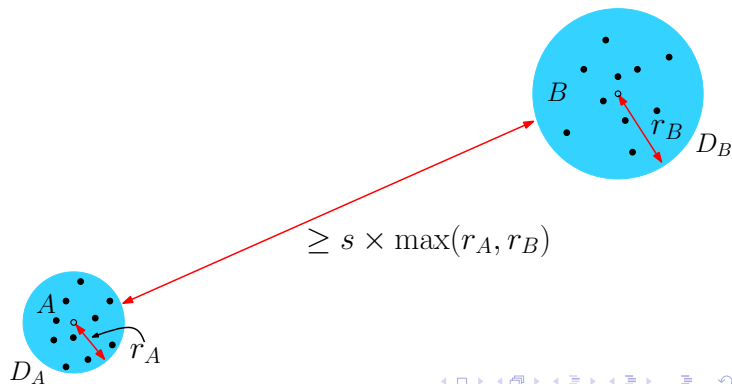
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- $A \subseteq D_A$ and $B \subseteq D_B$.
- $\mathbf{d}(D_A, D_B) \geq s \times \max(\text{radius}(D_A), \text{radius}(D_B))$.



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Well Separated Pair Decomposition:

Let $V \subset \mathbb{R}^d$ and $s > 0$. A WSPD for V with respect to s is a set $\{(A_i, B_i)\}_{i=1}^m$ of pairs of non-empty subsets of V such that

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- $\forall p, q \in V$, there is **exactly** one index i s. t.
 - $p \in A_i$ and $q \in B_i$ or
 - $q \in A_i$ and $p \in B_i$.

m : Size of WSPD.

Callahan & Kosaraju (1995)

For each set of n points, we can construct a WSPD of size $\mathcal{O}(s^d \cdot n)$ in $\mathcal{O}(n \log n)$ time using $\mathcal{O}(s^d \cdot n)$ space.

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WSPD Algorithm

Input: V and $t > 1$

Output: t -spanner $G(V, E)$

Set $\mathcal{W} :=$ WSPD of V w.r.t. $s := \frac{4(t+1)}{t-1}$;

Set $E = \emptyset$;

for each $(A_i, B_i) \in \mathcal{W}$ **do**

 Select an arbitrary node $u \in A_i$ and an arbitrary node
 $v \in B_i$;

 Add edge (u, v) to E .

end

return $G(V, E)$.

Time Complexity: $\mathcal{O}(n \log n)$.

Storage Complexity: $\mathcal{O}(n)$.

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-	Size	Weight	Degree	Time
Greedy spanner	$\mathcal{O}(n)$	$\mathcal{O}(wt(\text{MST}))$	$\mathcal{O}(1)$	$\mathcal{O}(n^2 \log n)$
Apx. greedy spanner	$\mathcal{O}(n)$	$\mathcal{O}(wt(\text{MST}))$	$\mathcal{O}(1)$	$\mathcal{O}(n \log n)$
Θ -graph	$\mathcal{O}(n)$	$\Theta(n \cdot wt(\text{MST}))$	$\Theta(n)$	$\mathcal{O}(n \log n)$
O. Θ -graph	$\mathcal{O}(n)$	$\mathcal{O}(n \cdot wt(\text{MST}))$	$\mathcal{O}(\log n)$	$\mathcal{O}(n \log n)$
WSPD spanner	$\mathcal{O}(n)$	$\mathcal{O}(\log n \cdot wt(\text{MST}))$	$\Theta(n)$	$\mathcal{O}(n \log n)$
Sink-spanner	$\mathcal{O}(n)$	$\mathcal{O}(n \cdot wt(\text{MST}))$	$\mathcal{O}(1)$	$\mathcal{O}(n \log n)$
Skip-list spanner	$\mathcal{O}(n)^*$	$\Theta(n \cdot wt(\text{MST}))^*$	$\Theta(n)$	$\mathcal{O}(n \log n)^*$

(*): Expected with high probability

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Traveling Salesperson Problem (TSP)

Find the **shortest** tour that visits each point exactly once and return to the starting point.



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Known results:

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Traveling Salesperson Problem (TSP)

Find the **shortest** tour that visits each point exactly once and return to the starting point.

Known results:

- The problem is NP-hard even in \mathbb{R}^d .
- A 2-approximation algorithm for metric spaces by Rosenkrantz *et al.* (1977).
- A 1.5-approximation algorithm by Christofides *et al.* (1976).
- A PTAS ($(1 + \varepsilon)$ -approx. Alg.) for geometric case by Arora (1998) and Mitchell (1999).
- A PTAS for geometric case using spanners by Rao and Smith (1998).

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Definition:

If G is a graph with vertex set P , then a tour of P in G is a (possibly non-simple) cycle in G that visits each point of P at least once.

Observation:

For any t -spanner G for P , there is a tour of P in G , whose weight is at most $t \cdot wt(\text{TSP}(P))$.

Theorem (Rao and Smith, 1998)

Given a $(1 + \varepsilon)$ -spanner of a set of n points with $\mathcal{O}(n)$ size and $\mathcal{O}(wt(\text{MST}))$ weight, we can compute a $(1 + \varepsilon)$ -approximation of $\text{TSP}(P)$ in $\mathcal{O}(n \log n)$ time.

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S. B. Rao and W. D. Smith, **Approximating Geometrical Graphs via “Spanners” and “Banyans”**, STOC'98, pp. 540–550, 1998.

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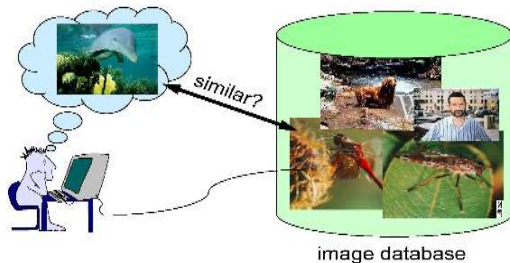
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Approximate proximity searching:

- Multimedia information retrieval,
- Data mining,
- Pattern recognition,
- Machine learning,
- Computer vision and
- Biomedical databases.



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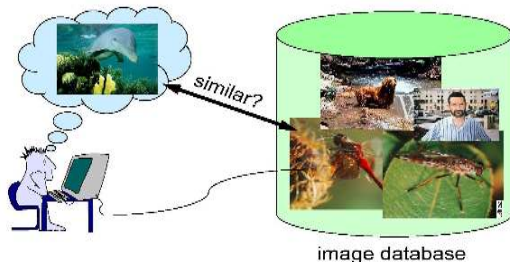
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What is the role of spanners?

- A metric shows the similarity between any two objects.
- But evaluating the distances is expensive.
- One way to speedup is computing the distance between any two objects and save them, but it needs $O(n^2)$ space (AESAs).
- A t -spanner can be used as a sparse data structure to reduce the space.

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- One way to speedup is computing the distance between any two objects and save them, but it need $\mathcal{O}(n^2)$ space (AESA).
- A t -spanner can be used as a sparse data structure to reduce the space.

G. Navarro, R. Paredes, and E. Chávez, **t-Spanners for metric space searching**, Data & Knowledge Engineering, pp. 820-854, 2007.

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D. Russel and L. Guibas, **Exploring Protein Folding Trajectories Using Geometric Spanners**, Pacific Symposium on Biocomputing, pp. 40-51, 2005.



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- Spanners among obstacles.
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