GEOMETRICAL PATTERN FEATURE EXTRACTION

BY PROJECTION ON HAAR ORTHONORMAL BASIS

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SUMMARY

The classification of a set of patterns is a problem that appears in very many fields. In general, the number of possible classes is unknown. To define a distance (or similarity) matrix on the set of patterns, we must summarize the available date in the form of a finite set of features with an information loss as small as possible. To evaluate distance coefficients, the best is to project the pattern on a total orthonormal basis ; on the condition of choosing a base matching the pattern properties which concern the classification problem to be solved. In the case of geometric patterns where the discontinuities play an essential role, Haar's discontinuous functions appears to be very promising as shown in the given examples. Morever, Haar's functions are well adapted to digital computation.

KEYWORDS

- Feature extraction
- Haar's functions
- Pattern classification

1 - PATTERN CLASSIFICATION

The classification of a set of patterns is a problem that appears in very many fields. In the most simple case the number of different classes possible is known a priori ; this is the case, for example, of printed characters of a given type. But in general their number is unknown.

In a general way, the problem of classification is the following : being given a set of patterns one makes certain measurements on these patterns and wants to determine subsets which are, with respect to these measurements, internally as similar as possible and externally as dissimilar as possible. More formally, we have to define a partition of the set of patterns ; that is, a set of sub-sets which are mutually exclusive and collectively exhaustive. It is not always possible to reach this ideal goal, sometimes the sub-sets are internally similar and externally dissimilar but neither exclusive nor exhaustive.

The logic diagram of figure 1. schematizes the ideal proceedure for defining such a partition. One uses a set of distance values (or similarity coefficients) in the form of a traingular matrix ; each distance (or similarity) being attached to a pair of pattern independently of the order in which one considers these patterns. The distances are to be evaluated from the rough data available on these patterns. If, in making the evaluation, one does not lose information the distance matrix will exactly represent the initial date. Unhappily this is rarely so. Moreover actual patterns are very often composite, the distance between two composite patterns is not very significant and it is necessary to extract first the elementary components of composite patterns.

2 - FEATURE EXTRACTION

In all that follows we will especially consider geometrical or more exactly topological patterns. A good example is a meteorological map. In this latter case, we are interested by ombilical points (either highest or lowest atmospheric pressure), and by evolution of these points. The problem to be solved is to summarize first the actual pattern by a set of features and after to classify a set of such maps.

For this kind of pattern, it is possible to associate with each pattern S a certain characteristic function s which contains all the rough date available concerning that pattern. The energy $/ E^{s2}$ dE associated with this characteristic function is finite because the actual patterns are of finite dimensions. The set of characteristic functions s constitute a Hilbert spa- and it is possible to define on L^2 a base orthonormal function i so that :

$$(E). \quad (E).dE = \quad if^* \; i$$

$\int$ (E). **(E).dE = 1

Thus it is natural to use as operators G. the group of scalar products which are the projections of pattern on the base i, and to define the main features g^ of the pattern S by :

$$g_i = \int s(E).\ \varphi_i(E).dE$$

The set of coefficients g. ~~is conver~~-gent and we have the Parseval equality for a certain base of orthononnal functions called total (and complete if the set is finite) :

$$\sum_{i=1}^{\infty} g_i^2 = \int_E s^2(E)\ dE$$

The above property is interesting as it permits evaluation of that which we might call the quality of pattern features extraction : the right side of the relation measures the pattern energy, we can evaluate as a function of the rank i to which we have carried the analysis what is the percentage of the energy of the pattern summed up in the set of i first computed features g by calculating the sum of their squares and decide to stop the analysis when the summation corresponds to 80 or 90 % of the total energy of the pattern to be summed up.

If it is possible to use distance between two patterns s(E) and (E), this quantity is the sum of the squares of the differences of the corresponding coefficients g_i and X^ :

$$D(s,\sigma) = \sum^{\infty}(g_i - \gamma_i)^2$$

This type of analysis is extremely classic ; the Fourier transformation is a particular case using the orthogonal properties of circular functions. Two spatial or temporal functions are identical if they have the same spectrum and we can compare several functions by calculating the sum of the squares of differences between the frequency components ; the greater this sum is the more distinct the functions will be. In place of the circular functions one also uses the orthogonal polynomials of Tchebicheff, Legendre, Hermite, Jacobi, Laguerre, etc. Bases are more or less efficiency ; theoritically, the best efficiency is achieved with a Karhunen-Loeve base of orthogonal eigen-functiona [6]. But, from a practical point of view it is sometimea preferable to use more classic orthogonal bases.

In a first part of our study, we have used a base of orthonormal functions deduced from Hermite's polynomials to characterize geometric unidimensional patterns of the Morse type signals dot-dash-space [3]• The use of Hermite*s polynomials for pattern recognition has been proposed by G.E. LOWITZ [4] . We have pointed outlhe interest of this proceedure but also the discrepancy between the discontinuous character of the patterns to be summed up and the continuous aspect of the Hermite Functions. In other words, an Hermite base is a bad one to summarize rough date if one admits that these rough data are essentially linked to the discontinuities of s.

It is more normal to think of using bases of discontinuous orthonormal functions which permit a priori a better analysis therefore a smaller information loss ; thus we can define the distances between patterns which will have more sense.

3 - HAAR'S ORTHONORMAL BASE

Haar's functions [2] constitute an example of such a base. Their use in pattern recognition proceedures has already been envisaged [1], Therefore our study has been modified and Hermites functions have been replaced by Haar's functions.

The goal is the automated analysis of maps. But, it is too difficult to front-attack this problem and we have used first the same one dimensional patterns as before.

Without losing anything of the generality one can, because the monodimensional patterns to be analysed are of finite energy and therefore of finite dimensions lead the study in the normalized range 0.1. Haar's 2^n functions of the n^{th} order are defined as follows (see figure 2).

$$\chi_n^{(k)}(x) = \begin{cases} 2^{n/2} & \text{on } [(2k-2)\alpha\ ,\ (2k-1)\alpha] \\ -2^{n/2} & \text{on } [(2k-1)\alpha\ ,\ 2k\alpha] \\ 0 & \text{for all others } x \in [0.1] \end{cases}$$

where $\alpha = 2^{-n-1}$ and $k = 1, 2, \ldots, 2^n$.

The amplitude $2^{n/2}$ result of normalization necessity :

$$\int_0^1 [\chi_n^k(x)]^2\ dx = 1$$

and we really have the orthogonality relationship :

$$\int_0^1 \chi_n^k(x).\chi_n^j(x)dx = 0 \text{ with } k \neq j$$

and :

$$\int_0^1 \chi_n^k(x).\chi_m^j(x)dx = 0 \text{ with } n \neq m \quad k = 1,2,\dots,2^n \quad j = 1,2,\dots,2^n$$

For each level n there exists 2^n Haar functions and therefore 2^n pattern features at the level n.

4 - NUMERICAL EXAMPLES

The interest in using a base of Haar's orthonormal functions is brought out in the examples that follow.

Some of the analyzed patterns are represented in figure 3. The amplitude can only take but two values 0 or h, h being such that the total energy of the pattern will be unity on the normalized range (0,1).

Figures 4, 5, 6 represent the Haar spectrum up to the 5th order, which is 63 coefficients per pattern. From this we can deduce a certain number of properties which are the following (for the unidimensional patterns analyzed).

a) a coefficient equal to 0 over a given interval signifies that over that interval the feature representing the signal is a constant, null or not or that it is symetrical relative to the mean of that interval.

b) a positive coefficient signifies that on the first half of the interval the signal is longer than on the second half, this indicates that at least a discontinuity exists over the interval. A negative coefficient leads to the inverse conclusions.

c) if all the coefficients are nuls to the M order, and if the sum of the squares of the coefficients calculated up to then is equal to the energy contained in the signal all the other coefficients of an order greater than M will be nuls, the calculated coefficients exactly sura up the pattern.

d) as soon as the quantity c< = 2 - J being the order of the Haar function - is less than the smallest interval appearing in the pattern, only the discontinuities will appear. This property is very important because it permits to state that from a certain number N, which is unknown a priori because we don't know the pattern, only the discontinuities remain ; which clearly appears in the examples shown.

e) the accuracy with which we represent the pattern is linked to the value of the interval, i.e. to the order of the Haar function where the analysis has reached.

Therefore it is possible to reconstruct the pattern from the coefficients of the N order, taking into account the discontinuities appearing on the intervals of a lenght greater than 2~N-1. As an example this is shown in figure 7 : at the 5^ order of the Haar functions the 5th, 10th, 13th, 18th, 23rd and 26th coefficients are different from zero, the interval lenght for this order is 1/32, thus we know that the pattern presents a discontinuity for the 5th, 10, 13th, 18th, 23rd and 26th interval of the reconstructed pattern of figure. 7.

The extraction of the main features of a monodimensional geometric pattern by projection on a Haar orthonormal base is then a proceedure that permits a very small information loss. In this way we can make the comparisons between structures, on one hand by considering the number of discontinuities brought out at the order where one has stopped and on the other hand by considering the value of the coefficients. For example, the patterns of figure 4. are very close to each other and have just two discontinuities, they are symetrical in relation to the middle of the interval (0.1). These properties appear in the values of the coefficients which are symetrical in relation to 0.5.

If the patterns to be analyzed have not the noiseless character of figure 3. but they are deformed, for example, by the presence of noise, the analysis will remain valid. As we can see in figures 6, 8 and 9 the nuls coefficients are no longer nuls, but certain of the most characteristic amongst them stand out clearly from the background noise of the others. The pattern signal to noise level is roughly 17 dB.

5 - CONCLUSION

The classification of a set of patterns by the projection on a total base of orthonormal functions permits to summarize the patterns in an efficient manner ; on the condition of choosing a base matched to the properties of the patterns which concern the classification. In the case of geometric patterns where the discontinuities play an essential role, the use of Haar's functions appears to be very promising as we have seen in the examples given above in the particular case of monodimensional patterns. Ve must add that Haar's functions are particularly well adapted to computation by digital computer.

For actual bidimensional patterns such as maps, it is possible to use either bidimensional Haar's function or a topological description of bidimensional space onto monodimensional one [5]. Ve are trying to evaluate what is the most efficient.

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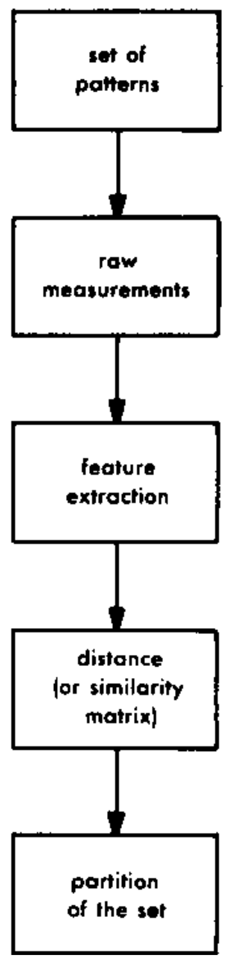

Fig. 1 - The general problem of pattern classification

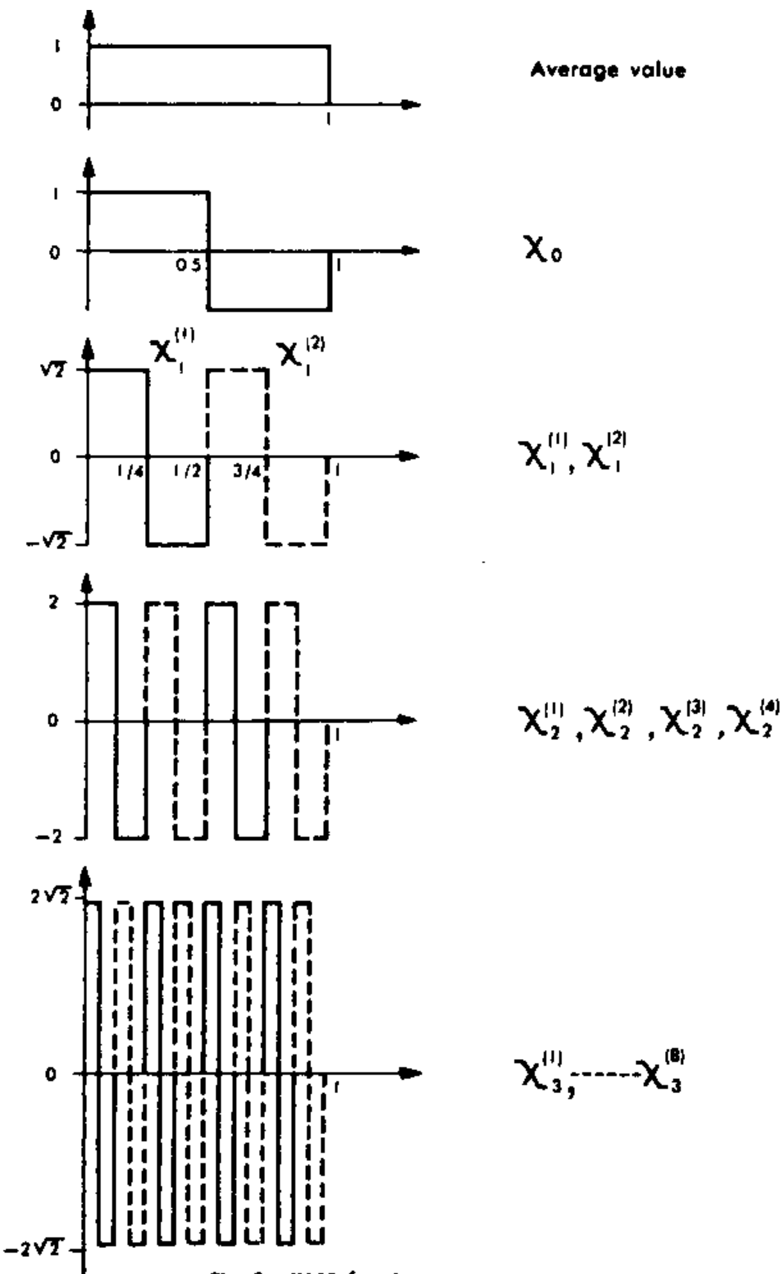

Fig. 2 - HAAR functions system

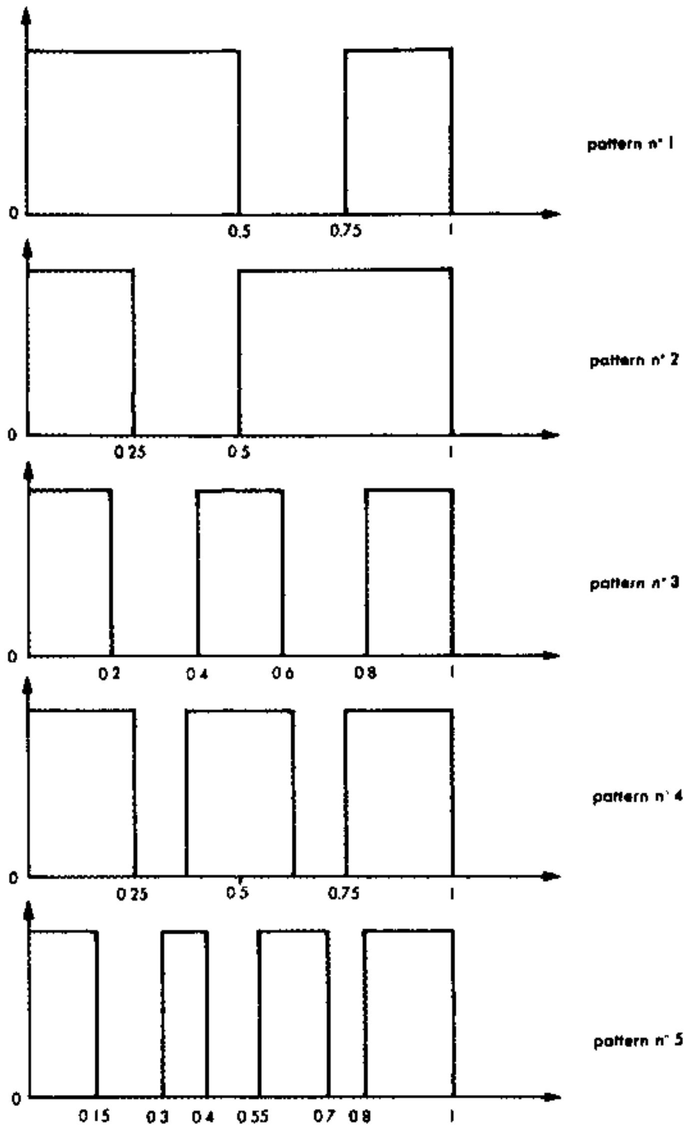

Fig. 3 - Analysed pattern

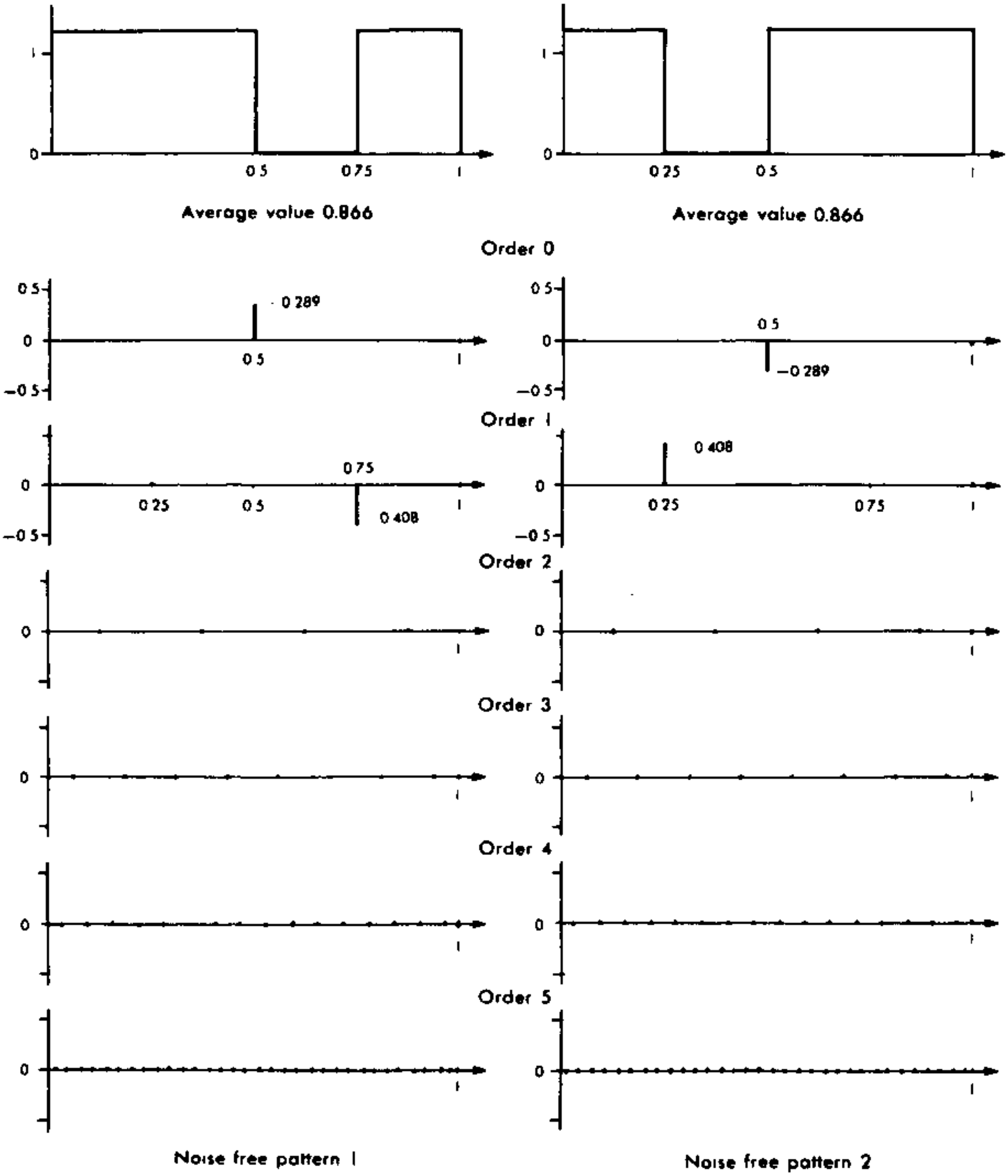

Fig. 4 - Analysis by HAAR functions

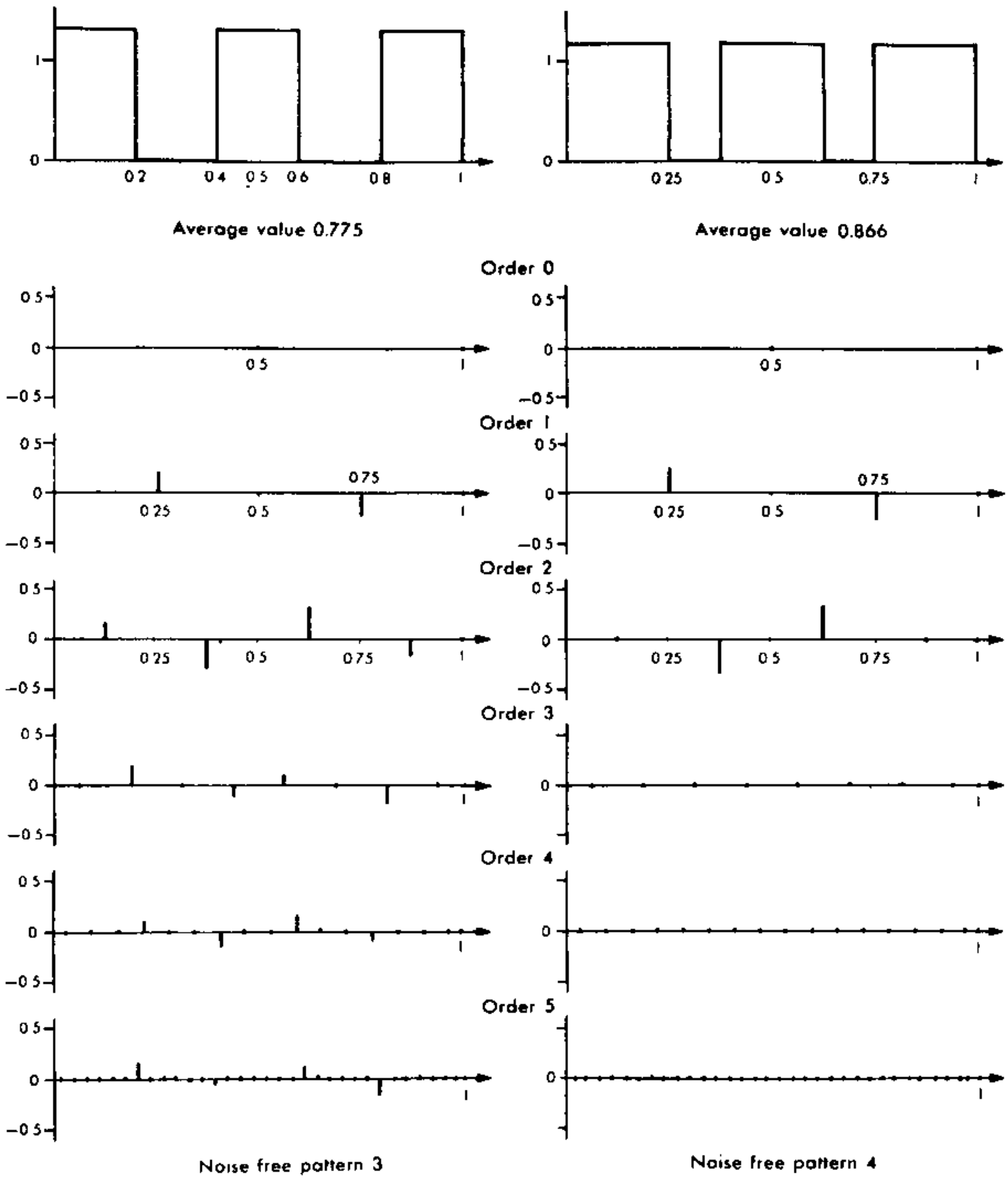

Fig. 5 - Analysis by HAAR functions

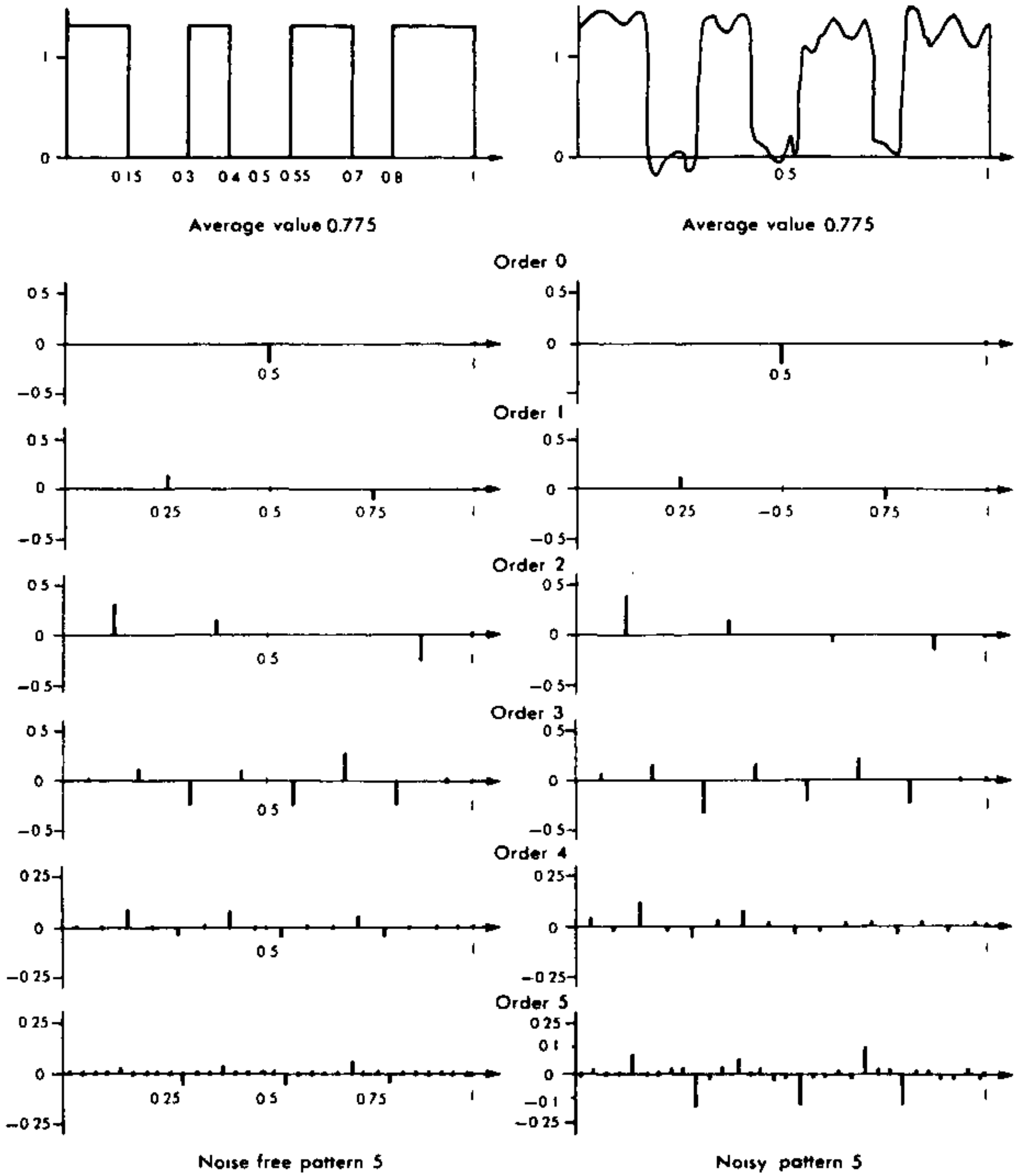

Fig. 6 - Analysis by HAAR functions

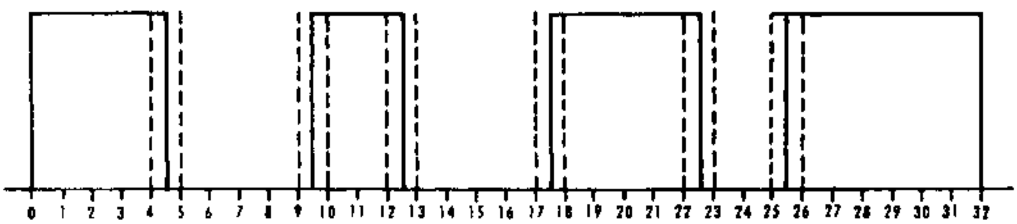

Fig. 7 - Reconstructed pattern

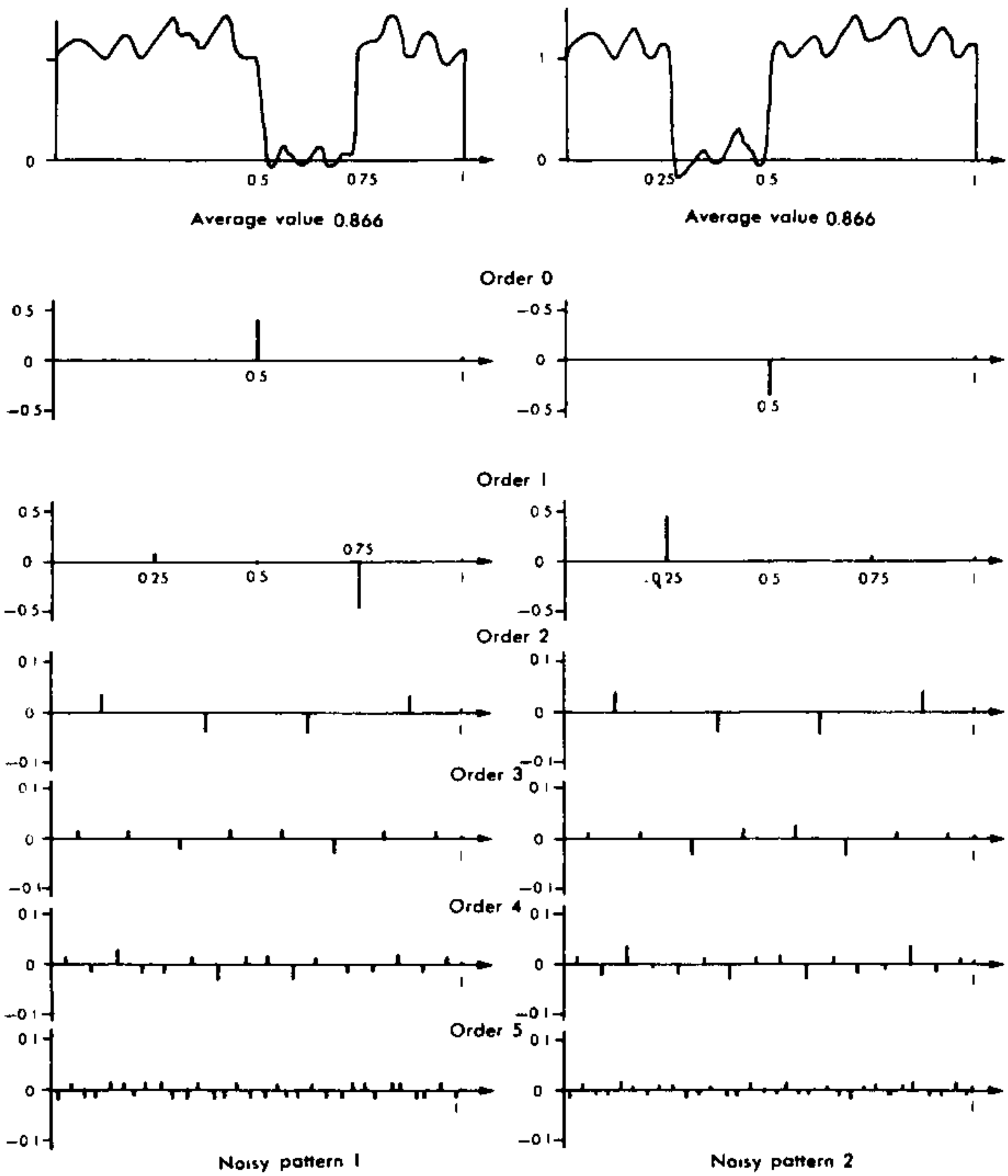

Fig. 8 - Analysis by HAAR functions

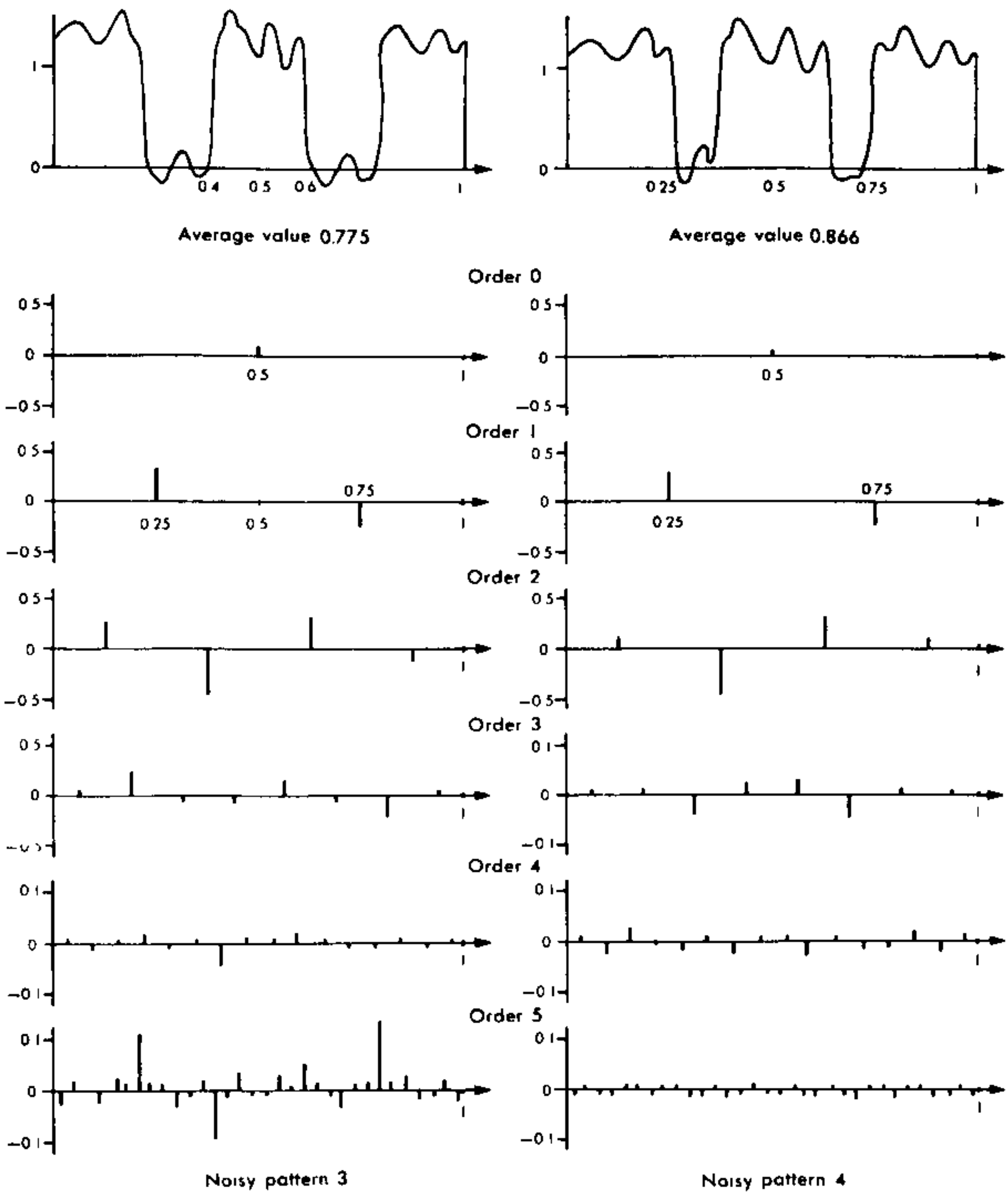

Fig. 9 - Analysis by HAAR functions