Geometric Spanners for Wireless Ad Hoc Networks

Khaled Alzoubi, Xiang-Yang Li, *Member*, *IEEE*, Yu Wang, *Member*, *IEEE*, Peng-Jun Wan, and Ophir Frieder, *Fellow*, *IEEE*

Abstract—We propose a new geometric spanner for static wireless ad hoc networks, which can be constructed efficiently in a localized manner. It integrates the connected dominating set and the local Delaunay graph to form a backbone of the wireless network. Priori arts showed that both structures can be constructed locally with bounded communication costs. This new spanner has these following attractive properties: 1) the backbone is a planar graph, 2) the node degree of the backbone is bounded from above by a positive constant, 3) it is a spanner for both hops and length, 4) it can be constructed locally and is easy to maintain when the nodes move around, and 5) moreover, the communication cost of each node is bounded by a constant. Simulation results are also presented for studying its practical performance.

Index Terms—Connected dominating set, clustering, Delaunay triangulation, spanner, unit disk graph, localized algorithm, wireless ad hoc networks.

1 Introduction

Wireless ad hoc networks have [1] drawn lots of attention in recent years due to their potential applications in various areas. We consider a wireless ad hoc network consisting of a set V of n wireless nodes distributed in a two-dimensional plane. Each wireless node has an omnidirectional antenna. This is attractive because a single transmission of a node can be received by all nodes within its vicinity. In the most common power-attenuation model, the power required to support a link between two nodes separated by distance r is r^α, where α is a real constant between 2 and 5, dependent on the wireless transmission environment. Here, we ignore the overhead cost of each node to receive and process the coming signal. By a proper scaling, assume that all nodes have the maximum transmission range equal to one unit. These wireless nodes define a *unit disk graph* $UDG(V)$ in which there is a link between two nodes if and only if their Euclidean distance is at most one. The number of links in the unit disk graph could be as large as $O(n^2)$, i.e., the square order of the number of network nodes.

Due to the nodes' limited resource in wireless ad hoc networks, scalability is crucial for network operations. One effective approach is to maintain only a linear number of links using a localized construction method. However, this sparseness of the constructed network topology should not compromise too much on the power consumptions on communications. In this paper, we study how to construct a sparse network topology efficiently for a set of wireless nodes such that every route in the constructed network topology is efficient. Here, a route is *efficient* if its length or hops or both is no more than a constant factor of the minimum needed to connect the source and the destination in the unit disk graph.

The movement of wireless nodes causes the network topology to change constantly, which makes the topology control and efficient routing in nonstatic wireless ad hoc networks difficult and challenging. Thus, we will assume that the nodes are static or can be viewed as static during a reasonable period of time. For example, the sensors in the sensor network do not move usually. In this paper, we study the topology control for a *static* wireless network. Notice that our algorithms do not need to update the network topology when nodes are moving, as long as no link used in the final network topology is broken. In other words, although the actual physical deployment of the network topology is no longer a planar graph when nodes are moving, the logical network topology is still a planar graph, which is crucial for some routing algorithm.

The simplest routing method is to flood the message, which not only wastes the rare resources of wireless node, but also diminishes the throughput of the network. One way to avoid flooding is to let each node communicate with only a selected subset of its neighbors [2], [3], [4], [5], or to use a hierarchical structure. Examples of hierarchical routing are dominating set-based routings [6], [7], [8], [9]. When each wireless node knows its geometry position and can quickly retrieve[1] the geometry information about the destination node of a routing request, several localized routing methods based on geometrical forwarding [4], [2] are proposed to avoid the flooding. Recently, Karp and

- K. Alzoubi is with the Department of Computer Science, Robert Morri College, Chicago, IL 60616. E-mail: alzokha@iit.edu.
- X.-Y. Li, Y. Wang, P.-J. Wan, and O. Frieder are with the Department of Computer Science, Illinois Institute of Technology, 10 W. 31st Street, Chicago, Il 60616. E-mail: {xli, wangyu1, wan, ophir}@cs.iit.edu.

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1. For example, for sensor networks collecting environmental data such as temperature, the data are typically sent to one specific node called sink. In this case, we can assume that the sink node is static and its position is known to all other nodes. The other way to get the location information of a node is to use GPS and location service.

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Kung [4] proposed a new protocol, *Greedy Perimeter Stateless Routing* (GPSR), which routes the packets on a planar subgraph of UDG and guarantees the delivery of the packet if there exists a path. Bose et al. [2] also proposed a similar method using Gabriel graph as planar subgraph. Relative neighborhood graph is also used in broadcasting [10]. These methods maintain some planar subgraph such as the relative neighborhood graph (RNG) or Gabriel graph (GG) as underlying network topology. The routing is based on geometry forwarding heuristics and the right-hand rule is used temporarily when a local minimum occurs. It was known that the RNG and GG are not spanners for UDG [11], [12]. Recently, Gao et al. [13] proposed a new method to construct sparse spanners. The method combines the node clustering algorithm with a new routing graph, called *Restricted Delaunay Graph* (RDG). Although their clustering algorithm [14] achieves a constant approximation in expectation, the approximation constant is too large for having any practical meaning. Additionally, the method of constructing RDG is not communication efficient.

Consequently, we focus on constructing a sparse network topology, i.e., a subgraph of $UDG(V)$, which has the following desirable features.

Sparseness. The topology should be a sparse graph, i.e., the total number of links in this network topology is linear with the total number of wireless nodes. This enables most of algorithms, e.g., routing algorithm based on the shortest path, to run on this topology more efficiently in term of both time and power consumption.

Spanner. The topology is a spanner of $UDG(V)$ in terms of both length and hops. Given a weighted graph G, let $d_G(u,v)$ be the weight of the shortest path connecting u and v in G. A subgraph $G' \subseteq G$ is a spanner of G if there is a positive real constant t such that, for any two nodes u and v, $d_{G'}(u,v) \leq t \cdot d_G(u,v)$. The constant t is called the *length stretch factor* if the weight is the Euclidean length. It is called the *hops stretch factor* if the weight of a link is one. It is called the *power stretch factor* if the weight of a link is the power needed to support the communication of this link.

Bounded degree. Each node has a bounded degree. Consequently, each node needs to hold and process a constant number of neighbors. This is attractive since every wireless node has limited computational resources, storage and, more importantly, limited power.

Planar. The topology is a planar graph (i.e., no two edges cross each other in the graph). Some routing algorithms, such as right-hand routing and *Greedy Perimeter Stateless Routing* (GPSR) [4], require the topology be planar.

Efficient Localized Construction. The network topology can be constructed and maintained in a localized manner due to the limited resources of the wireless nodes. Here, a distributed algorithm constructing a graph G is a *localized algorithm* if every node u can exactly decide all edges incident on u based only on the information of all nodes within a constant hops of u (plus a constant number of additional nodes' information if necessary). More importantly, the number of messages sent by each node shall be bounded. In addition it is anticipated that each node needs only time complexity $O(d \log d)$ to construct the underlying topology, where d is the number of one-hop neighbors.

A trade-off can be made between the sparseness of the topology and the power efficiency. However, not all sparse subgraphs are good candidates for the underlying network topology. There are two sets of structures used for wireless networks: flat structures and hierarchical structures. The flat structures used previously, include the relative neighborhood graph, Gabriel graph, Yao structure, and the Delaunay triangulation. On the other hand, the hierarchical structures used typically are based on dominating set or connected dominating set, or their extensions such as d-dominating set [15].

In [4], Karp and Kung used two planar subgraphs: the *relative neighborhood graph* and the *Gabriel graph*. Bose et al. [12] proved that the length stretch factors of these two graphs are $\Theta(n)$ and $\Theta(\sqrt{n})$, respectively. Recently, some researchers [16], [17] proposed to construct the wireless network topology based on Yao graph (also called θ-graph). It is known that the length stretch factor and the node out-degree of the Yao graph are bounded by some positive constants. But, as Li et al. mentioned in [17], all these three graphs cannot guarantee a bounded node degree, e.g., the node in-degree of the Yao graph could be as large as $O(n)$. In [11], [17], Li et al. further proposed to use another sparse topology, *Yao and Sink*, that has both a constant bounded node degree and a constant bounded length stretch factor. It is a spanner for length or power, but not for hops. It is easy to construct a configuration of a set of nodes, for example, n nodes evenly distributed on a unit segment, such that the Yao structure is not a hop spanner. In addition, all these graphs related to Yao graph are not planar graphs.

Many researchers proposed to use the *connected domination set* (CDS) as a virtual backbone for hierarchical routing in wireless as hoc networks [7], [18], [8], [19]. Efficient distributed algorithms for constructing connected dominating sets in ad hoc wireless networks were well-studied [20], [21], [22], [23], [24], [7], [8], [25]. The notion of cluster organization has been used for wireless ad hoc networks since their early appearance. Baker et al. [21], [22] introduced a "fully distributed linked cluster architecture" mainly for hierarchical routing and demonstrated its adaptivity to the network connectivity changes. The notion of the cluster has been revisited by Gerla et al. [26], [27] for multimedia communications with the emphasis on the allocation of resources, namely, bandwidth and channel, to support the multimedia traffic in an ad hoc environment. In [14], Gao et al. proposed a randomized algorithm for maintaining the discrete mobile centers, i.e., dominating sets. They showed that it is an $O(1)$ approximation to the optimal solution with very high probability, but the constant approximation ratio is quite large. Recently, Alzoubi et al. [20] proposed a method to approximate *minimum connected dominating set* (MCDS) within eight whose time complexity is $O(n)$ and message complexity is $O(n \log n)$. Alzoubi [28] continued to propose a localized method to approximate the MCDS using linear number of messages. Existing clustering methods first choose some nodes to act as coordinators, i.e., clusterhead of the clustering process. Then, a cluster is formed by associating the clusterhead with some (or all) of its neighbors. Previous methods differ on the

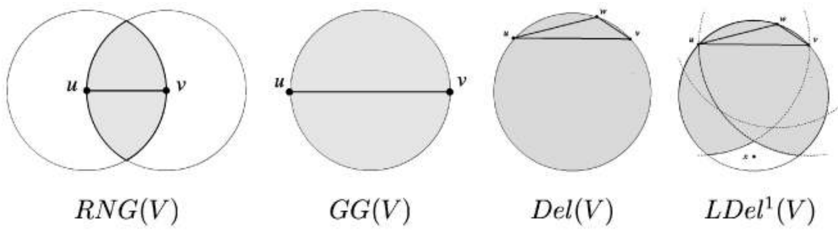

Fig. 1. Definitions of various topologies. The shaded area is empty of nodes inside.

criterion for the clusterhead selection, which is either based on the lowest (or highest) ID among all unassigned nodes [22], [27], based on the maximum node degree [26], or based on some generic weight [23] (node with the largest weight will be chosen as clusterhead). Notice that any maximal independent set is always a dominating set. Several clustering methods essentially compute a maximal independent set as the final clusterheads.

The constructed backbone is not always a planar graph, while the planarity is required by several geometry-based localized routing algorithms. To meet these requirements that may look impossible, we construct a hybrid sparse structure with all properties we listed before. We first propose a new method to approximate MCDS. Notice, we can use any method that can approximate the MCDS efficiently such as those by Alzoubi [28], or by Baker [21], [22]. We then build the local Delaunay graph [29] on top of the approximated MCDS. We show that the constructed backbone is a planar graph and each node has a bounded degree. All ordinary nodes are connected to their dominators. We show that the constructed subgraph CDS' is spanner for both length and hops and has at most $O(n)$ edges. The total communication cost of this method is $O(n)$, which is within a constant factor of the optimum. Moreover, the communication cost of *each* node is bounded by a constant. The computation cost of each node is at most $O(d \log d)$, where d is the number of its one-hop neighbors. We also conduct experiments to show that this topology is efficient in practice. To the best of our knowledge, this is the first one to generate planar backbone while the communication cost of *each* wireless node is bounded by a constant. This is more attractive since the communications in wireless networks are the most power consuming operations.

The rest of the paper is organized as follows: In Section 2, we provide preliminaries necessary for describing our new algorithms, and briefly review the literature related to network topology design issues. Section 3 presents our new spanner formation algorithms based on CDS and LDel graphs. In addition, we prove some properties of the new spanner. Section 4 presents the experimental results. We conclude our paper in Section 5 by pointing out some possible future research directions.

2 Geometry Definitions and Notations

In this section, we give some geometry definitions and notations that will be used in our presentation later. We assume that all wireless nodes are given as a set V of n points in a two-dimensional space. Each node has some computational power. These nodes induce a *unit disk graph* $UDG(V)$ in which there is an edge between two nodes if and only if their distance is at most one. Hereafter, we always assume that $UDG(V)$ is a connected graph. We call all nodes within a constant k hops of a node u in the unit disk graph $UDG(V)$ as the k*-local nodes* or k*-hop neighbors* of u, denoted by $N_k(u)$, which includes u itself. We always assume that the nodes are almost-static in a reasonable period of time.

Various proximity subgraphs of the unit disk graph can be used in ad hoc wireless networks [4], [11], [16], [17], such as the *relative neighborhood graph*, the *Gabriel graph*, and the *Yao graph*. None of these graphs are hop-spanners. In contrast, we use a *connected dominating set* (CDS) as a virtual backbone of the wireless network and use *localized Delaunay graph* (LDel) to make the backbone planar. See Fig. 1 for an illustration of when an edge is included in a graph defined.

A subset S of V is a *dominating set* if each node u in V is either in S or is adjacent to some node v in S. Nodes from S are called dominators, while nodes not from S are called dominatees. A subset C of V is a *connected dominating set* (CDS) if C is a dominating set and C induces a connected subgraph. Consequently, the nodes in C can communicate with each other without using nodes in $V - C$. A dominating set with minimum cardinality is called minimum dominating set, denoted by MDS. A connected dominating set with minimum cardinality is denoted by MCDS.

A subset of vertices in a graph G is an *independent set* if, for any pair of vertices, there is no edge between them. It is a *maximal independent set* if no more vertices can be added to it to generate a larger independent set. It is a *maximum independent set* (MIS) if no other independent set has more vertices.

We continue with the definition of the Delaunay triangulation. Assume that no four vertices of V are cocircular. A triangulation of V is a *Delaunay triangulation*, denoted by $Del(V)$, if the circumcircle of each of its triangles does not contain any other vertices of V in its interior. Delaunay triangulation of a two-dimensional point set is a planar graph and can be constructed in time $O(n \log n)$. Keil and Gutwin [30], [31] showed the *Delaunay triangulation* is a planar spanner with the length stretch factor as most $\frac{4\sqrt{3}}{9}\pi \approx 2.42$.

However, the main drawback of applying the Delaunay triangulation in the ad hoc wireless environment is that it cannot be constructed locally. Some edges of the Delaunay triangulation could be much longer than the transmission range. For even a triangle whose edges are all shorter than the transmission range, it is still expensive to test whether its circumcircle (could be infinitely large) is empty of other vertices inside. To address this drawback, Li et al. [29] defined a new geometry structure, called *k-localized Delaunay graph* (LDel^k) and presented a distributed algorithm to construct it efficiently. A triangle $\triangle uvw$ satisfies *k-localized Delaunay property* if its circumcircle, denoted by $disk(u,v,w)$, does not contain any vertex from $N_k(u) \cup N_k(v) \cup N_k(w)$ inside and all edges of the triangle $\triangle uvw$ have length no more than one unit. Triangle $\triangle uvw$ is called a *k-localized Delaunay triangle*. An edge uv is a Gabriel edge if the disk using uv as diameter does not contain any vertex inside and $||uv|| \leq 1$. The *k-localized Delaunay graph* over a vertex set V, denoted by $LDel^{(k)}(V)$, has exactly all Gabriel edges and the edges of all *k-localized* Delaunay triangles. Li et al. [29] proved that local Delaunay triangulation LDel^k is a planar graph for $k \geq 2$ and has thickness 2 if $k = 1$. Here, a graph G has *thickness* t if G can be decomposed into t planar graphs, but not $t-1$ planar graphs.

Notice that, the definition of *k-localized Delaunay graph* (LDel^k) by Li et al. [29] is different from the definition of *Restricted Delaunay graph* (RDG) by Gao et al. [13]. Let $UDel(V) = Del(V) \cap UDG(V)$, i.e., the edges in Delaunay triangulation with length at most one unit. Gao et al. [13] called *any* planar graph containing $UDel(V)$ as an RDG. They gave a method to construct an RDG. However, their method is not communication efficient, nor computation efficient. The worst time communication cost is equal to the number of links in the unit disk graph, which could be $O(n^2)$.

3 New Spanner Formation Algorithms

We begin this section by proposing the localized planar backbone formation algorithms based on the *connected dominating set* and the *localized Delaunay triangulation*.

3.1 Formation of Backbone

Previous algorithms for building CDS typically have two phases: clustering and finding connectors (or called gateways). The clustering algorithm basically finds a subset of nodes such that the rest of the nodes are visible to at least one of the cluster-heads. By definition, any algorithm generating a maximal independent set is a clustering method. Various methods can then be used to connect the cluster-heads to form a connected graph. For the completeness of presentation, we will review some priori arts on building CDS, MCDS, and localized Delaunay graph. We will interchange the terms cluster-head and dominator. The node that is not a cluster-head is also called *ordinary* node or *dominatee*. A node is called a *white* node if its status is yet to be decided by the clustering algorithm. Initially, all nodes are white. The status of a node after the clustering method finishes could be *dominator* or *dominatee*.

3.1.1 Clustering

Many algorithms for clustering have been proposed in the literature [20], [28], [21], [22], [23], [7], [27], [24], [15], [32], [33], [8]. All algorithms assume that the nodes have distinctive identities (denoted by ID hereafter). We will typically review the ones by Baker [21], [22] and Alzoubi et al. [28], [6]. For the sake of general description of these priori arts, we will summarize them using our own words. The well-known methods for building a dominating set typically use two messages, IamDominator and IamDominatee, and have the following procedures: A white node claims itself to be a dominator if it has the smallest ID among all of its white neighbors, if there is any, and broadcasts IamDominator to its one-hop neighbors. A white node receiving IamDominator message marks itself as dominatee and broadcasts IamDominatee to its one-hop neighbors. The set of dominators generated by the above method is actually a maximal independent set since no two adjacent nodes will be marked as dominators. Here, we assume that each node knows the IDs of all its one-hop neighbors, which can be achieved by requiring each node to broadcast its ID to its one-hop neighbors initially. This protocol can be easily implemented using synchronous communications as done in [21], [22]. If the number of neighbors of each node is known a priori, then this protocol can also be implemented using asynchronous communications. Here, knowing the number of neighbors ensures that a node does get all updated information of its neighbors so it knows that whether itself has the smallest ID among all white neighbors.

After clustering, one dominator node can be connected to many dominatees. However, it is well-known that a dominatee node can only be connected to at most *five* dominators in the unit disk graph model. For the completeness of presentation, we include a short proof here.

Lemma 1. *For every dominatee node v, it can be connected to at most five dominator nodes in the unit disk graph model.*

Proof. For the sake of contradiction, assume that a node v has six dominator neighbors. We know in the unit disk centered at v, there must be two dominator neighbors w and u, the angle $\angle wvu$ is at most $\frac{\pi}{3}$. So, the distance between w and u must be no more than one unit, which means that there is an edge between w and u in UDG. This is a contradiction with the definition of maximal independent set. □

Generally, it is well-known that, for each node (dominator or dominatee), there are at most a constant number of dominators that are at most k units away.

Lemma 2. *For every node v, the number of dominators inside the disk centered at v with radius k-units is bounded by a constant ℓ_k.*

Proof. Because any two dominators are at least one unit away, the half-unit disks centered at dominators are disjoint with each other. In addition, all such dominators should be in the disk centered at v and with radius k. Then, ℓ_k is bounded by how many disjoint half-unit disks we can park in the disk centered at v with radius $k + 0.5$. See Fig. 2. We have $\ell_k \leq \frac{\pi(k+0.5)^2}{\pi(0.5)^2} = (2k+1)^2$ using an area argument. When $k = 2, 3, 4$, we have $\ell_k \leq 25, 49, 81$. □

The bounds on l_k can be improved by a tighter analysis. The above lemma implies that, for every node v, the number of dominators within k hops is bounded by a constant ℓ_k.

Almost all proposed clustering methods are similar to this synchronous protocol. The differences of previous methods approximating MCDS lye in how to find gateways to connect these clusterheads and whether providing

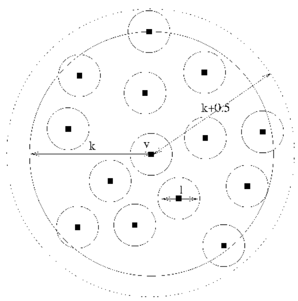

Fig. 2. For every node v, the number of dominators within k units is bounded by a constant ℓ_k.

performance guarantees. For example, the basic algorithm for constructing a CDS proposed in [26] does not guarantee that the constructed clusters are connected. As it agreed, in some cases, it needs *Distributed Gateway* (DG) to connect some clusters that are nonoverlapping. But, how to choose the DGs was not specified. Additionally, no performance guarantee was proven. In [21], [22], they consider in detail how to select the gateway nodes to connect the clusters based on cases of overlapping clusters and nonoverlapping clusters. Here, two clusters (headed by two different clusterheads) are said to be overlapping if there is at least on common dominatee node; they are said to be non-overlapping if there are two dominatee nodes (one from each cluster) are connected. However, they did not prove the message complexity of their protocols, nor the approximation ratio of the generated connected dominating set. Additionally, as they agreed, it may generate two or perhaps more gateway pairs for some nonoverlapping clusters pair. On the other hand, Alzoubi et al. [6], [28] proposed a localized method to find connectors using total $O(n)$ messages and showed that the constructed CDS is within a constant factor of optimum. This property enables us to build a planar spanner in total linear number of messages, which is crucial for wireless ad hoc networks since the communication is the most energy-consumption operation. Actually, we will show that a modification of the method by Baker et al. also has linear number of messages, and the size of the constructed structure is also within a constant factor of optimum. We will discuss in detail of these two methods, which will be the first phase of our hybrid method building planar backbone.

3.1.2 Finding Connectors

The second step of connected dominating set formation is to find some *connectors* (also called *gateways*) among all the dominatees to connect the dominators. Then, the connectors and the dominators form a *connected dominating set* (or called backbone).

Given a dominating set S, let $VirtG$ be the graph connecting all pairs of dominators u and v if there is a path in UDG connecting them with at most three hops. It is well-known that the graph $VirtG$ is connected. This observation is a basis of several algorithms [21], [22], [26], [27] for CDS, although no proof was given in these previous results. It is natural to form a connected dominating set by finding connectors to connect any pair of dominators u and v if they are connected in $VirtG$. This strategy was used in several previous methods, such as [20], [28], [21], [22], [27]. Let $\Pi_{UDG}(u,v)$ be the path connecting two nodes u and v in UDG with the smallest number of hops. Let's first consider how to connect two dominators within three hops. The method by Alzoubi et al. [6], [28] chose the connectors as follows: 1) If the path $\Pi_{UDG}(u,v)$ has two hops, then u finds the dominatee node that comes first to the notice to connect u and v, and 2) if the path $\Pi_{UDG}(u,v)$ has three hops, then u finds the node, say w, that comes first to the notice such that w and v are two hops apart. Then node w selects the node that comes first to the notice to connect w and v. Thus, basically, node u will decide the next node on the path to connect to another node v.

Notice that, the above approach is different from the one adopted by Baker et al. [21], [22]. In their protocols, they let the dominatee nodes decide whether they will serve as the connectors (gateways) or not. For example, if a dominatee node finds that it is dominated by two nonadjacent dominators, say u and v, it claims itself as a candidate of the connectors for u and v. The node with the highest ID among nodes in the intersection area covered by nodes u and v is chosen as the gateway node for the node pair u and v. In other words, they let the nodes in this intersection area elect the one with the highest ID, but no detailed protocol is given to do so. For the case of nonoverlapping clusters, a pair of adjacent dominatees (one from each cluster) need to claim them as the candidates for the gateways of these two clusters. They always select the pair of dominatees with the largest sum of identity numbers. In case of a tie, the pair involving the node with the highest ID number is chosen. However, unlike the case of overlapping clusters, here we may end up with two or perhaps more gateway pairs. The existence of one pair may not be known to both partners of the other pair [21]. This cannot be avoided without increasing the communications [21]. We modify the method by Baker et al. and show that it does approximate CDS using linear number of communications. We then discuss in detail the approach to optimize the communication cost and the memory cost. It uses the following primitive messages (some messages are used in forming clusters):

- IamDominator(u): Node u tells its 1-hop neighbors that u is a dominator.
- IamDominatee(u,v): Node u tells its 1-hop neighbors that u is a dominatee of node v.
- TryConnector(u,w,v,i): Node w proposes to its 1-hop neighbors that it could be one of the connectors to connect two dominators u and v. Integer i specifies whether it is the first or the second node on the path to connect u and v. If uv are two hops apart, then set $i = 0$.
- IamConnector(u,w,v,i): Node w tells its 1-hop neighbors that it is the connector to connect two nodes u and v.

Notice that the message IamDominator(u) is only broadcasted at most once by each node; the message IamDominatee(u,v) is only broadcasted at most five times by each node u for all possible dominators v from Lemma 1. From Lemma 2, we know that TryConnector(u,w,v,i) are also broadcasted at most a constant number of times by each node for all possible dominators u and v.

Lemma 3. *Each node has to send out at most a constant number of messages in forming a connected dominating set.*

Each node uses the following link lists.

- Dominators: It stores all dominators of u if there is any. Notice that, if the node itself is a dominator, no value is assigned for Dominators.

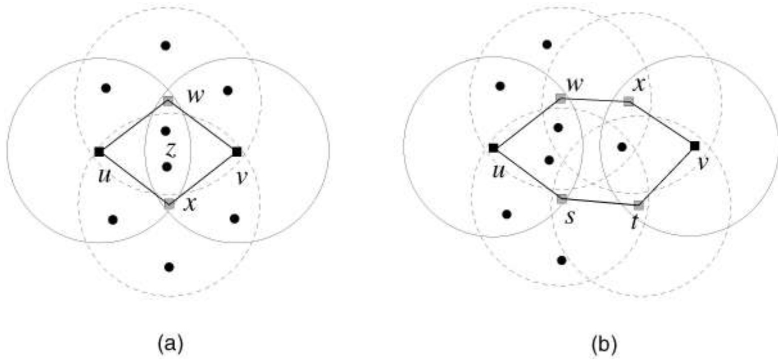

Fig. 3. Multiple connectors found for one pair dominators. (a) Two-hop away dominators and (b) three-hop away dominators.

- 2HopDominators: It stores all dominators v that are two hops apart from u.

From Lemma 2, for each node u, there are at most ℓ_k number of dominators v that are k hops apart from u. The size of each list is bounded by ℓ_1 and ℓ_2, respectively. Then, we are in the position to discuss the distributed algorithm for finding connectors, which are built on the framework of Baker et al. [21], [22] and Alzoubi et al. [28], [6]. Assume that a maximal independent set is already constructed by a cluster algorithm.

Algorithm 1: Finding Connectors

1. Every dominatee w broadcasts message IamDominatee(w, v) indicating that w is a dominatee of v.
2. Every node x stores its two-hop away dominators from the messages IamDominatee(w, v) broadcasted by its neighbor w. Additionally, for each two-hop away dominator node v, it stores a unique neighbor node w that connects it and v. Node w could be the one with the smallest ID or the largest remaining battery power.
3. Every dominatee node w broadcasts to its one-hop neighbors a message TryConnector$(u, w, v, 0)$ for every dominators pair u and v (stored at Dominators).
4. If node w has the smallest ID among all its neighbors that sent TryConnector$((u, *, v, 0))$, then node w broadcasts IamConnector$(u, w, v, 0)$.
5. Every dominatee node w broadcasts to its one-hop neighbors TryConnector$(u, w, v, 1)$ for its dominator u and a two-hop away dominator v. To save communications and decrease the number of connectors produced, we can let node w broadcast such message if u has smaller ID than v.
6. Similarly, if w has the smallest ID among all its neighbors sending TryConnector$(u, *, v, 1)$, then node w broadcasts a message IamConnector$(u, w, v, 1)$ to its one-hop neighbor.
7. Node w also sends message to the dominatee node x selected by w to connect v, asking it to be connector. After receiving such a message, dominatee node x broadcasts to its one-hop neighbors a message IamConnector $(u, x, v, 2)$ for the two-hop away dominator u and its dominator v.

It is possible that there may be multiple paths selected to connect two dominators u and v. See Fig. 3 for an illustration. However, this increases the robustness of the backbone.

For each two-hop away dominators pair u and v, there are at most two nodes claiming it to be connectors for them. This is because we can put at most two nodes inside the lune defined by u and v such that they cannot hear each other directly. Notice, if two nodes can hear each other (i.e., neighbors), then they cannot both have the smallest ID among all its neighbors that sent TryConnector$(u, *, v, 0)$. Thus, there are at most two connectors introduced for two dominators that are two-hop apart. See Fig. 3a for an illustration.

For two dominators that are three hops away, it is obvious that there are at most five nodes sent out IamConnector$(u, *, v, 1)$. Moreover, each such sent message will trigger at most another node to send out message IamConnector$(u, *, v, 2)$. Thus, there are at most five connectors introduced for two dominators. (This number can be improved by tighter analysis, but here we are interested to show that it is bounded by a constant.) Consequently, the total number of connectors introduced is at most a constant factor of the number of dominators in the graph. It is well-known that MDS in UDG can be approximated within constant 5. So, the above method will generate a CDS whose size is within a constant factor of the minimum. Additionally, it is obvious that the number of communications by each node is bounded by a constant: There are a constant number of dominator pairs (u, v); one within two hops and one within one hop of a dominatee node and, for each pair, the communications is at most two. There is one message for claiming itself as connector candidate, and one message for claiming itself (if necessary) as connector.

Additionally, instead of using the smallest ID to determine which node will serve as the connector for dominators pair u and v, the following criteria may perform better practically:

1. The node w that broadcasts the message TryConnector first for the same node pair serves as the connector, and the other node (of neighbors w) will not try to broadcast the message TryConnector for the same node pair. This strategy is feasible since only one node (within the transmission range) can send message at any specific time for wireless ad hoc networks.
2. The node with the largest weight will serve as the connector. Here, the weight could be its remaining battery power, the reciprocal of its moving speed (so connector will not be recomputed frequently), or

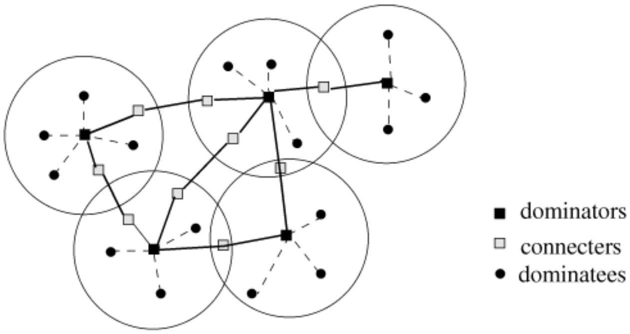

Fig. 4. An example of backbone.

some other characteristics important for wireless ad hoc networks. For more such criteria, see [35], [23].

The method discussed here only uses two hops information by the dominatee nodes to try and see whether they could serve as connectors or not. Notice that, the method by Alzoubi et al. [28], [6] needs the three hops neighbors information to select connectors by the dominator nodes. It is interesting to see the practical performance differences of these two methods in mobile environment. We expect the method proposed here to perform better in terms of updating structures in mobile environment.

The graph constructed by the above algorithm is called a CDS graph (or *backbone* of the network). If we also add all edges that connect all dominatees to their dominators, the graph is called extended CDS, denoted by CDS'. In Fig. 4, we present an example of CDS', where the solid lines in the graph forms the CDS graph, the square nodes are dominators or connectors, while the circular nodes are dominatees. The set of dominators and connectors forms a *connected dominating set*. Connected dominating set CDS induces a graph: two nodes are connected if and only if their distance is no more than one unit. The induced graph is called induced connected dominating set graph (ICDS). Obviously, the CDS is a subgraph of ICDS. If we also add all edges that connect all dominatees to their dominators, the graph is called extended induced CDS, denoted by ICDS'. Later, we will prove that both CDS' and ICDS' are the hop and distance spanners; both CDS and ICDS have a bounded node degree. Graphs ICDS and ICDS' can be constructed using only one message each node (to tell its neighbors whether it is dominator, dominatee, or connector node) if CDS is constructed.

3.2 The Properties of Backbone

This section will show that the CDS' graph is a sparse spanner in terms of both hops and length, meanwhile CDS has a bounded node degree. Generally, we showed this for any graph that connects two dominator nodes.

Lemma 4. *The node degree of CDS is bounded by a constant.*

Proof. Consider any node u. There are two cases: u is a dominator node or a connector node.

For a dominator u, it can only be connected to some connectors w, which must have some dominators v that are one-hop or two hops away from w. From Lemma 2, we know that the number of this kind of dominators v is bounded by ℓ_3. When v and u are two hops apart, there are at most two connectors introduced for them. When v and u are three hops apart, there are at most 10 connectors introduced for them and at most five of them are connected to u. So, the degree of u is also bounded by $2\ell_2 + 5\ell_3$.

For a connector w, it can be connected to at most ℓ_1 dominator nodes and to some connectors. Each of these connectors p (within transmission range of w) should be directly connected to some dominator q, then the number of this kind of dominators q is bounded by ℓ_2. And, for each such dominator, it introduces at most $2\ell_2 + 5\ell_3$ connectors. So, the degree of w is bounded by a constant $\ell_1 + \ell_2(2\ell_2 + 5\ell_3)$. □

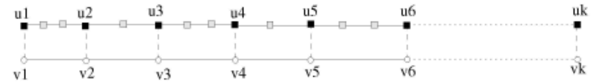

Fig. 5. CDS' is a hop-spanner.

The above lemma immediately implies that CDS is a sparse graph, i.e., the total number of edges is $O(k)$, where k is the number of dominators. Moreover, the graph CDS' is also a sparse graph because the total number of the links from dominatees to dominators is at most $5(n-k)$. Notice that we have at most $n-k$ dominatees, each of which is connected to at most five dominators. However, the degree of some dominator node in CDS' may be arbitrarily large since some dominator node could have many dominatees.

After we construct the backbone CDS and the induced graph CDS', if a node u wants to send a message to another node v, it follows the following procedure. If v is within the transmission range of u, node u directly sends a message to v. Otherwise, node u asks its dominator to send this message to v (or one of its dominators) through the backbone. Then, we show that CDS' (plus all implicit edges connecting dominatees that are no more than one unit apart) is a good spanner in terms of both hops and length. In the following proofs, we use $\Pi_{G_h}(s,t)$ and $\Pi_{G_l}(s,t)$ to denote the shortest hop path and the shortest length path in a graph G from node s to node t. Let $l(\Pi)$ and $h(\Pi)$ be the length and the number of hops of path Π, respectively. The following proof was similar to that presented by Gao et al. [13]. Alzoubi et al. also gave a similar proof for their method. However, our proof shows that, given any two nodes s and t, there is a *unique* path such that its length is no more than a constant factor of $l(\Pi_{UDG_l}(s,t))$, and its hops is no more than a constant factor of $h(\Pi_{UDG_h}(s,t))$.

Lemma 5. *The hops stretch factor of* CDS' *is bounded by a constant* 3.

Proof. Assume the shortest hop path from s to t in UDG is $\Pi_{UDG_h}(s,t) = v_1 v_2 ... v_k$, where $v_1 = s$ and $v_k = t$, as illustrated by Fig. 5. We construct another path in CDS' from s to t and the number of hops of this path is at most $3k+2$.

For each node v_i in the path $\Pi_{UDG_h}(s,t)$, let u_i be its dominator if v_i is not a dominator, else let u_i be v_i itself. Notice that there is a three-hop path $u_i v_i v_{i+1} u_{i+1}$ in the original UDG. Then, from Algorithm 1, we know there must exist one or two connectors connecting u_i and u_{i+1}. Obviously, nodes u_1 and u_k are connected by a path $\Pi_{CDS'}(u_1, u_k)$ in CDS' using at most $3k$ hops. It implies that nodes s and t are connected by a path $\Pi_{CDS'}(s,t)$

(link su_1 followed by $\Pi_{CDS'}(u_1, u_k)$, followed by link $u_k t$) with at most $3k + 2$ hops in CDS′. Thus, the hops stretch factor of CDS′ is bounded by 3 (with an additional constant 2). □

Lemma 6. *The length stretch factor of CDS' is bounded by a constant* 6.

Proof. Given any two nodes s and t such that $||st|| > 1$, we will show that the path $\Pi_{CDS'}(s,t)$ constructed in the proof of Lemma 5 has length at most six times the length of $\Pi_{UDG_l}(s,t)$.

First, for any path Π, $l(\Pi) \leq h(\Pi)$ because the length of every link is no more than one unit. From Lemma 5, we also know that $h(\Pi_{CDS'}(s,t)) \leq 3k+2$, where k is the minimum number of hops needed to connect s and t, i.e., $k = h(\Pi_{UDG_h}(s,t))$. Then,

$$l(\Pi_{CDS'}(s,t)) \leq h(\Pi_{CDS'}(s,t)) \leq 3k+2.$$

Notice that, in the shortest path $\Pi_{UDG_l}(s,t) = w_1 w_2 ... w_m$, the sum of each two adjacent links $w_{i-1}w_i$ and $w_i w_{i+1}$ must be larger than one; otherwise, we can use link $w_{i-1}w_{i+1}$ instead of $w_{i-1}w_i w_{i+1}$ to find a shorter path from the triangle inequality $||w_{i-1}w_{i+1}|| \leq ||w_{i-1}w_i|| + ||w_i w_{i+1}||$. Therefore,

$$l(\Pi_{UDG_l}(s,t)) > \lfloor h(\Pi_{UDG_l}(s,t))/2 \rfloor.$$

Notice that $h(\Pi_{UDG_l}(s,t)) \geq h(\Pi_{UDG_h}(s,t)) = k$. So, $k < 2l(\Pi_{UDG_l}(s,t)) + 2$. Then,

$$l(\Pi_{CDS'}(s,t)) \leq 3k+2 \leq 6l(\Pi_{UDG_l}(s,t)) + 6.$$

Consequently, the length stretch factor of CDS′ is bounded by 6 (with an additional constant 6). Here, we are only interested in nodes s and t with $||st|| > 1$. □

Similarly, we can show that ICDS has a bounded node degree. As CDS′ is a subgraph of ICDS′, the hop and length stretch factors of ICDS′ are also at most 3 and 6, respectively.

Several routing algorithms require the underlying topology be planar. However, the backbone CDS can be a nonplanar graph. Notice in the formation algorithm of CDS, we do not use any geometry information. The resulting CDS may be a nonplanar graph. Even using some geometry information, the CDS still is not guaranteed to be a planar graph. Here, we give an example illustrated by Fig. 6. The lengths of link u_1u_2, u_2u_3, u_3u_4, v_1v_2, v_2v_3, v_3v_4 are all one unit, while the lengths of link u_2v_2 and u_2v_3 are longer than one. For dominator nodes u_1 and u_4, there is only one three-hop path $u_1u_2u_3u_4$. So, the link u_2u_3 must be in CDS. For the same reason, v_2v_3 must be in CDS. Clearly, u_2u_3 intersects v_2v_3.

3.3 Local Delaunay Triangulation on Induced Graph CDS′

Several localized routing heuristics have been proposed recently for wireless ad hoc networks. Some routing algorithms such as GPSR [4], [2] require the underlying network topology to be planar. However, we know that CDS is not guaranteed to be a planar graph, so do its supergraphs CDS′, ICDS, and ICDS′. Thus, we cannot directly apply the geometry forwarding-based routing algorithms on the backbone CDS or any of its supergraphs. When each node knows its geometry position, however, we

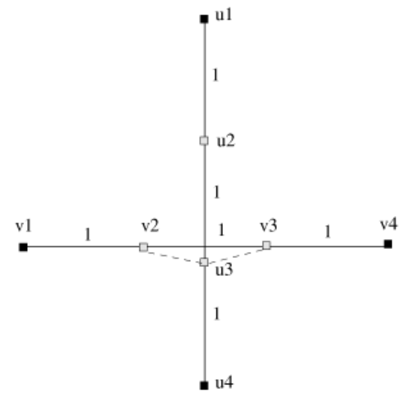

Fig. 6. CDS should be nonplanar.

can apply the *Localized Delaunay Triangulation* [29] on top of the ICDS graph to planarize the ICDS graph without losing the spanning properties. Hereafter, we assume that each wireless node knows absolute or relative positions of itself and each of its neighbors. A broad variety of location dependent services will become feasible in the near future. Although the commercial Global Position System (GPS) has accuracy around 10 meters, the modern systems have an accuracy up to three meters [36]. Indoor location systems are based on the proximity of fixed objects with known coordinates (e.g., sensors), measuring angle of arrival, and time delays of signals. Active Badge system, for example, has accuracy within 9 cm of their true position [36], with work in progress to improve accuracy. If no indoor or outdoor location service is available, the distance between neighboring nodes can be estimated on the basis of incoming signal strengths or time delays.

3.3.1 Review of Local Delaunay Triangulation

For the completeness of the presentation, we review the algorithms proposed in [29] to construct the local Delaunay triangulation. For a set of nodes V, the algorithm first constructs a graph of $LDel^{(1)}(V)$ and then makes it planar efficiently.

Algorithm 2: Localized Delaunay Triangulation

1. Each wireless node u broadcasts its location and listens to the messages from other nodes.
2. Assume that every wireless node u gathers the location information of $N_1(u)$. Node u computes the Delaunay triangulation $Del(N_1(u))$ of its 1-neighbors $N_1(u)$, including u itself.
3. Node u finds all Gabriel edges uv and marks them as *Gabriel edges*. Notice that here $||uv|| \leq 1$, and the disk using uv as diameter is empty.
4. Node u finds all triangles $\triangle uvw$ from $Del(N_1(u))$ such that all three edges of $\triangle uvw$ have length at most one unit. If angle $\angle wuv \geq \frac{\pi}{3}$, node u broadcasts a message proposal(u,v,w) to form a 1-localized Delaunay triangle $\triangle uvw$ in $LDel^{(1)}(V)$ and listens to the messages from other nodes.
5. When node v receives a message proposal(u,v,w), v accepts the proposal of constructing $\triangle uvw$ if $\triangle uvw$ belongs to the Delaunay triangulation $Del(N_1(v))$ by broadcasting message accept(u,v,w); otherwise, it rejects the proposal by broadcasting message reject(u,v,w). Node w performs similarly.

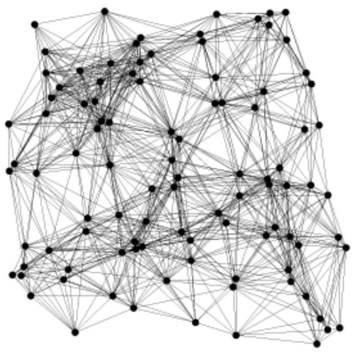

Fig. 7. A unit disk graph.

6. Node u accepts the triangle $\triangle uvw$ if both nodes v and w accept the message proposal(u, v, w). Node v and w perform similarly.

It is easy to show that the total communication cost of the above algorithm is $O(n)$, where n is the number of total input nodes. The computation cost of each node is $O(d \log d)$ (from computing the Delaunay triangulation of $N_1(u)$). It was proven in [29] that the above algorithm does generate $LDel^{(1)}(V)$. It was also proven in [29] that, if two 1-Delaunay triangles xyz and uvw intersect, then either the circumcircle of xyz contains one of the vertices in $\{u, v, w\}$ or the circumcircle of uvw contains one of the vertices in $\{x, y, z\}$. We then make this graph $LDel^{(1)}(V)$ planar as follows:

Algorithm 3: Planarize $LDel^{(1)}(V)$

1. Each wireless node u broadcasts the Gabriel edges incident on u and the triangles $\triangle uvw$ of $LDel^{(1)}(V)$ and listens to the messages from other nodes.
2. Assume node u gathered the *Gabriel edges* and 1-localized Delaunay triangles information of all nodes from $N_1(u)$. For two intersected triangles $\triangle uvw$ and $\triangle xyz$ known by u, node u removes the triangle $\triangle uvw$ if its circumcircle contains a node from $\{x, y, z\}$.
3. Each wireless node u broadcasts all remaining triangles incident on u and listens to the broadcasting by other nodes.
4. Node u keeps all triangles $\triangle uvw$ if both v and w have triangle $\triangle uvw$ remaining.

It was proven that the $LDel^{(1)}(V)$ has thickness 2, i.e., can be decomposed to two planar graphs. Thus, it has at most $6n$ edges. Then, it is easy to show that the total communication cost of planarizing $LDel^{(1)}(V)$ is $O(n)$.

3.3.2 Properties of $LDel(ICDS)$

Applying Algorithms 2 and 3 on ICDS, we get a planar graph called $LDel(ICDS)$. Moreover, we will prove that ICDS has a bounded node degree and so does $LDel(ICDS)$. It was proven in [29] that $LDel(G)$ is a spanner if G is a unit disk graph. Notice that ICDS is a unit disk graph defined over all dominators and connectors. Consequently, $LDel(ICDS)$ is a spanner in terms of length. So here, we only need to prove that $LDel(ICDS)$ has a bounded hops stretch factor.

Lemma 7. *The hops stretch factor of $LDel(ICDS)$ is bounded by a constant.*

Proof. It was proven before that ICDS is a hop-spanner because ICDS contains CDS as a subgraph and CDS is a hop-spanner. Thus, we only have to show that, for any link uv in ICDS, there is a path in $LDel(ICDS)$ connecting u and v using a constant number of hops.

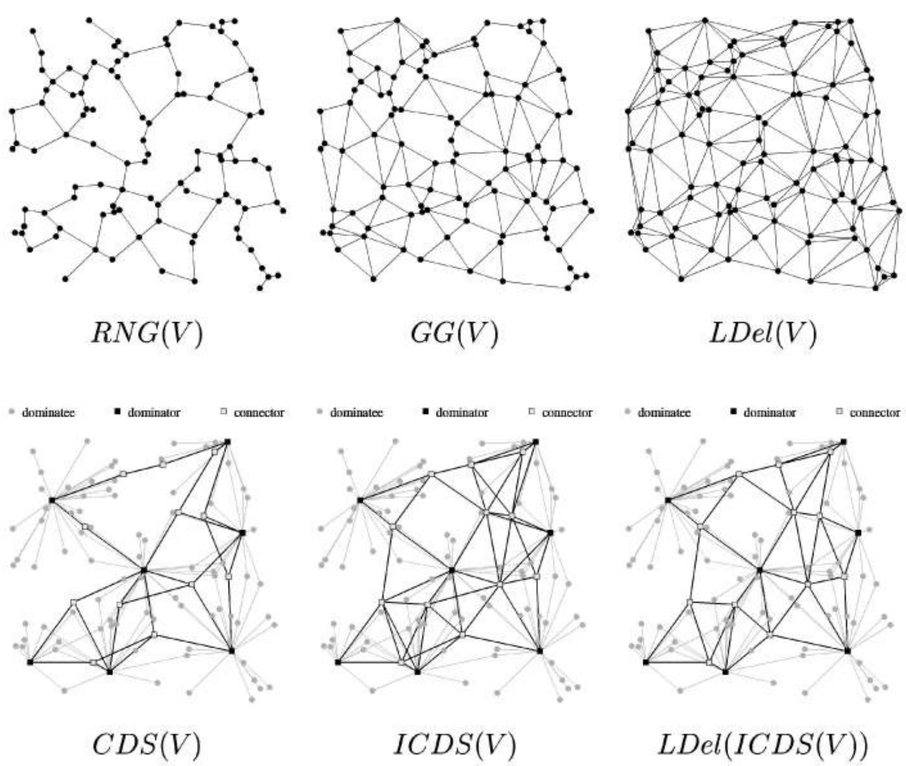

Fig. 8. Different network topologies.

TABLE 1
Topology Quality Measurements

	d_a	d_m	l_a	l_m	h_a	h_m	e
UDG	21.4	42	-	-	-	-	1069
RNG	2.37	4	1.32	4.49	3.62	16	119
GG	3.56	9	1.12	2.08	2.58	8	178
LDel	5.56	12	1.05	1.44	1.95	5	276
CDS	1.09	16	-	-	-	-	54.4
CDS'	3.34	41	1.27	5.04	1.37	3.5	170
ICDS	1.72	16	-	-	-	-	85.8
ICDS'	4.03	41	1.23	4.17	1.32	3	201
LDel(ICDS)	1.20	9	-	-	-	-	60.0
LDel(ICDS')	3.51	38	1.23	4.20	1.40	4	176

It was proven in [29] that the length stretch factor of $LDel(G)$ is at most 2.5 for any unit disk graph G. Therefore, we know that there is a path in $LDel(ICDS)$ with length at most 2.5 to connect u and v. Then, all nodes in that path are inside the disk centered at u with radius 2.5. There are two types of nodes inside this disk: dominators or connectors. Inside this disk, obviously, there are at most a constant number $\ell_{2.5} < 36$ of dominators, which is from Lemma 2. We then show that there are at most a constant number of connectors inside the disk also.

For connectors, it is either connected to a dominator node inside the disk or is connected to a dominator node outside the disk, but the distance of that dominator node to u is at most 3.5. From Lemma 2, we know the number of dominators that can connect to a connector inside that disk is at most $\ell_{3.5}$. Notice that there are at most $5\ell_3$ connectors connected to a dominator node. Thus, there are at most $5\ell_3 * \ell_{3.5}$ connectors inside the disk.

Then, the total number of links in a path connecting u and v in graph $LDel(ICDS)$ is bounded by the number of dominators and connectors inside that disk, which is at most $5\ell_3 * \ell_{3.5} + \ell_{2.5}$. Then, we know that $LDel(ICDS)$ is a hop-spanner. Notice that, although $5\ell_3 * \ell_{3.5} + \ell_{2.5}$ is very large here, the bound can be reduced by using more careful analysis. □

Notice that $LDel(ICDS)$ has thickness of 2 which implies that the average node degree of $LDel(ICDS)$ is at most 12. Moreover, we will show that the node degree of ICDS is bounded by a constant, so does the node degree of $LDel(ICDS)$.

Lemma 8. *The node degree of ICDS is bounded by a constant.*

Proof. For any dominator node u, it can only connect to connectors, which are introduced by some dominator nodes within three hops of u. Notice that, some connectors (within the transmission range of u) may be introduced by some dominators pair (x, y) with $x \neq u$, $y \neq u$. However, x and y are still within three hops of u. Each dominator can introduce at most $5\ell_3$ connectors. Thus, the degree of a dominator node is at most $5\ell_3\ell_3$.

For a connector node w, it can connect to both connectors and at most five dominators. The connectors are introduced by some dominator nodes within two hops of w. There are at most ℓ_2 such dominators, each of them can introduce at most $5\ell_3$ connectors. Thus, the degree of a connector node is at most $5\ell_2\ell_3 + 5$.

Thus, the node degree in ICDS is bounded by $5\ell_3 * \ell_3$ due to $5\ell_3 * \ell_3 > 5\ell_2 * \ell_3 + 5$. □

It immediately implies that the graph $LDel(ICDS)$ has a bounded node degree $5\ell_3 * \ell_3$. Notice that this implies that the number of messages sent by the dominator node or connector node is bounded by a constant during the generation of local Delaunay graph on the backbone.

4 SIMULATIONS

After building the planar backbone, we can run *Dominating-Set-Based Routing* [8] on it. When routing a message on the planar backbone (such as $LDel(ICDS)$), we can use some other variant routing algorithms, such as Greedy Perimeter

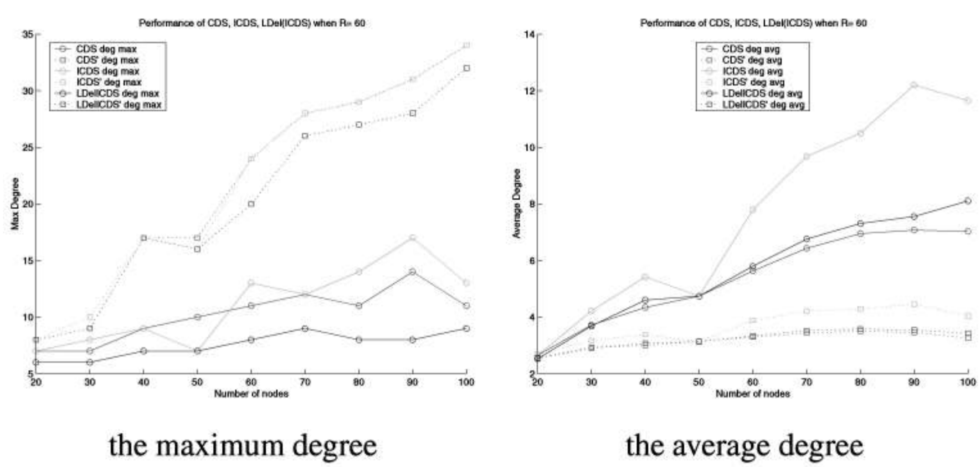

Fig. 9. The relation of the graph degree with the node density.

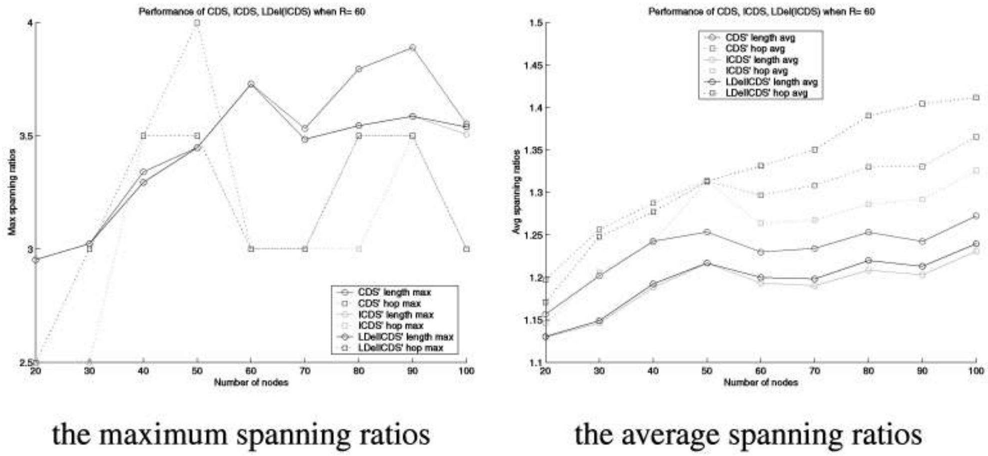

Fig. 10. The relation of the spanning ratios with the node density.

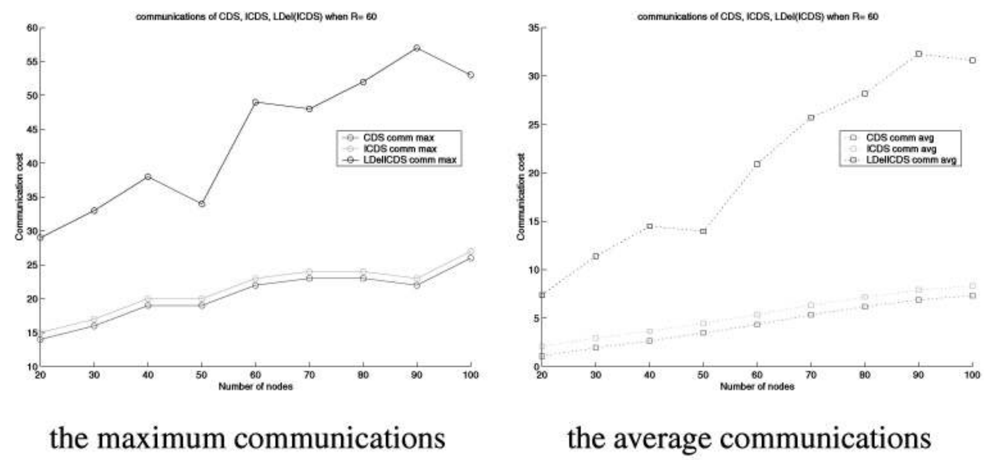

Fig. 11. The relation of the communication cost with the node density.

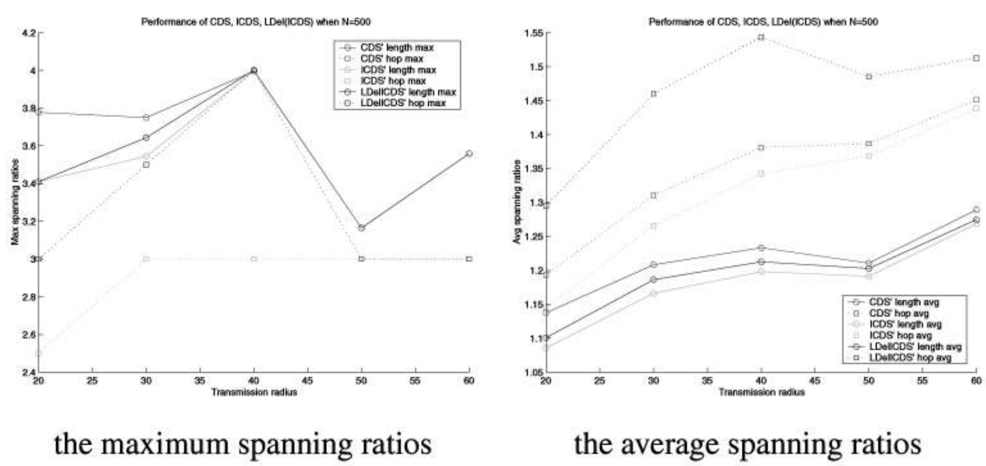

Fig. 12. The relation of the spanning ratios with the transmission radius.

Stateless Routing (GPSR) [4], [2]. Because the routing on planar graphs was already studied, we will concentrate on studying the structural properties of the constructed planar backbone $LDel(ICDS)$: the stretch factors, the maximum and average node degree of the graph, and the communication cost to build these structures.

In our experiments, we randomly generate a set V of n wireless nodes on a $200m$ by $200m$ square, i.e., randomly and uniformly choosing nodes' $x\text{-coordinate}$ and $y\text{-coordinate}$ values. The transmission radius of all wireless nodes is an experimental parameter. We then generate the $UDG(V)$, and test the connectivity of $UDG(V)$. If it is connected, we construct different topologies from V, such as CDS, CDS',

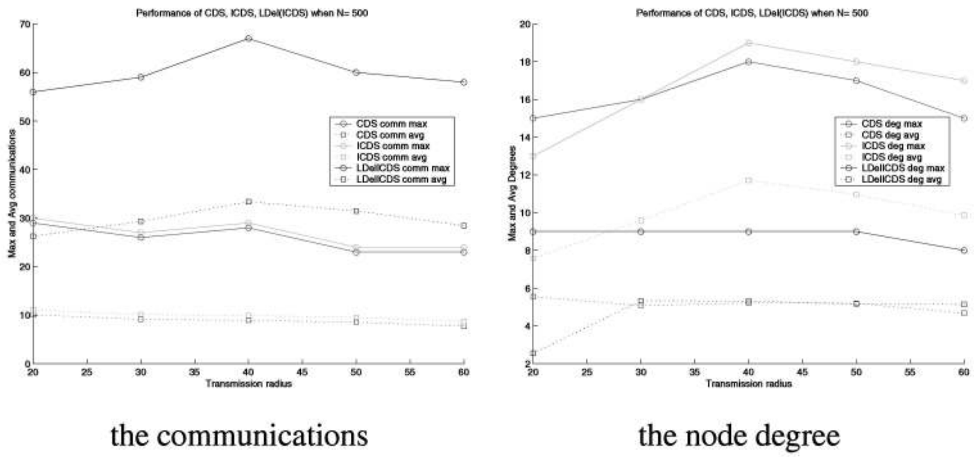

Fig. 13. The relation of the communication cost and the node degree with the transmission radius.

ICDS, ICDS′, LDel(ICDS), and so on. Then, we measure the stretch factors, degree bound of these topologies, and the communication cost to construct them. Given n, we generate 10 vertex sets V of size n and then generate the graphs for each of these 10 vertex sets. The average and the maximum are computed over all these vertex sets. Fig. 8 gives all different topologies defined in this paper for the unit disk graph illustrated by Fig. 7. The transmission radius of each node here is set as $35m$.

In Table 1, l_a and l_m are the average and the maximum length stretch factor over all nodes and all graphs respectively; h_a and h_m are the average and the maximum hop stretch factor over all graphs, respectively. Additionally, d_a and d_m denote the average and the maximum node degree, and e is the average number of edges over all graphs. Here, the maximum node degrees of CDS′, ICDS′, and LDel(ICDS′) are large because the backbone nodes have many links to the dominatee nodes when the graph is dense. As we expected, they are almost equal to the maximum node degree of the unit disk graph. The maximum node degree of the backbone graph such as CDS, ICDS, and LDel(ICDS) does not depend on the node density. Graph LDel(ICDS) has the lowest maximum degree because it removes the some crossing links in other graphs.

We further conduct some experiments to study the relations of the spanning ratios and the communication cost with the node density, the diameter of the original unit disk graph. The variation of the diameter of the graph is achieved by varying the transmission radius. Figs. 9, 10, and 11 illustrate the relations of the node degree, the spanning ratio, and the communication cost with the node density. Here, transmission range is always set as $60m$. Remember that we generate nodes in a $200m$ by $200m$ square region. The communication cost is computed based on the number of messages each node needs to send. Here, the messages could be IamDominator, IamDominatee, TryConnector(u, w, v, i), IamConnector(u, w, v, i), proposal(u, v, w), accept(u, v, w), reject(u, v, w), and so on. We found that the maximum communication cost of each node (around 25 messages) to build CDS or ICDS is considerably smaller than our theoretical upper bound. We also found that the difference between the maximum communication cost of each node to build LDel(ICDS′) and the communication cost to build CDS is almost fixed. Notice that the difference is actually the cost of building local Delaunay graph on top of the ICDS. This is due to the fact that the maximum degree of the ICDS graph is always bounded by a constant and the communications to build LDel by a node depends on its degree. Each node has to process the proposal messages sent by its neighbors, which implies that the number of accept and reject messages sent by a node are related to its degree.

Figs. 12 and 13 illustrate the relations of the spanning ratios and the communication costs with the transmission radius of the node. Here, the number of the wireless nodes is fixed as 500.

5 Summary and Future Work

In this paper, we present a new algorithm to construct a sparse spanner as network backbone: the local Delaunay graph over a connected dominating set graph. A communication efficient localized algorithm was presented for approximating the minimum connected dominating set within a constant factor of the minimum. The constructed connected dominating set is efficient for both length and hops and has at most $O(n)$ edges while each node has a bounded degree. Then, we apply the localized Delaunay graph on the induced graph ICDS to generate a planar graph without sacrificing the constant hop and length stretch factor properties. We showed that the constructed topology $LDel(ICDS)$ has all the desirable features we listed in Section 1. This topology can be constructed locally and is easy to maintain when the nodes move around. All our algorithms have the message complexity $O(n)$. Moreover, we showed that the number of messages sent by *each* node is bounded by a constant. We also conducted extensive simulations to study the spanning ratios of these structures and the communication cost to construct them when the static nodes are randomly placed in a square region. Notice that, recently, Gao et al. also proposed a similar method. However, their algorithms are not communication nor computation efficient and are impossible to be implemented in a localized manner.

There are many interesting open problems left for further study. Remember that, we use the following assumptions on wireless network model: omnidirectional antenna, single transmission received by all nodes within the vicinity of the transmitter, all nodes have the same transmission range, and nodes are static for a reasonable period of time. The problem will become much more complicated if we relax some of these assumptions, although some preliminary follow-up works [37], [38], [39] were done recently. Another interesting open problem is to study the dynamic updating of the planar backbone efficiently when nodes are moving in a reasonable speed. It is interesting to see the practical performance differences of three methods approximating MCDS by Baker et al., Alzoubi et al., and the one proposed here, in a mobile environment. Further, future work is aimed at lowering the constant bounds given in this paper by using a tighter analysis.

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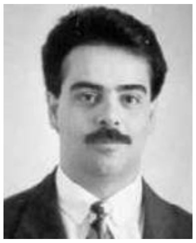

Khaled M. Alzoubi received the PhD degree in computer science from the Illinois Institute of Technology in 2002, and the BS degree in mathematics and computer science from the University of Illinois at Chicago in 1987. He has been an assistant professor of computer science at Robert Morris College since 1998. His research interests include distributed algorithms and wireless networks.

Xiang-Yang Li received the MS degree (2000) and the PhD degree (2001) degree in computer science from the University of Illinois at Urbana-Champaign. He received the BS degree in computer science and business management from Tsinghua University, China, in 1995. He has been an assistant professor of computer science at the Illinois Institute of Technology since 2000. His research interests span the computational geometry, wireless ad hoc networks, optical networks, and cryptography. He is a member of the ACM and IEEE.

Yu Wang received the BEng degree and the MEng degree from Tsinghua University, Beijing, in 1998 and 2000, both in computer science. He joined the Department of Computer Science, Illinois Institue of Technology in 2000, as a PhD student. His current research interests include wireless networks, algorithm design, mobile computing, and artificial intelligence. He is a member of the ACM and IEEE.

Peng-Jun Wan received the PhD degree in computer science from the University of Minnesota in 1997, the MS degree in operations research and control theory from the Chinese Academy of Science in 1993, and the BS degree in applied mathematics from Qsinghua University in 1990. He has been an assistant professor in computer science at Illinois Institute of Technology since 1997. His research interests include optical networks and wireless networks.

Ophir Frieder is the IITRI professor of computer science at the Illinois Institute of Technology. His research interests span the general area of distributed information systems. He is a member of ACM and a fellow of the IEEE.