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GEOMETRICALLY NON-LINEAR FINITE ELEMENT RELIABILITY ANALYSIS OF STEEL PLANE FRAMES WITH INITIAL IMPERFECTIONS

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Abstract. The random load carrying capacity of steel plane frames with bracing stiffness is studied. The load carrying capacity is evaluated using the geometrically non-linear FEM analysis. The incremental stiffness matrix of a slightly curved element utilized in the non-linear incremental analysis is listed. Initial imperfections are considered as random variables. Statistical analysis and Sobol sensitivity analysis are performed using the Latin Hypercube Sampling method. The effect of initial random imperfections on the load carrying capacity is studied, whilst assuming constant slenderness of the columns. The evaluation parameters are the pair of non-random values of elastic bracing stiffness, and system length of the columns. The paper illustrates that the load carrying capacity is very sensitive to initial crookedness of the columns in the event that the non-sway (symmetric) and sway (anti-symmetric) buckling modes coincide. In this case, the design load carrying capacity obtained from statistical analysis according to the EN 1990 (2002) standard is relatively very small (of low safety). Results show that the reliability of design of a steel frame according to EUROCODE 3 (1993) is significantly misaligned. The significance of the first and the second buckling forces as indicators of sensitivity of the load carrying capacity to the imperfections is discussed.

Keywords: frame, steel, stability, strength, imperfections, failure, nonlinear, limit states, reliability, design.

1. Introduction

Columns and beams are the most fundamental members that require consideration in stability design of steel structures. There is a long history of investigation into steel column buckling problems, and hence an extensive body of past research literature, both theoretical and experimental (Bjorhovde 2010). The design of slender columns and frames is currently based on the critical load for elastic buckling, and on the reduction factor (derived from the critical load) for the relevant buckling curve. Such analysis, however, does not provide uniform safety factors for columns or frames of different types, e.g., see (Bazant, Xiang 1997).

The influence of random initial imperfections on the random load carrying capacity of a symmetric portal frame, see Fig. 1, is analysed in the presented article. The consideration of geometrical non-linearity is inevitable. The value of stiffness K of cross elastic bracing determines whether it is a frame with sway columns (K is small) or non-sway columns (K is large). A further computational parameter is the system length of column h. K and h are non-random parameters. K and h for which the non-dimensional slenderness of columns is constant were considered.

The presented article is connected with the results of stochastic analysis of reliability of steel portal frames (Kala 2011a). The load carrying capacity is studied using methods of stochastic and sensitivity analyses (Saltelli et al. 2004). Pertinent to the ultimate limit state, the design buckling resistance of a compression column $N_{b,Rd}$ (design load carrying capacity) of EUROCODE 3 (1993) is verified by means of statistical analysis according to standard EN 1990 (2002). EN 1990 (2002) describes the principles and requirements for safety, serviceability and durability of structures. It is based on the limit state concept used in conjunction with the partial factor method.

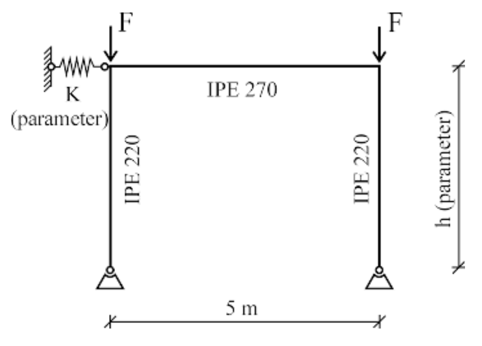

Fig. 1. Symmetric rectangular frame

2. Critical loads and buckling mode of perfect frame

Let us consider a perfect frame, Fig. 1, with constant value of non-dimensional slenderness of columns $\bar{\lambda} = 0.9$. Let us seek all pairs K and h for which $\bar{\lambda} = 0.9$ holds. The result is illustrated in Fig. 2.

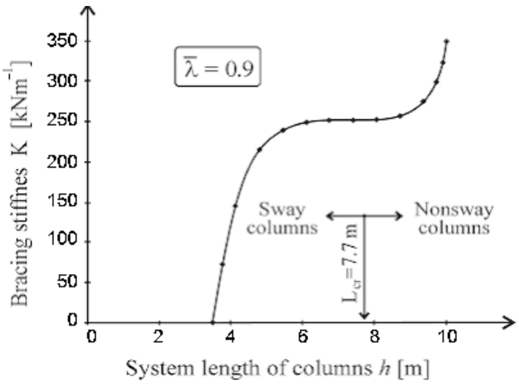

Fig. 2. Bracing stiffness K vs. system length h

Since $\bar{\lambda}$ is constant, hence the first critical load of the perfect frame is $F_{cr1} = 968.1$ kN, and corresponding buckling length of the columns $L_{cr} = 7.703$ m. The second critical load F_{cr2} decreases with increasing system length h and it holds that $F_{cr2} \geq F_{cr1}$, see Fig. 3.

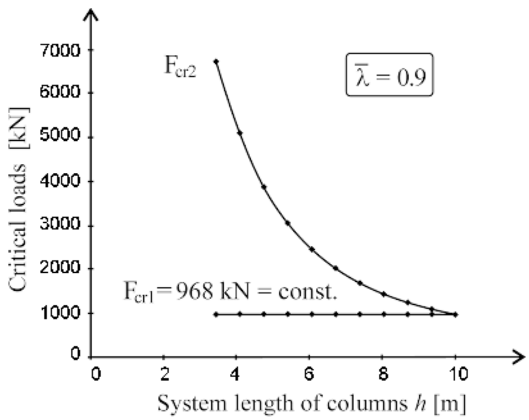

Fig. 3. Critical loads F_{cr1}, F_{cr1} vs. h

A certain critical bracing stiffness $K = 350$ kNm^{-1} and height $h = 10$ m exist for which the critical loads for the sway (anti-symmetric) and non-sway (symmetric) buckling modes coincide $F_{cr1} = F_{cr2}$=968.14 kN, see Fig. 4.

From the perspective of dimensioning, we may distinguish between frames with sway columns if $L_{cr} < h$ and

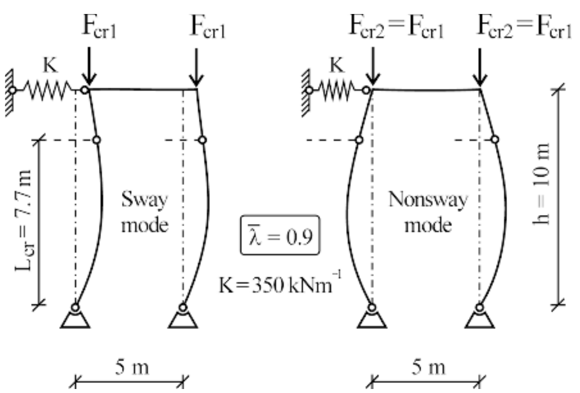

Fig. 4. Sway and non-sway buckling modes

frames with non-sway columns if $L_{cr} > h$, see Fig. 2. Let us denote $P_E = (\pi^2/h^2)EI$ = Euler load (critical load of a pin-ended column) where E is Young's modulus of the columns and I is the second moment of area of the columns. For sway columns, it holds that $P_E \leq F_{cr1,}$ and for non-sway columns $0 \leq P_E$.

For $\bar{\lambda} = 0.9$, it should hold that $h \leq 10$ m. If $h > 10$ m, then $\bar{\lambda} > 0.9$ even if maximum stiffness of $K = \infty$ is considered, (i.e. movement of the column tip is completely prevented). For $h > 10$ m and $K = \infty$, we obtain symmetric frame buckling, and the value of $\bar{\lambda}$ cannot be brought closer to the value of 0.9 through increase of the stiffness K. It thus holds that for all $\bar{\lambda} = 0.9$ $K \in \langle 0; 350 \rangle$ kNm^{-1}, see Fig. 2.

The dependences of K versus h for other $\bar{\lambda}$ values were determined analogously, see Fig. 5. The point in the middle of the curve has h equal to the buckling length for corresponding $\bar{\lambda}$, see Fig. 5. In Fig. 5 the part of the curve is worth noticing for which the value of K is approximately constant. In practice, it means that, for certain values of K, we can find more values of h such that $\bar{\lambda}$ is approximately constant.

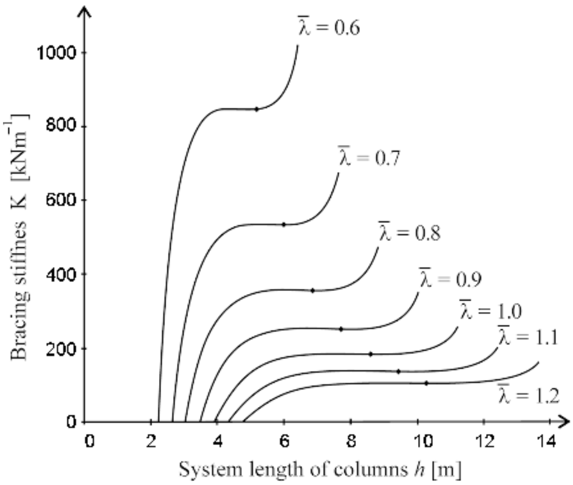

Fig. 5. Bracing stiffness K vs. h

3. Problem formulation

EUROCODE 3 (1993) determines $N_{b,Rd}$ with the aid of stability analysis with buckling length. The aim of the study is the verification of the design buckling resistance $N_{b,Rd}$ statistically evaluated according to EN 1990 (2002). Let us assume that $\bar{\lambda}$ is constant, $N_{b,Rd}$ is therefore also constant.

EN 1990 (2002) enables the evaluation of the design load carrying capacity using statistical analysis of the random load carrying capacity. The random variability of the load carrying capacity is due to the random variability of initial imperfections. The design load carrying capacity for $\beta_d = 3.8$ is, in practice, obtained as 0.1 percentile (Kala *et al.* 2009, 2010; Kala 2011a). Let us denote the 0.1 percentile of the load carrying capacity as $N_{b,0.1}$. Let us select constant $\overline{\lambda}$ and consider a set of pairs K and h, see Fig. 5. For each pair K and h, statistical analysis of the load carrying capacity is worked out, and design value $N_{b,0.1}$ is evaluated.

In the event that $N_{b,0.1}$ is higher than $N_{b,Rd}$, then analysis according to EUROCODE 3 (1993) is safe but not very economical, on the contrary, the analysis is economical but not very safe. In accordance with EN 1990 (2002), the design may be considered as optimally safe and economical if $N_{b,Rd} = N_{b,0.1}$.

4. Initial random imperfections

Statistical characteristics of out-of-plumb inclinations (sway imperfections) Θ_1, Θ_2 and initial crookedness of the columns (bow imperfections) e_1, e_2 were derived in Kala (2011a, b), see Fig. 6.

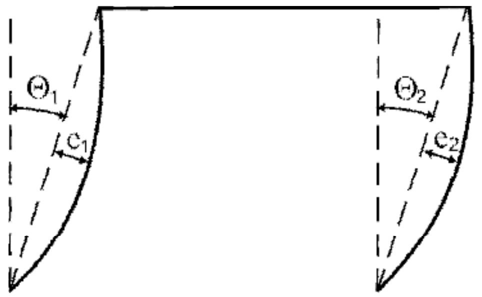

Fig. 6. Sway and bow random imperfection

Experimentally obtained cross-section geometry and material characteristics of steel products manufactured by a dominant Czech producer (Melcher *et al.* 2004; Kala *et al.* 2009) were used in the presented study. For non-measured quantities (e.g., Young's modulus), the study was based on data obtained from technical literature; e.g., statistical characteristics of Young's modulus are given in Soares (1988).

Input variables X_i of the left frame column include yield strength f_{y1}, cross-sectional height h_1, cross-sectional width b_1, web thickness t_{w1}, flange thickness t_{f1}, and Young's modulus E_1. Input variables of the right frame column are f_{y2}, h_2, b_2, t_{w2}, t_{f2}, E_2. Input variables of the cross beam are f_{y0}, h_0, b_0, t_{w0}, t_{f0}, E_0.

The input random variables (input random imperfections) are clearly listed in Table 1. All input random variables are statistically independent.

Table 1. Input random variables

Member	Symbol	Mean	St. deviation
LC	h_1*	220.20 mm	0.9731 mm
LC	b_1*	111.53 mm	1.0855 mm
LC	t_{w1}*	6.22 mm	0.2304 mm
LC	t_{f1}*	9.13 mm	0.4219 mm
LC	E_1**	210 GPa	10.5 GPa
LC	f_{y1}*	297.3 MPa	16.8 MPa
LC	e_1*	0	0.76533 *h*
CB	h_0*	270.24 mm	1.194 mm
CB	b_0*	136.88 mm	1.3322 mm
CB	t_{w0}*	6.96 mm	0.2577 mm
CB	t_{f0}*	10.13 mm	0.4678 mm
CB	E_0**	210 GPa	10.5 GPa
CB	f_{y0}*	297.3 MPa	16.8 MPa
RC	h_2*	220.20 mm	0.9731 mm
RC	b_2*	111.53 mm	1.0855 mm
RC	t_{w2}*	6.22 mm	0.2304 mm
RC	t_{f2}*	9.13 mm	0.4219 mm
RC	E_2**	210 GPa	10.5 GPa
RC	f_{y2}*	297.3 MPa	16.8 MPa
RC	e_2*	0	0.76533 *h*
System	Θ_1**	0	(Kala 2011b)
System	Θ_2**	0	(Kala 2011b)

Continue of Table 1

* Histogram, ** Gauss pdf,

LC left column, CB cross beam, RC right column

5. Geometrical non-linear analysis

A comprehensive description of the major methodologies of non-linear finite element analysis for solid mechanics, as applied to continua and structures, is listed in (Belytschko *et al.* 2001). Information for structural engineers covering the complete field of finite element analysis in solid mechanics is listed, e.g. (Bathe 1982; Němec *et al.* 2010). Let us describe some aspects of the geometrical non-linear analysis that was applied.

The frame geometry was meshed using beam elements with initial curvature in the form of a 3rd degree parabola, see Fig. 7. The solution is based on the assumption that the material is perfectly elastic, and that shear deformations are negligible.

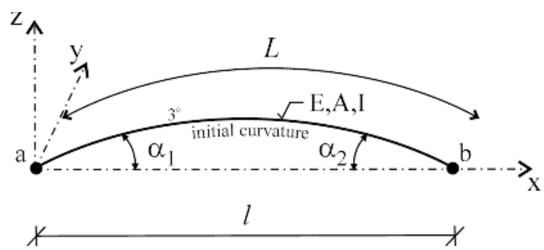

Fig. 7. Beam element with small initial curvature

If the structure is loaded with such a large load that deformations and rotations of elements are large but relative deformations are small, it is expedient to add the load effect in small incremental steps, so that linear relations may be used in each loading step. Let us describe the first two steps of the Euler incremental method.

Let us consider the first loading step and a frame with initial imperfections. The frame deformation is evaluated according to the linear theory of FEM. Further FEM analysis is applied to the frame with deformed geometry, and internal forces and moments are obtained. Conditions of equilibrium are thus fulfilled on the deformed structure.

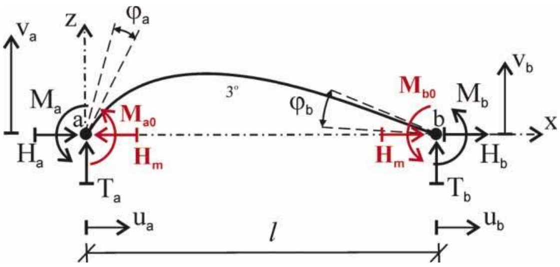

Fig. 8. Vectors f^T and u^T

Let us consider the second loading step. Each beam element is loaded by internal forces and moments from the first loading step. Stiffness of the i^{th} element is influenced by force H_m and moments M_{a0}, M_{b0}, see Fig. 8. The displacement vector of end nodes of the beam element has the form (1), and nodal forces and bending moments are written in the vector (2) (see Fig. 8):

$$u^T = \{u_a, v_a, \varphi_a, u_b, v_b, \varphi_b\}; \quad (1)$$

$$f^T = \{H_a, T_a, M_a, H_b, T_b, M_b\}. \quad (2)$$

The dependence between vectors (1) and (2) is described using the incremental stiffness matrix k_f:

$$k_f u = f. \quad (3)$$

The incremental stiffness matrix k_f is composed of the elastic stiffness matrix k_{f0} and the geometric stiffness matrix k_G (see Eqs (4), (5) and (6)).

$$k_f = k_{f0} + k_G. \quad (4)$$

The elastic stiffness matrix k_f is rewritten for utilization in the geometrical non-linear solution, so that it contains the tensile stiffness EA/L and bending stiffness EI/L of the unloaded member of length L, area A, the second moment of area I, and Young modulus E, see Fig. 7.

$$k_{f0} = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L}c_3 & -\frac{EA}{L}c_1 & -\frac{EA}{L} & -\frac{EA}{L}c_3 & -\frac{EA}{L}c_2 \\ & \frac{E\left(Al^2c_3^2+12I\right)}{l^2L} & \frac{E\left(Ac_1l^2c_3+6I\right)}{lL} & \frac{EA}{L}c_3 & -\frac{E\left(Al^2c_3^2+12I\right)}{l^2L} & \frac{E\left(Ac_2l^2c_3+6I\right)}{lL} \\ & & \frac{AE}{L}l^2c_1^2+\frac{4EI}{L} & \frac{EA}{L}c_1 & -\frac{E\left(Ac_1l^2c_3+6I\right)}{lL} & \frac{AE}{L}l^2c_1c_2+\frac{2EI}{L} \\ & & & \frac{EA}{L} & -\frac{EA}{L}c_3 & \frac{EA}{L}lc_2 \\ & & & & \frac{E\left(Al^2c_3^2+12I\right)}{l^2L} & \frac{E\left(Ac_2l^2c_3+6I\right)}{lL} \\ & & & & & \frac{AE}{L}l^2c_2^2+\frac{4EI}{L} \end{bmatrix}, \quad (5)$$

where:

$$c_1 = \frac{2}{15}\alpha_1 - \frac{1}{30}\alpha_2;\ c_2 = \frac{2}{15}\alpha_2 - \frac{1}{30}\alpha_1;\ c_3 = c_1 + c_2.$$

$$k_G = \frac{M_{a0}+M_{b0}}{l^2}\begin{bmatrix} 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \frac{H_m}{l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (6)$$

where force H_m and moments M_{a0}, M_{b0} are known from the previous loading step. As soon as frame deformation from the second loading step is evaluated, the linear solution of the FEM ($k_f = k_{f0}$) is then used to evaluate the internal forces and moments of the deformed frame. The procedure is analogous for further loading steps of the Euler method. In practice, step-by-step Euler Newton-Raphson iterative procedures are used; see, e.g., Reddy (2004), Wriggers (2008).

The geometrical non-linear solution was worked out and programmed by the author of the presented paper (Kala 2005). Each column was meshed using ten beam elements, and the cross-beam was meshed using three beam elements.

Initial axial crookedness of the columns and cross beam of the analyzed frame gradually increase with increasing load until ultimate limit state has been reached. The first criterion (i) for the load carrying capacity is the loading at which plasticization of the flange is initiated (Kala 2005). The second criterion (ii) for the load carrying capacity is given by the load corresponding to a decrease of the determinant of the stiffness matrix to zero which occurs at high yield strength values of slender columns with small initial imperfections Θ_1, Θ_2, e_1, e_2 (Kala 2005). Increase in load leads to decrease of the determinant of stiffness matrix until either criterion (i) or (ii) occurs. In other words, the ultimate one-parametric loading (elastic resistance) is defined as the lowest value of load carrying capacities (i) and (ii).

6. Statistical analysis

Realizations of input variables from Table 1 were computed applying the Latin Hypercube Sampling (LHS) method, which is a method of type Monte Carlo (McKey *et al.* 1979). LHS method is used to simulate a real experiment. The output is the random load carrying capacity. The load carrying capacity was determined with an accuracy of 0.1 percent in each simulation run.

The statistical analysis was evaluated using 300 thousand simulation runs. $N_{b,0.1}$ was evaluated from the basic probability definition in a manner described in (Kala 2009). Fig. 9 illustrates an example of random

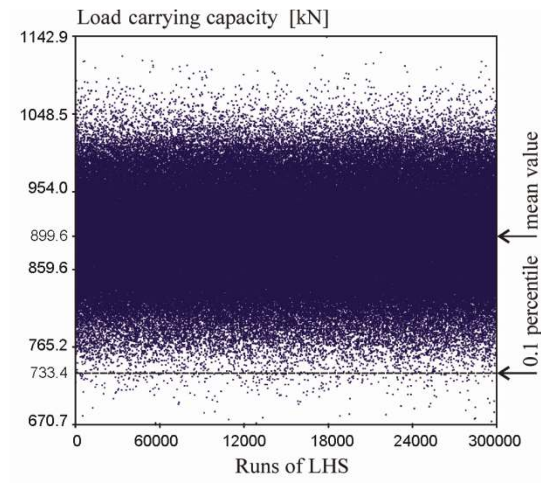

Fig. 9. Load carrying capacity for $\overline{\lambda} = 0.6$, $h = 2.175$

realizations of the load carrying capacity for $\overline{\lambda} = 0.6$, $h = 2.175$ m and $K = 0$. Three hundred random realizations of the load carrying capacity have values lower than 733.4 kN. Practically, $N_{b,0.1} = 33.4$ kN is obtained as the 300th lowest value in the organized ascending file (see Kala (2009)).

Results of the statistical analysis from Fig. 9 are depicted in Fig. 10. The course of mean values, standard deviations and $N_{b,0.1}$ were approximated by cubic polynomials from ten values of h. The procedure is analogous for other values of $\overline{\lambda}$, see Figs 10 to 16.

The design value of load carrying capacity evaluated according to EN 1990 (2002) as $N_{b,0.1}$ is lower (safer) than the mean value. It is apparent from Figs 10 to 16 that the course of $N_{b,0.1}$ is concave. This is mainly due to the concave course of mean values and convex course of standard deviations. The comparison of $N_{b,0.1}$ and $N_{b,Rd}$ points out the misalignment of reliability of steel frames designed according to EUROCODE 3 (1993). The higher the value of $N_{b,0.1}$, the higher the reliability of design according to EUROCODE 3 (1993).

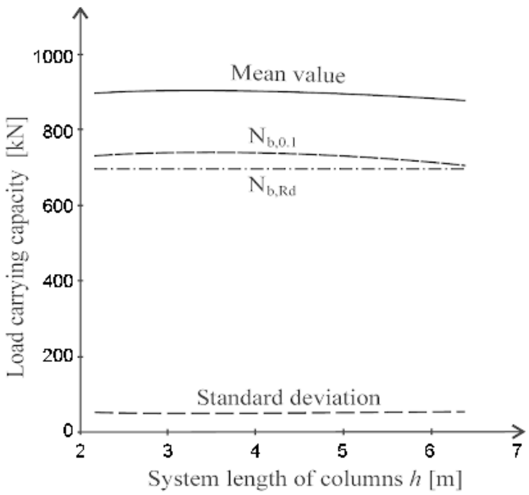

Fig. 10. Load carrying capacity for $\overline{\lambda} = 0.6$

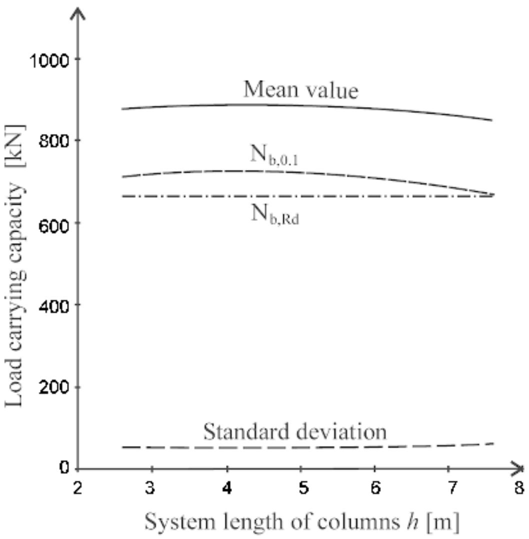

Fig. 11. Load carrying capacity for $\overline{\lambda} = 0.7$

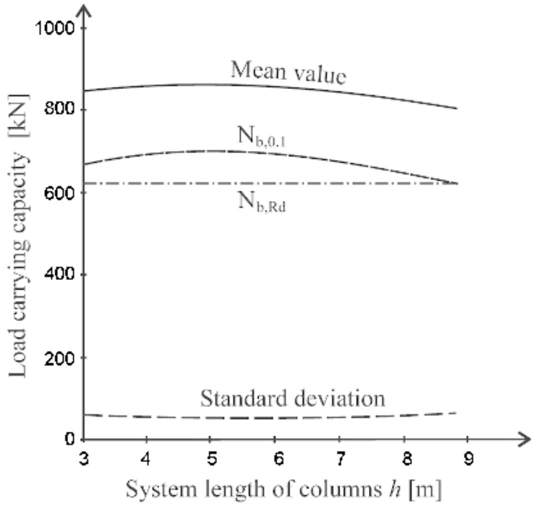

Fig. 12. Load carrying capacity for $\overline{\lambda} = 0.8$

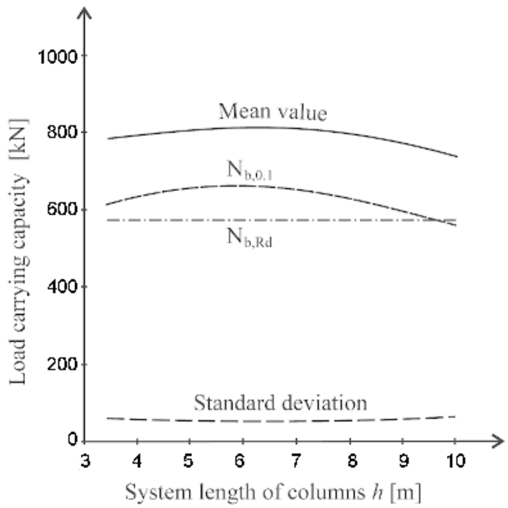

Fig. 13. Load carrying capacity for $\overline{\lambda} = 0.9$

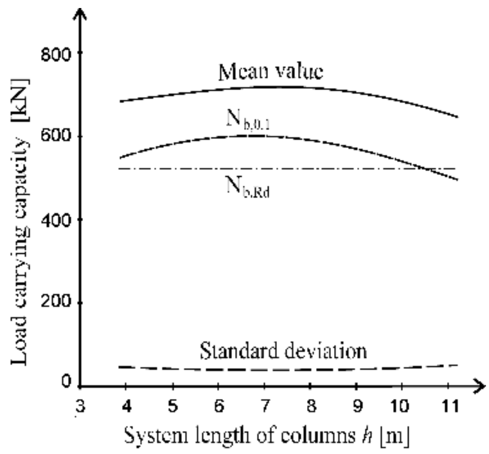

Fig. 14. Load carrying capacity for $\overline{\lambda} = 1.0$

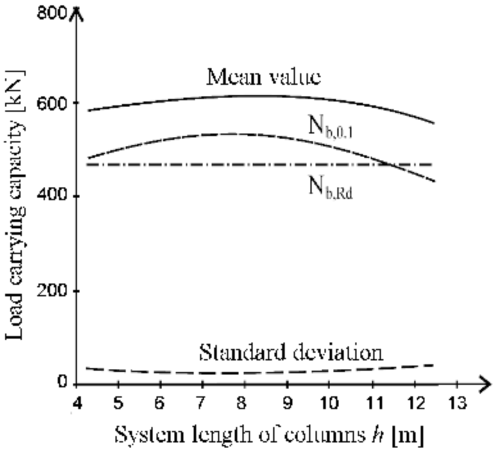

Fig. 15. Load carrying capacity for $\overline{\lambda} = 1.1$

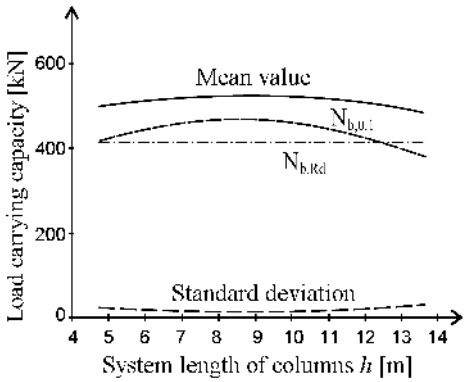

Fig. 16. Load carrying capacity for $\overline{\lambda} = 1.2$

The differences between the mean value and $N_{b,0.1}$ are, to a certain degree, influenced by the plot of the standard deviation of the load carrying capacity. The standard deviation of the load carrying capacity (output) is dependent on the standard deviations of variables from Table 1 (inputs). One of the important tasks of the probabilistic analysis of reliability is the identification of input random variables that have a dominant influence on the variance of output.

The effect of sway and bow imperfections on the load carrying capacity is cardinal for $\overline{\lambda} \approx 0.9$ (Kala 2011a). Let us consider $\overline{\lambda} \approx 0.9$ and respective pairs *K* and *h* (see Fig. 2), and let us study the sensitivity of the load carrying capacity to initial imperfections.

7. Sensitivity analysis

The sensitivity analysis is a study of how variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, on different sources of variation, and of how the given model depends upon the information fed into it (Saltelli *et al.* 2004). Different understanding of sensitivity analysis is used in different modelling communities, see, e.g., Edalat *et al.* (2010), Keitel *et al.* (2011). The imperfection sensitivity in buckling problems has been the subject of numerous investigations (Szymczak 2003; Mang *et al.* 2009; Melcher *et al.* 2009; Kala 2009; Mang *et al.* 2011). The main motivation for these studies is the fact that initial imperfections often bring about a significant reduction of buckling resistance.

One of the most perfect methods of sensitivity analysis is the Sobol sensitivity analysis (Sobol' 1993; Saltelli *et al.* 2004). In our study, the sensitivity analysis of load carrying capacity (random output *Y*) to input imperfections (random inputs X_i) was evaluated according to Eqs (7) and (8):

$$S_i = \frac{V(E(Y|X_i))}{V(Y)}. \quad (7)$$

S_i measures the first order (e.g., additive) effect (the so-called main effect) of X_i (input imperfections) on the model output *Y* (load carrying capacity). The sum of all S_i is equal to 1 for additive models, and less that 1 for non-additive models.

The second order sensitivity index S_{ij} is the interaction term (8) between factors X_i, X_j. It captures that part of the response of *Y* to X_i, X_j which cannot be written as a superposition of effects separately due to X_i and X_j:

$$S_{ij} = \frac{V(E(Y|X_i, X_j))}{V(Y)} - S_i - S_j. \quad (8)$$

An important distinction between Sobol and classical sensitivity is that the Sobol sensitivity analysis detects interactions of input variables through the second and higher order terms, whilst classical sensitivity methods give only derivatives with respect to single variables. The sensitivity indices of other higher orders can be calculated analogously. The case with statistically independent input random variables is studied. For a system with *M* factors, there may be interaction terms up to the order *k*, i.e. (Saltelli *et al.* 2004):

$$\sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \sum_i \sum_{j>i} \sum_{k>j} S_{ijk} + ... + S_{123...M} = 1. \quad (9)$$

The sensitivity indices were evaluated applying the LHS method. The conditional random arithmetical mean $E(Y|X_\mathrm{i})$ was evaluated for 500 simulation runs; the variance $V(E(Y|X_\mathrm{i}))$ was calculated for 500 simulation runs, as well. The variance $V(Y)$ of load carrying capacity is calculated under the assumption that all the input imperfections are considered to be random ones; one million runs were applied. The second order sensitivity indices (8) were calculated analogously. Interactions of all further higher orders are grouped in a single term (see Figs 17 and 18).

It is apparent from Fig. 19 that if $h \to 10$ m ($F_{cr2} \to F_{cr1}$) then the sensitivity of the load carrying capacity to initial imperfections e_1 and e_2 rises sharply. For $h = 10$ m $S_{e1}+S_{e2}+S_{e1,e2} = 0.84$, which shows that the influence of bow imperfections e_1 and e_2 on the load carrying capacity is very high. The sum of second order interaction terms between bow and sway imperfections is $S_{e1,\theta1}+S_{e2,\theta1}+S_{e1,\theta2}+S_{e2,\theta2} = 0.03$. The example shows that

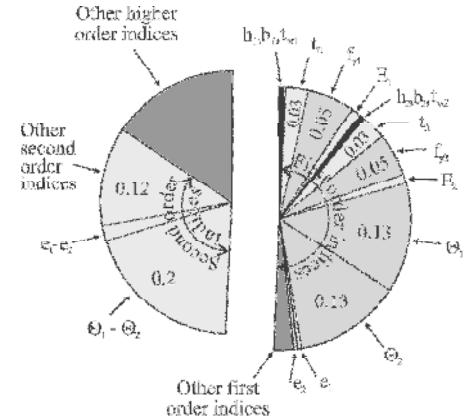

Fig. 17. Sensitivity analysis for $\overline{\lambda} = 0.9$, h=3.45

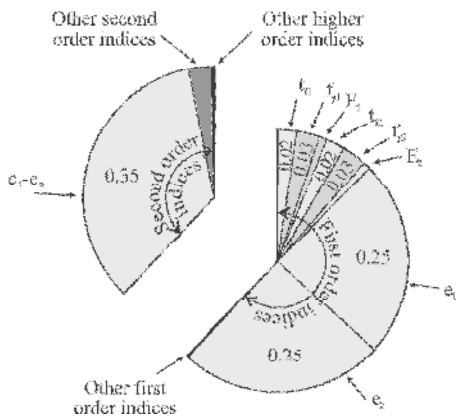

Fig. 18. Sensitivity analysis for $\overline{\lambda} = 0.9$, h=10

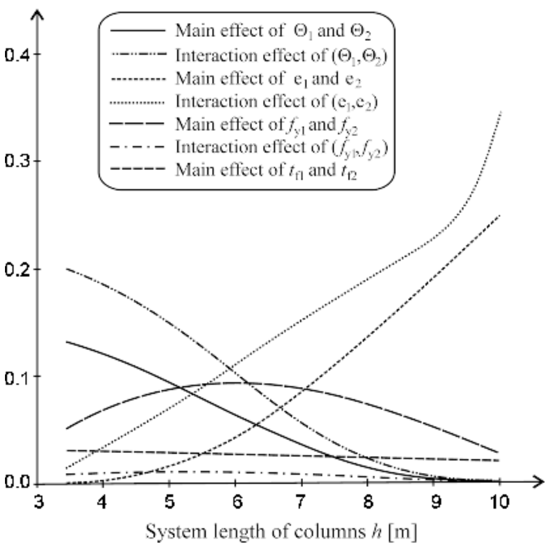

Fig. 19. Sensitivity analysis for $\overline{\lambda} = 0.9$

the sensitivity of load carrying capacity to initial bow imperfections e_1 and e_2 increases sharply if $F_{cr2} \rightarrow F_{cr1}$. The Sobol sensitivity analysis supplements results (Bazant, Xiang 1997), where a similar problem was analyzed for non-random imperfections.

The courses of the main effects of f_{y1} and f_{y2} (see Fig. 19) and $N_{b,0.1}$ (see Fig. 13) are concave. The design of the frame according to EUROCODE 3 (1993) is most reliable when $N_{b,0.1}$ is maximal. This occurs when the load carrying capacity is very sensitive to the yield strength and thus relatively insensitive to imperfections e_1, e_2, Θ_1, Θ_2.

Let us note that the statistical analysis of the load carrying capacity requires precise statistical information on e_1 and e_2 obtained from ample samples, which however are not always available (e.g., Kala *et al.* 2010). Experimental data is absent, particularly if sway imperfections are studied. Epistemic uncertainty of bow e_1, e_2 and primarily sway Θ_1, Θ_2 imperfections is higher than other imperfections (e.g., Kala 2007; Kala 2011a). Therefore if the load carrying capacity is very sensitive to bow and/or sway imperfections, we can expect higher risks related to the uncertainty of ultimate limit state.

8. Stability and reliability of structures

The requirement to analyze the influence of geometric imperfections on limit states is encountered mainly in long compressed steel bars. But even short bars may fail from the point of stability. This occurs if the steel bars are composed of thin-walled elements susceptible to local loss of stability. Current research is aimed mainly at FEM simulations (e.g., Kotelko *et al.* 2008, 2011; Loughlan *et al.* 2011; Pasternak, Kubieniec 2010; Pavlovčič *et al.* 2010; Melcher *et al.* 2009). In all cases, the stability failure (in comparison with e.g. failure of a tensed bar) is very tricky because it is not signalled beforehand by increase in deformation, and it occurs suddenly and without warning.

Stability problems may occur in load bearing structures as well as in geotechnical tasks during analysis of stability of hillsides and rock walls. Input data is obtained from experimental research (e.g., Amšiejus *et al.* 2009, 2010). In the case of steel-concrete structures, stability occurs mainly in concrete columns reinforced by high strength steel bars (Kliukas *et al.* 2010) and composite columns (Guezouli *et al.* 2010; Kala *et al.* 2010). Characteristics of concrete and steel-concrete structures include long-term effects such as creep and shrinkage, which may cause loss of strength (Au, Si 2011).

In the building industry, stability is an important phenomenon regarding primarily reliability. Structural reliability is secured, to a basic degree, by standards for design. The general principles for reliability of different structures are provided by the international standard ISO 2394. The normative principles and application rules are listed in standard EN 1990 (2002) in Europe, and in ASCE/SEI 7-05 (2005) in the USA. Current approaches in the verification of reliability emanate mainly from probabilistic methods (Kala 2007; Karmazínová *et al.* 2009; Kudzys *et al.* 2010) and advanced optimization approaches (e.g., Atkočiūnas *et al.* 1997; Kala 2008; Gottvald 2010; Leng *et al.* 2011; Atkočiūnas, Venskus 2011; Jankovski, Atkočiūnas 2011). Different risk assessments of construction projects are discussed and compared (e.g., Zavadskas *et al.* 2010). Generally, the degree of reliability of slender bars loaded in compression could be more differentiated. However, the current trend is aimed mainly at transparency and simplicity of computational approaches of standards for design.

9. Conclusions

The design buckling resistance $N_{b,Rd}$ evaluated according to EUROCODE 3 (1993) was verified with the design value $N_{b,0.1}$ obtained from statistical analysis according to EN 1990 (2002). Different values of $N_{b,0.1}$ were evaluated for constant $\overline{\lambda}$ values. The differences between $N_{b,Rd}$ and $N_{b,0.1}$ indicate that the reliability of design of steel frames according to EUROCODE 3 (1993) is significantly misaligned. The lowest values of $N_{b,0.1}$ were obtained for $F_{cr1} = F_{cr2}$, when the non-sway (symmetric) and sway (anti-symmetric) buckling modes coincide, see Figs 10 to 16. The second lowest $N_{b,0.1}$ values were obtained for zero bracing stiffness $K = 0$. The course of $N_{b,0.1}$ is concave and has a maximum which (in comparison with $N_{b,Rd}$) corresponds to the relatively most reliable design according to EUROCODE 3 (1993). Parameter $\overline{\lambda}$ was selected by seven constants, so that the most frequently occurring values in engineering practice were represented.

The Sobol sensitivity analysis was used to study a frame with $\overline{\lambda} = 0.9$. The sensitivity of the load carrying capacity to imperfections e_1 and e_2 increases if $F_{cr2} \rightarrow F_{cr1}$, see Fig. 19. Maximum sensitivity (as well as the lowest value of $N_{b,0.1}$) was obtained for $F_{cr1} = F_{cr2}$, see Fig. 18 (and Fig. 13). The study indicates a certain significance of the second critical load F_{cr2} as another indicator of the sensitivity of frames to imperfections. The coincidence of the critical loads of two different buckling modes increases the sensitivity on the imperfections e_1, e_2. Let us note that F_{cr2} is not taken into consideration during common design of steel structures according to EUROCODE 3 (1993), based on the rationale that perfect structures can theoretically buckle only according to the first buckling mode.

The probability that a frame with initial random imperfections is symmetrical is zero from the mathematical point of view. The deformed geometry of the frame upon attaining the load carrying capacity is a random variable which is dependent on the initial random imperfections. The final deformation of column axis for the ultimate limit state has a random shape and is not affine to the initial imperfections. The frame deformation for $F_{cr2} \rightarrow F_{cr1}$ upon reaching the limit state is formatively close to the non-sway mode (see right part of Fig. 4). On the contrary, for $K = 0$, the deformations are formatively close to the sway mode (see left part of Fig. 4).

Imperfections that have high influence on the load carrying capacity deserve special attention, especially in cases when $N_{b,0.1}$ is low (low safety). With regard to the system reliability, it is important to identify imperfections that interact and which may thus generate extreme values. An example is the second order index $S_{e1,e2} = 0.35$, see Fig. 18. Within the interval $h \in \langle 9; 10 \rangle$, functions K and $S_{e1,e2}$ are increasing and convex, whilst functions S_{e1} and S_{e2} are increasing and approximately linear, see Figs 2 and 19. The acceleration of increase $S_{e1,e2}$ at the ends of the interval are interesting and should be further studied.

During the analysis of reliability and economy of design of steel frames, it is necessary to identify those cases when the design values evaluated according to EN 1990 (2002) are dangerously low, and to study the imperfections that cause them. Reliable design can be achieved either through change of reliability indicators of design standard EUROCODE 3 (1993), or increasing the quality of production. In the case of initial imperfections Θ_1, Θ_2, e_1, e_2 it is generally recommended to decrease their deviation, which can be achieved through more consistent control in production.

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PLIENINIŲ RĖMŲ SU DEFEKTAIS GEOMETRIŠKAI NETIESINĖ BAIGTINIŲ ELEMENTŲ PATIKIMUMO ANALIZĖ

Z. Kala

S a n t r a u k a

Tiriama plieninio plokščio rėmo su standžiaisiais ryšiais laikomoji galia. Ji vertinama atliekant geometriškai netiesinę BEM analizę. Aptariama šiek tiek išlinkusio elemento laipsniškai didėjanti standumo matrica, atliekant netiesinį iteracinį skaičiavimą. Atsitiktiniu dydžiu laikomas pradinis defektas. Statistinė ir Sobolio (*Sobol*) jautrumo analizė atliekama pritaikant LHS metodą (*Latin Hypercube Sampling Method*). Nagrinėjamas pradinio atsitiktinio defekto poveikis laikomajai galiai darant prielaidą, kad pastovus dydis yra liauna kolona. Vertinimo kriterijus yra ne atsitiktinių didžių pora, t. y. tampriai standūs ryšiai ir konstrukcijos kolonų aukštis. Straipsnyje aptariama kolonos pradinio kreivio įtaka laikomajai galiai, atsižvelgiant į klupumo formas, kai nelinksta (simetrinė apkrova) ir linksta (nesimetrinė apkrova). Laikomosios galios projektavimo apkrovos, šiuo atveju gautos iš statistinės analizės pagal EN 1990 (2002) standartą, yra palyginti nedidelės. Gauti rezultatai rodo, kad plieninio rėmo patikimumas pagal Eurocode 3 (1993) labai nesutampa. Nagrinėjama defektų įtaka laikomajai galiai atsižvelgiant į pirmą ir antrą klupimo jėgą.

Reikšminiai žodžiai: rėmas, plienas, stiprumas (stipris), defektai, suirimas, netiesinis, ribiniai būviai, patikimumas, projektavimas.

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