Geometrical approach in physical understanding of the Goos–Haenchen shift in one- and two-dimensional periodic structures

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The Goos–Haenchen shift of a totally reflected beam at the planar interface of two dielectric media, as if the incident beam is reflected from beneath the interface between the incident and transmitted media, has been geometrically associated with the penetration of the incident photons in the less-dense forbidden transmission region. This geometrical approach is here generalized to analytically calculate the Goos–Haenchen shift in one- and two-dimensional periodic structures. Several numerical examples are presented, and the obtained results are successfully tested against the well-known Artman's formula. The proposed approach is shown to be a fast, simple, and efficient method that can provide good physical insight to the nature of the phenomenon. © 2008 Optical Society of America

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Since the pioneering work of Goos and Haenchen on the total internal reflection (TIR) from the interface of two different dielectric media in 1947 [1], the lateral shift of the barycenter of a totally reflected beam from what is predicted by the ray optics [hereafter referred to as the Goos–Haenchen shift (GHS)], has been subjected to both theoretical [2–5] and experimental investigations [6–8]. Although originally limited to the simplest case where the incident light illuminates the interface between two isotropic, homogeneous dielectric media under TIR, the phenomenon has more recently been studied in complex structures, e.g., in singly and doubly negative materials [9–11], absorptive media and metals [12,13], nonlinear structures [14], and one- and two-dimensional photonic crystals (1DPC, 2DPC) [15,16]. The latter case is further investigated in this Letter, where an insightful geometrical approach is presented to extract the GHS.

The most general formula for extraction of the lateral displacement of the barycenter of a beam reflected from the planar interface of an isotropic, homogeneous dielectric material with the refractive index n_0 (region I in Fig. 1), and an arbitrarily inhomogeneous yet isotropic material with refractive index $n(x,y)$ (region II in Fig. 1) reads as [5]

$$\Delta = \frac{\int_x x|\Psi^r(x,0)|^2 \mathrm{d}x}{\int_x |\Psi^r(x,0)|^2 \mathrm{d}x}. \tag{1}$$

Here, the barycenter of the incident field distribution is assumed to be located at $x=0$, $\Psi^r(x,y)$ indicates the reflected field distribution, and Δ denotes the barycenter shift of the reflected beam, i.e., the GHS (see Fig. 1). By using the Parseval–Plancherel lemma under the stationary phase approximation, the abovementioned general formula for x-independent refractive index profile $n(x,y)=n(y)$ can be further simplified to the well-known Artman's formula [5]: $\Delta=-(k_0 n_0 \cos\theta)^{-1}\mathrm{d}\varphi/\mathrm{d}\theta$, where k_0 denotes the wavenumber at free space, θ is the angle between the normal and the incident beam axis, and ϕ is the phase of the reflection coefficient associated with the principal space harmonic coinciding with the beam axis.

In the simplest case where the transmitted medium is a homogeneous dielectric material with the refractive index $n(x,y)=n_1<n_0$, the GHS can be associated with the penetration of the incident photons in region II for $y>0$. This is as if the incident beam is effectively reflected from the plane lying at a distance d beneath the boundary, and the GHS can be written as [17]

$$\Delta = 2d\tan(\theta), \tag{2}$$

where d is the reciprocal of the imaginary part of the normal wave-vector component of the principal space harmonic in region II: $d=(\mathrm{Im}[k_y])^{-1}=[k_0(n_1^2-n_0^2\sin^2\theta)^{1/2}]^{-1}$. Inasmuch as $\mathrm{Im}[k_y]$ is inversely proportional to the spatial damping rate of the principal space harmonic along the y axis in the transmission region $y>0$, d is referred to as the photonic skin depth (PSD) [18].

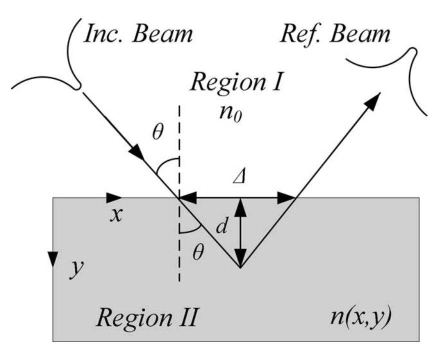

Fig. 1. Geometrical interpretation of the GHS.

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Although limited to the simplest case of the planar interface between two dielectric homogeneous materials, the rather simplistic formula for extraction of the GHS given in Eq. (2) is here generalized by extending the concept of the PSD in 1DPCs. The expected photonic skin depth (EPSD), which is hereafter denoted by d_{exp}, is defined as

$$d_{\mathrm{exp}} = \frac{\int_y y|\psi^t(x,y)|^2 \mathrm{d}y}{\int_y |\psi^t(x,y)|^2 \mathrm{d}y}, \tag{3}$$

where $\Psi^t(x,y)$ indicates the transmitted field distribution. Although the introduced EPSD, d_{exp} in Eq. (3), differs by a factor of 2 from the reciprocal of the imaginary part of the normal wave-vector component in homogeneous media and is not appropriate for GHS extraction in such a special case, it is defined in such a way that properly considers the oscillatory behavior of electromagnetic fields in periodic structures when field oscillation cannot be neglected. It should be noticed, however, that the oscillatory nature of the field plays a critical role unless the wavelength of light is much larger than the period of the structure when the periodic refractive index profile can be effectively replaced with its spatial average. The spatial probability density function of photons in region II is governed by the intensity of the transmitted field $|\Psi^t(x,y)|^2$, and the EPSD is in fact the mathematical expectation of the photonic penetration in the forbidden transmission region. In reference to Fig. 1, the EPSD can now be geometrically linked to the GHS, which can be similarly written as

$$\Delta = 2d_{\mathrm{exp}} \tan(\theta). \tag{4}$$

To numerically justify the presented heuristic approach in extraction of the GHS, a typical 1DPC is here considered. The refractive index of the incident medium is $n_0=2.5$, and the 1DPC is a stratified structure whose refractive index profile reads as

$$n(x,y) = n(y) = \begin{cases} 1.0 & p\Lambda < y \leq (4p+1)\Lambda/4 \\ 2.5 & (4p+1)\Lambda/4 < y \leq (p+1)\Lambda \end{cases} ;p \in \mathbb{Z}^+. \tag{5}$$

This structure is illuminated by an S-polarized wave with the free-space wavelength $\lambda=3\Lambda$ and the incident angle θ. For $32° < \theta < 52°$ the incident beam lies within the photonic band gap (PBG), the Floquet wavenumber becomes complex, and the incident power is completely reflected back. For $\theta > 66°$ the transverse component of the incident wave vector $k_0 n_0 \sin\theta$ is large enough to ensure the TIR, and the incident power is also totally reflected back.

Here, three different methods are employed to extract the GHS, and the obtained results are plotted versus the incident angle θ in Fig. 2. First, the Artman's formula is applied to extract the exact GHS of sufficiently spectrally narrow incident beams (solid

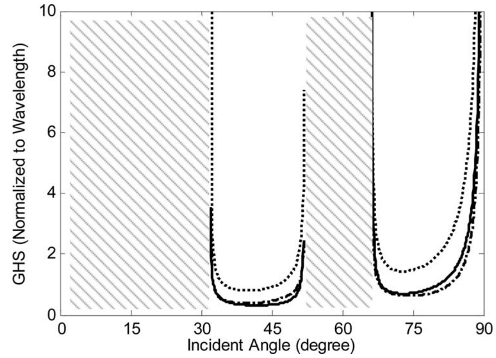

Fig. 2. The GHS at the interface of the considered 1DPC: the Artman's formula (solid curve), the proposed geometrical approximation by using the EPSD (dashed curve), the proposed geometrical approximation by using the PSD (dotted curve).

curve). Second, the geometrical approximation based on the EPSD given in Eq. (4) is employed (dashed curve). Third, the geometrical approximation based on the PSD introduced in [18] and given in Eq. (2) is used (dotted curve). This figure clearly demonstrates that the proposed EPSD, in contrast to the PSD introduced in [18], is good enough to geometrically approximate the GHS. The failure of the PSD, which does not match with the EPSD except for the exponentially decaying field distribution, is partly due to the oscillatory nature of the electromagnetic fields within each unit cell.

The proposed geometrical approach based on the EPSD in Eq. (3) can be further extended to extract GHS in 2DPC. The transmitted field, whose intensity would be x dependent, should be averaged along the x axis and then be used in finding the EPSD:

$$d_{eff} = \frac{\int_y y\left|\int_x \Psi^t \mathrm{d}x\right|^2 \mathrm{d}y}{\int_y \left|\int_x \Psi^t \mathrm{d}x\right|^2 \mathrm{d}y}. \tag{6}$$

It is also possible to spatially average the refractive index profile along the x direction, substitute the 2DPC for its low-frequency equivalent 1DPC, and then apply the proposed geometrical approach based on the one-dimensional EPSD given in Eq. (3). Although accurate for a 2DPC whose periodicity along the x axis is smaller than the wavelength of light [16], the latter method based on the refractive index averaging is not as accurate as the former one based on the transmitted field averaging given in Eq. (6). This is numerically demonstrated in the following examples.

As the first numerical example, the GHS at the interface between the incident medium with refractive index $n_0=1.5$ and a square lattice 2DPC whose constitutive parameters, in accordance with the inset of Fig. 3, read as $n_1=1$ and $n_2=1.5$ is calculated. The structure is illuminated by an E-polarized sufficiently spectrally narrow beam at the free-space wavelength $\lambda=2L$, and the results are shown in Fig. 4. The solid, dotted, and dashed curves are plotted by

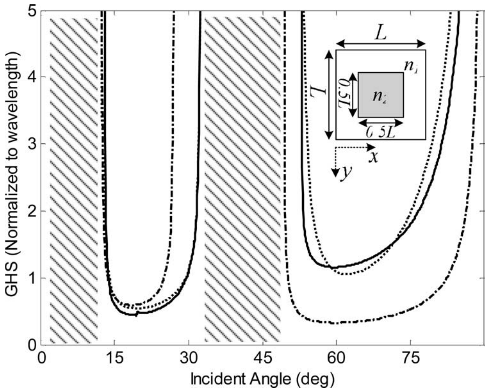

Fig. 3. The GHS at the interface of a 2DPC whose basic cell is shown in the inset: the Artman's formula (solid curve), the proposed geometrical approximation by using the one-dimensional EPSD based on the refractive index averaging (dashed curve), and the proposed geometrical approximation by using the two-dimensional EPSD based on the field averaging (dotted curve).

using the Artman's formula, the two-dimensional EPSD based on the field-averaging method given in Eq. (6), and the one-dimensional EPSD calculated by refractive index averaging and substituting the 2DPC for its low-frequency equivalent 1DPC, respectively. Here, the rigorous approach is followed and the reflection coefficient in the Artman's formula is calculated by using the Legendre polynomial expansion method [19]. The proposed two-dimensional EPSD outsmarts the one-dimensional EPSD based on the refractive index averaging and is in good agreement with what the Artman's formula predicts.

As another example, a typical 2DPC with the lattice constant a is illuminated by an E-polarized Gaussian beam at the incident angle $\theta = 35°$, the free-space wavelength $\lambda = 2.6a$. The finite-difference time-domain (FDTD) method is then employed to extract

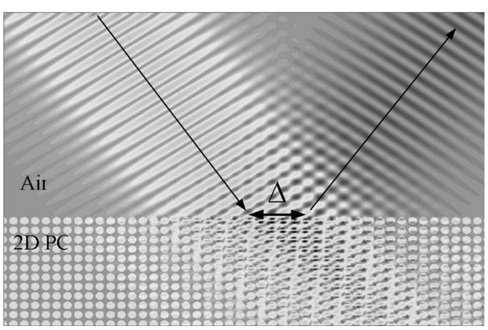

Fig. 4. FDTD simulation of a Gaussian plane wave incident upon the interface of a typical sqaure 2DPC.

the GHS. As shown in Fig. 3, the considered 2DPC is formed of dielectric circular rods with electric permittivity $\varepsilon = 4$ and radius $r/a = 0.4$. The Artman's formula, the proposed geometrical approximation based on the two-dimensional EPSD given in Eq. (6), the proposed geometrical approximation based on the one-dimensional EPSD computed after substituting the 2DPC for its low-frequency equivalent 1DPC, and the FDTD simulation result in $\Delta = 2.1\lambda$, 2.2λ, 2.5λ, and 1.9λ, respectively. Once again, the proposed field averaging is shown to be superior to the conventional refractive index averaging.

In conclusion, a heuristic approach is presented to geometrically link the GHS with the proposed EPSD in one- and two-dimensional periodic structures. Different examples are provided, and the accuracy of the proposed formula is numerically attested against the well-known Artman's formula and the FDTD simulation. The proposed approach is, however, analytical and much faster.

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