Geometry Variations Analysis of TiO_2 Thin-Film and Spintronic Memristors

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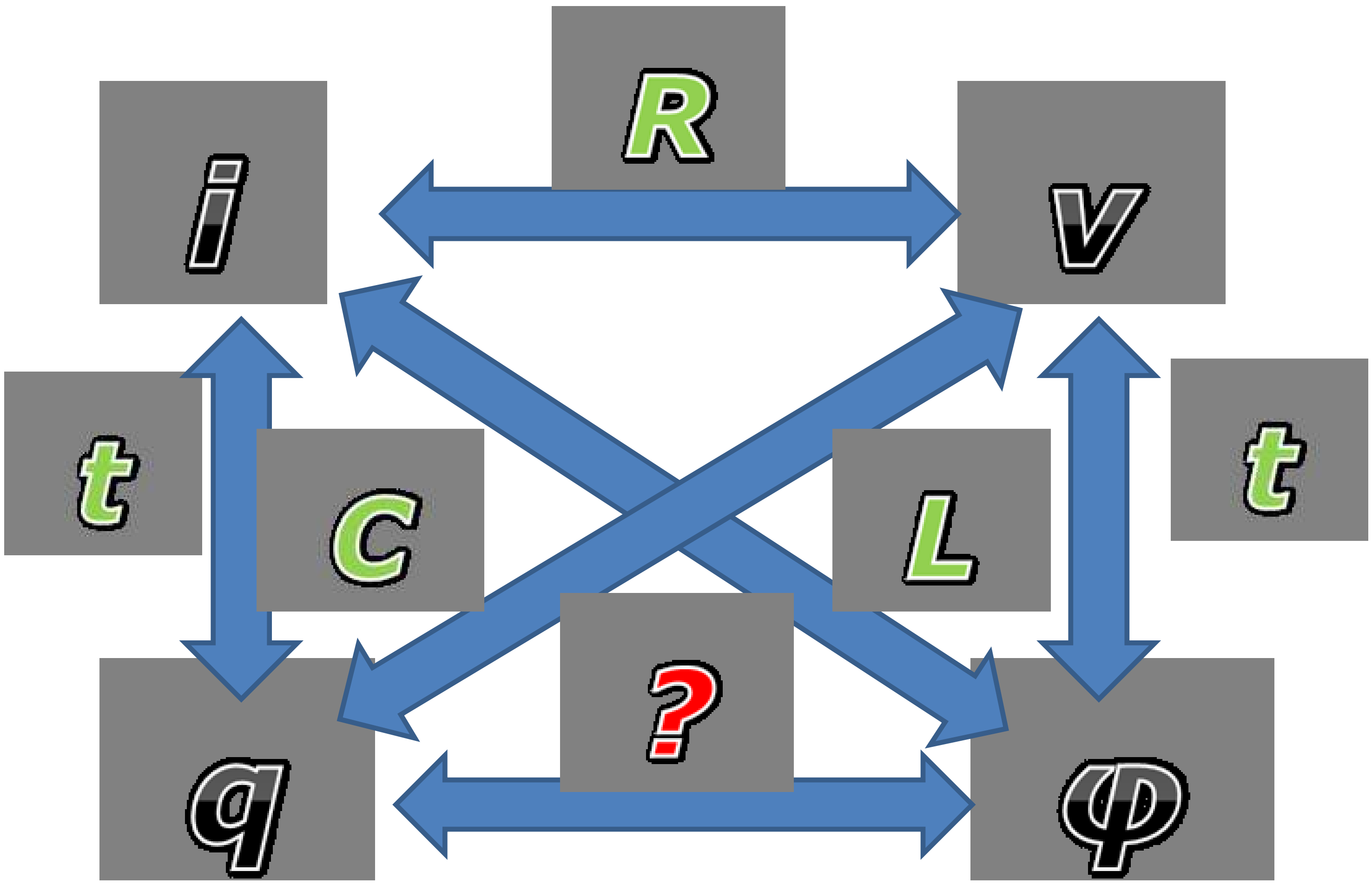
WHAT IS MEMRISTOR



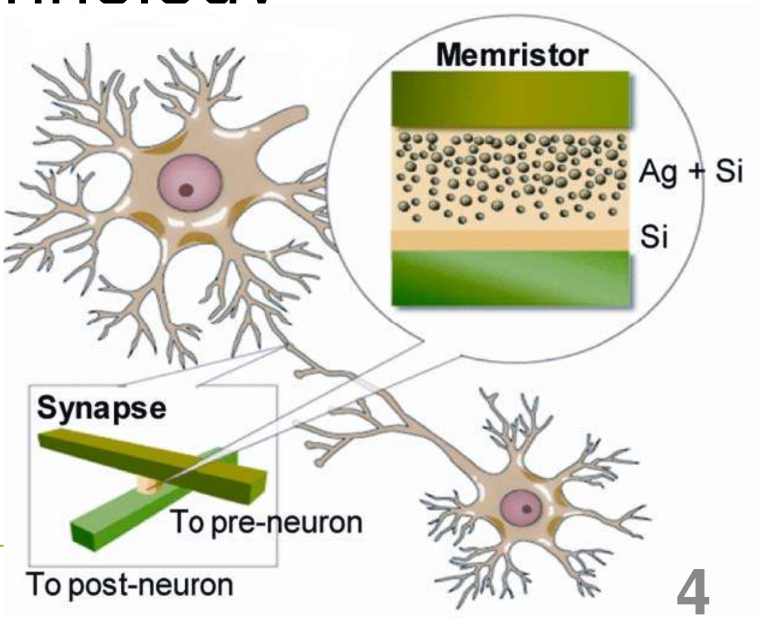
WHAT IS MEMRISTOR

- Memristor is a resistor with memory
  - $M(t) = \phi(t)/q(t)$, unit Ω
  - Intrinsic state to remember the history
  - Passive, AC
- Predicted in 1971, Found in 2008.

PROSPECT OF MEMRISTOR

- Memristor features
  nano-size, non-volatility, reconfigurable

- Potential Applications
  - High density storage technology
    DRAM 18 $Gbits/cm^2$
    Memristor 100 $Gbits/cm^2$
  - Reconfigurable computation
  - Neural network



OUTLINE

- ❑ Motivation
- ❑ Memristor examples
  - TiO_2 thin-film memristor
  - Spintronic memristor
- ❑ Memristor model with geometry variations
- ❑ Statistical analysis
- ❑ Performance analysis
- ❑ Summary

MOTIVATIONS

- Improve fabrication
  - Measurement
  - Predict margin and actual performance
- More applications
  - Multi-level memory: Levels?
  - Reconfigurable computation: Accuracy?
  - Neural network: Fuzzy operation?

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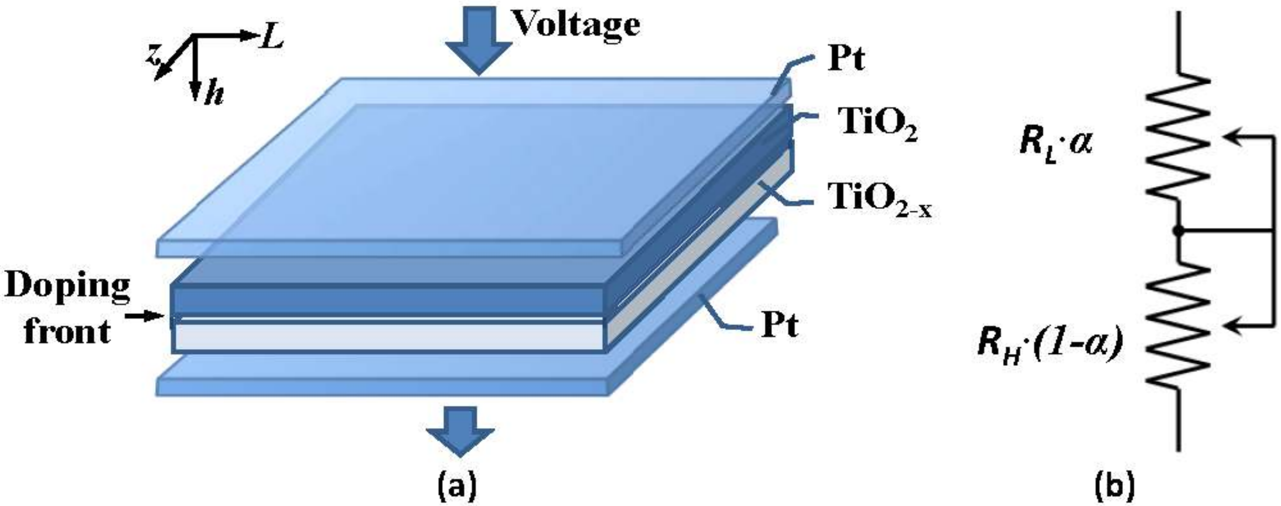
MEMRISTOR EXAMPLES

- ❑ TiO_2 thin-film memristor

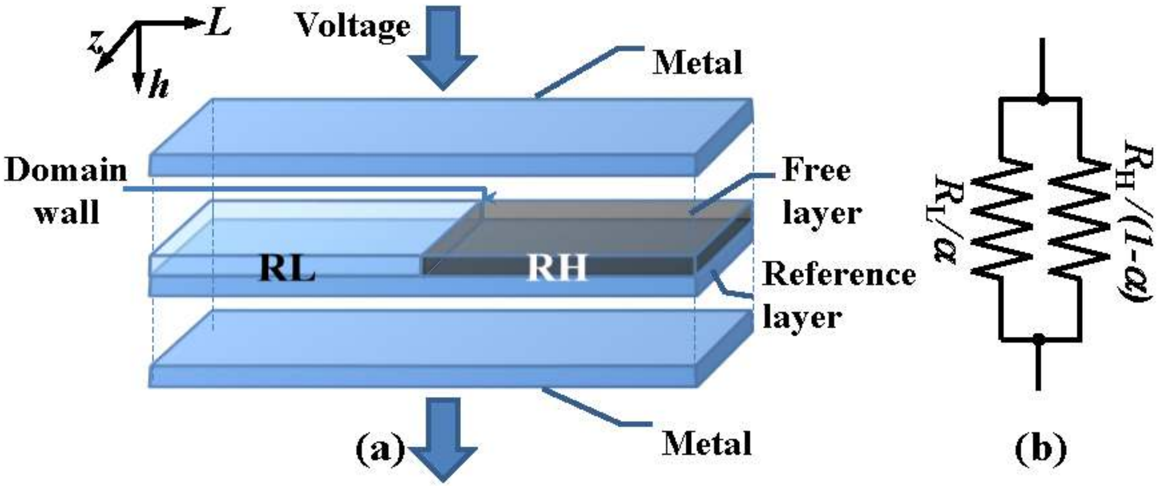


$$M(a) = a \cdot R_L + (1-a) \cdot R_H$$

$$\frac{da(t)}{dt} = \mu_v \cdot \frac{R_L}{h^2} \cdot \frac{V(t)}{M(a)}$$

MEMRISTOR EXAMPLES

- TMR based spintronic memristor



$$M(a) = \frac{R_H \cdot R_L}{R_H \cdot a + R_L \cdot (1-a)}$$

$$\frac{da(t)}{dt} = \frac{\Gamma_v}{l} \cdot J_{eff}(t), J_{eff} = \begin{cases} J(t) = \dfrac{V(t)}{M(a) \cdot l \cdot z} & , J(t) \geq J_{cr} \\ 0 & , J(t) < J_{cr} \end{cases}$$

DIFFERENCES

- ❑ Typical
- ❑ Many differences
  - Principles
  - Equivalent circuits
  - Sizes
- ❑ Make model a general solution

	Length (*L*)	**Width (*z*)**	**Thickness (*h*)**
TiO_2	50 nm	50 nm	10 nm
Spintronic	200 nm	10 nm	7 nm

OUTLINE

NYU-poly
POLYTECHNIC INSTITUTE OF NYU

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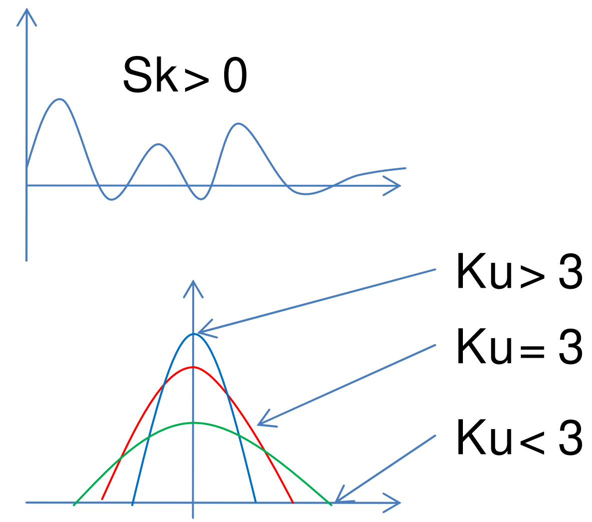
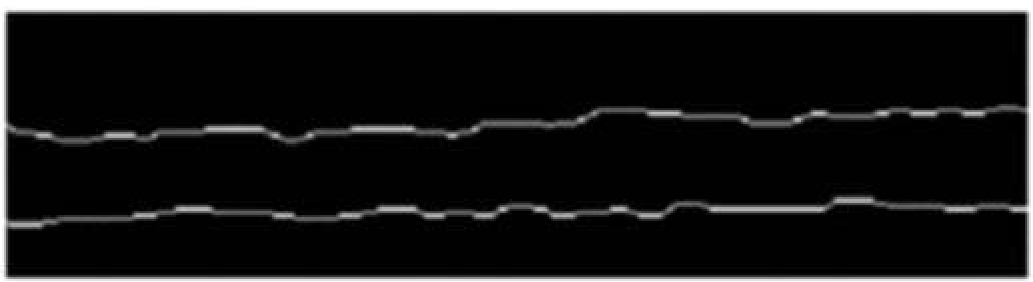
GEOMETRY VARIATIONS

- Line Edge Roughness (LER)
  - Beyond the capability of analytic model
  - The most difficult part
- Thickness fluctuation (TF)
  - Follows Gaussian distribution
- Random doping (RDD)
  - Not significant, not considered

LER CHARACTERIZATION

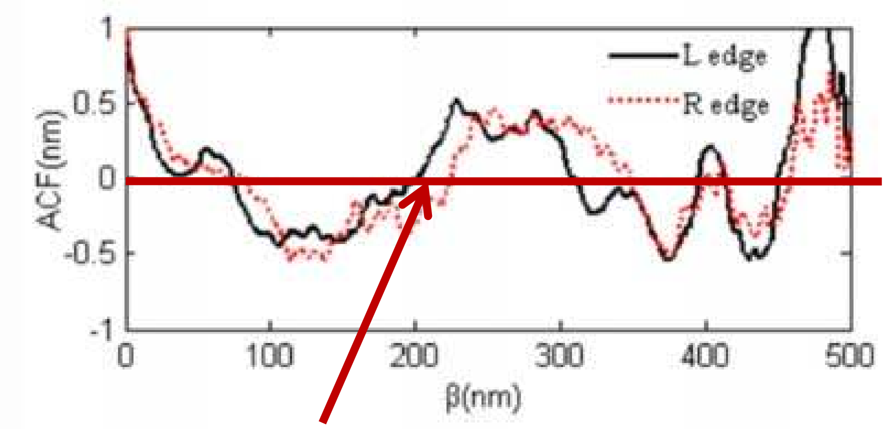
- LER's characteristics*:
  - Root Mean Square (RMS)
  - Skewness (Sk)
  - Kurtosis (Ku)
  - Power spectral density (PSD)
  - Auto-correlation function (ACF)



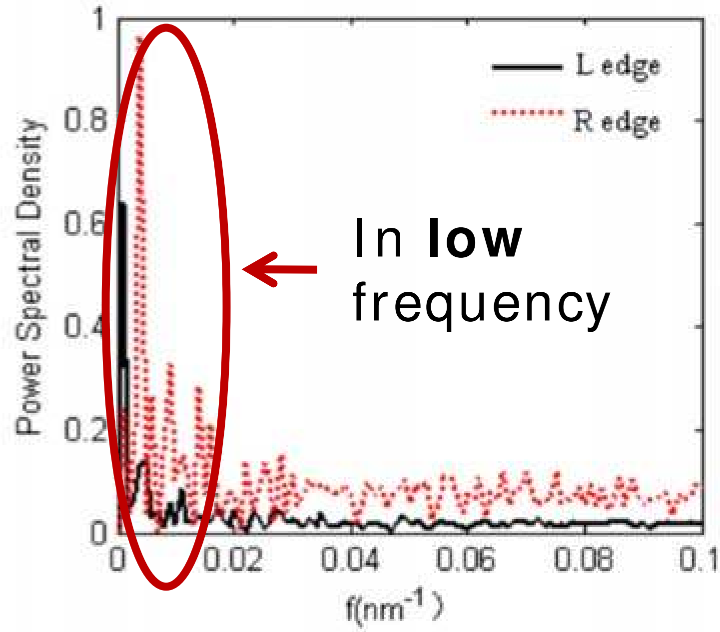


* Z. Jiang, "Characterization of Line Edge Roughness and Line Width Roughness of Nano-scale Typical Structures," 2009.

LER CHARACTERIZATION



oscillate around **zero**

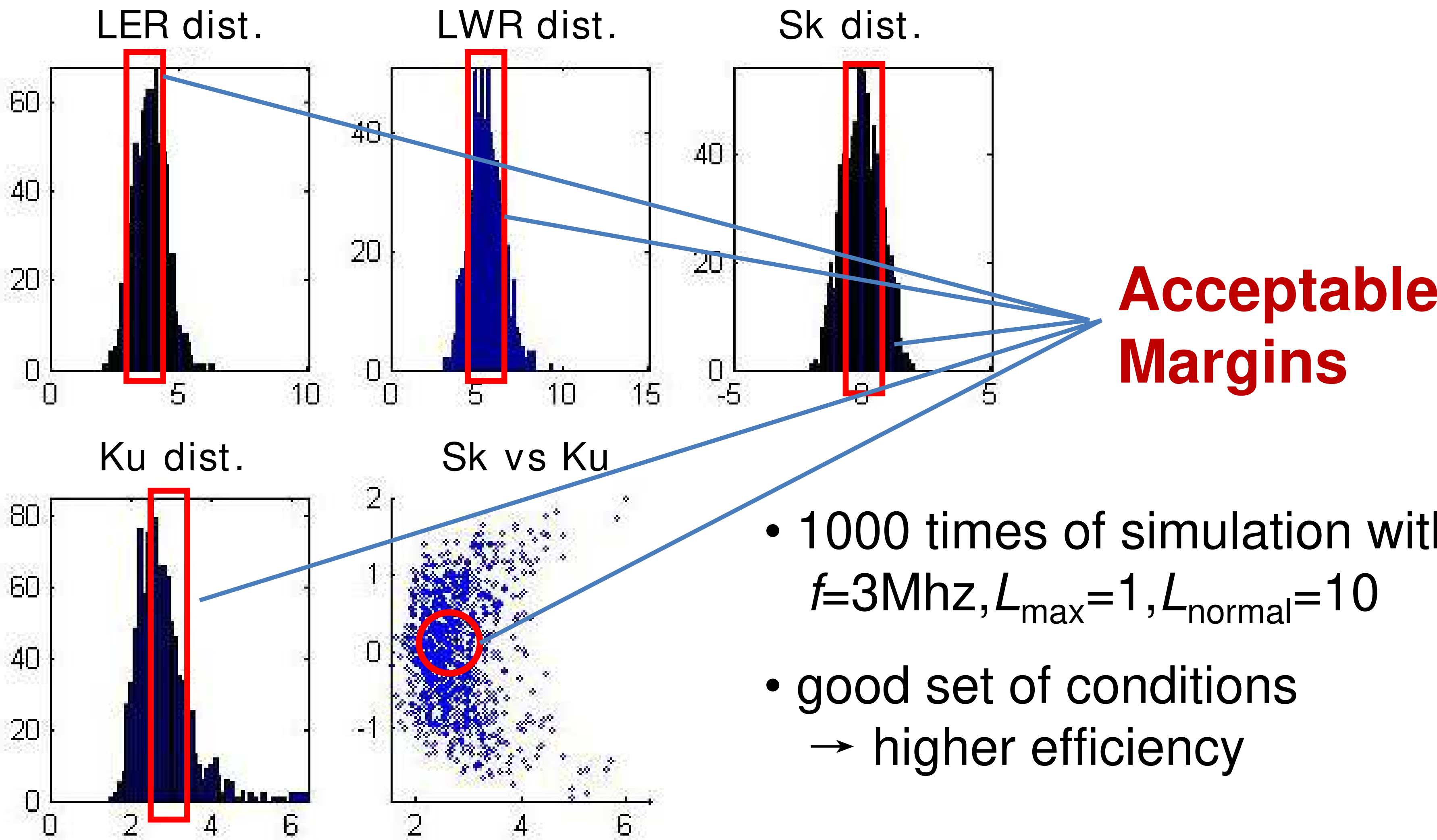


$$\Delta L = L_{max} \cdot \sin(f \cdot x) + L_{normal} \cdot p$$

LER Noise ***Low Frequency Noise*** ***Gaussian Noise***

* from nano-scale resist pattern fabricated with CABL-9000C EBL system of Crestec. Co. of Japan with the accelerating voltage 30kV

LER SIMULATION RESULTS

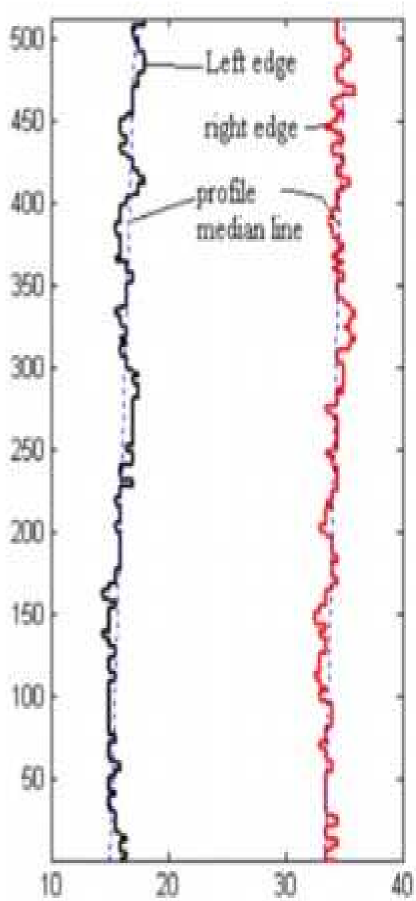


- 1000 times of simulation with f=3Mhz, L_{max}=1, L_{normal}=10
- good set of conditions → higher efficiency

LER CHARACTERIZATION

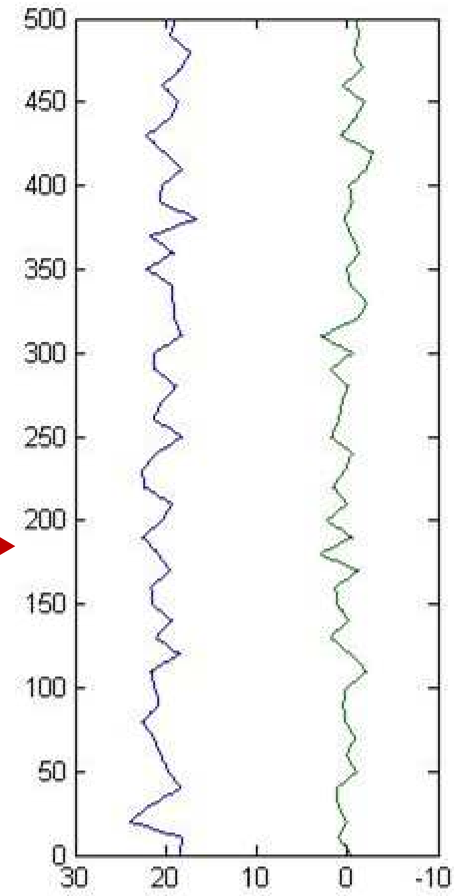
Left line:
3 σ_{LER} :3.52
Sk:0.1703
Ku:2.9458

Real Data →



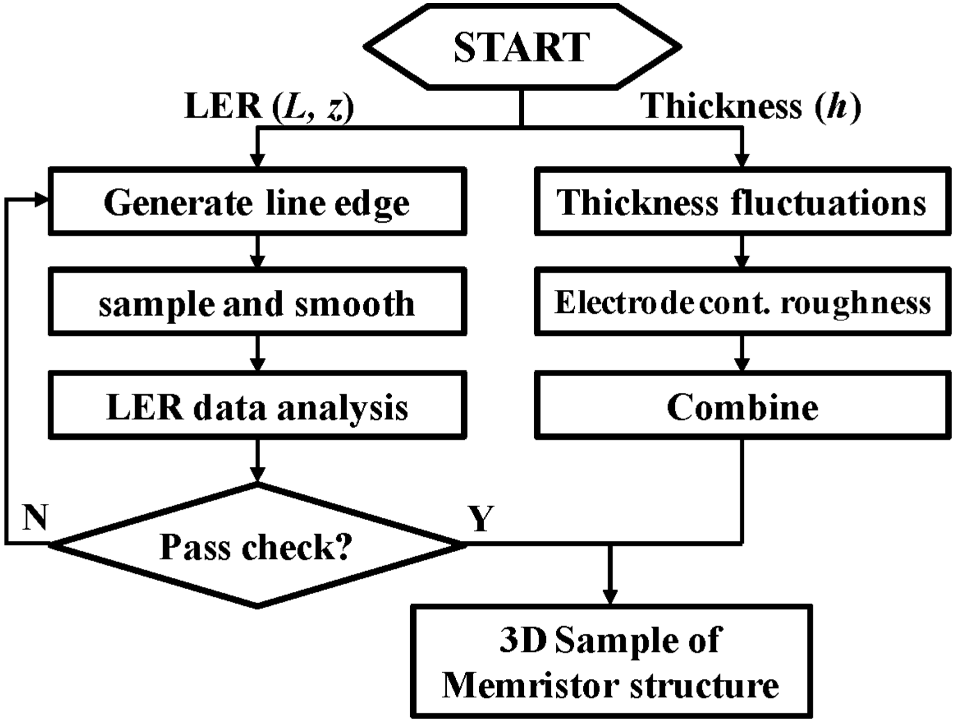
Right line:
3 σ_{LER} :2.65
Sk:0.1127
Ku:3.0212

Simulation Result →

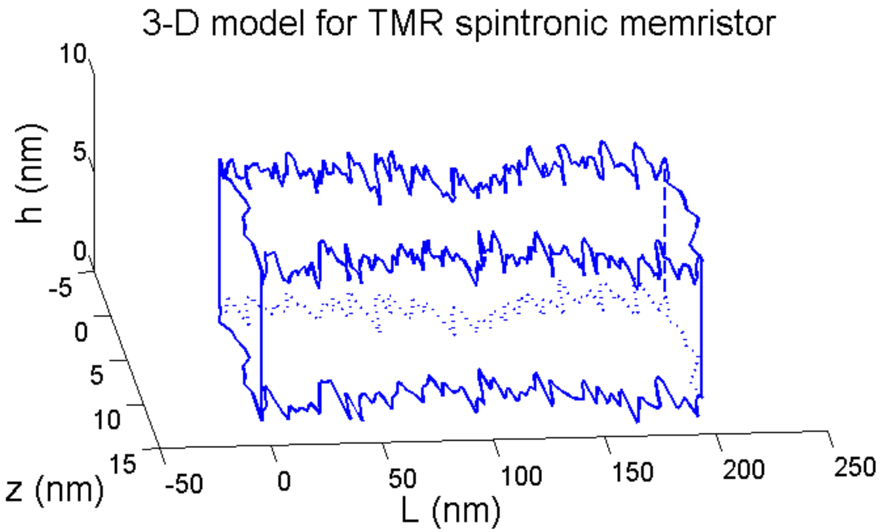


* from nano-scale resist pattern fabricated with CABL-9000C EBL system of Crestec. Co. of Japan with the accelerating voltage 30kV

GEOMETRY VARIATIONS GENERATION FLOW



Generation Flow



An Example of Spintronic Memristor

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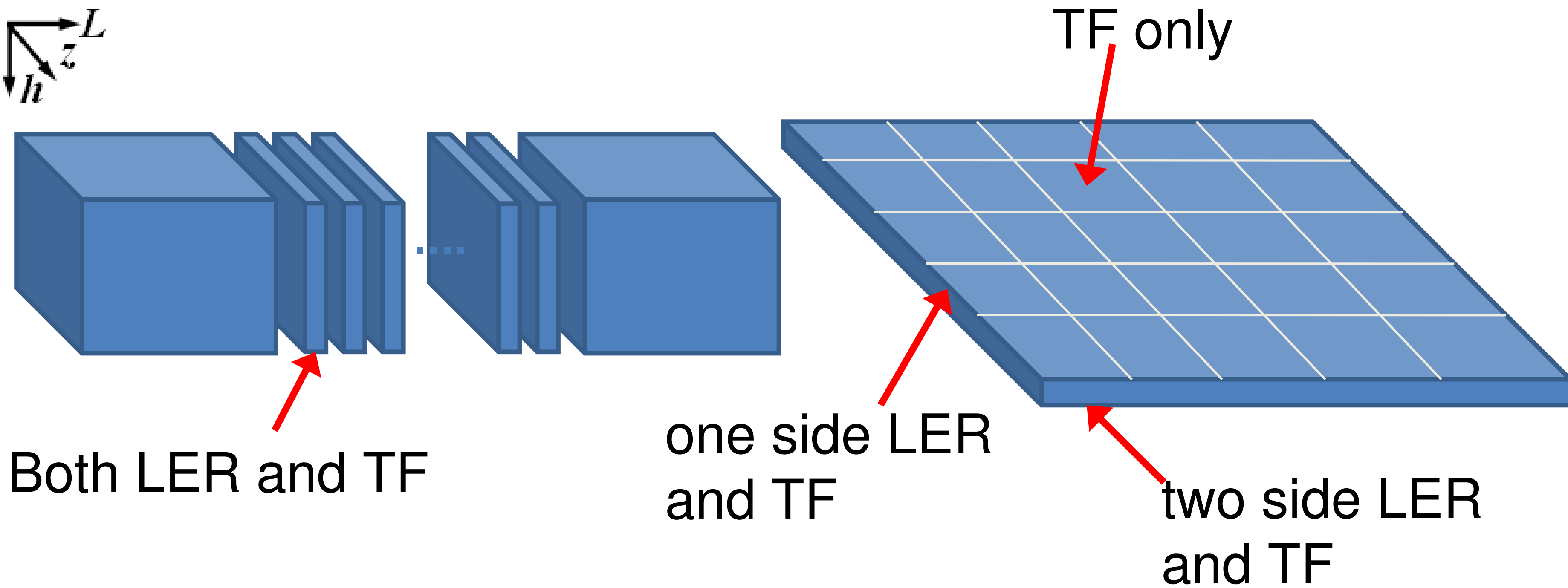
STATISTICAL ANALYSIS

Spintronic memristor **TiO_2 memristor**



- ***Compute state of each filament***
- ***Combine them to achieve the overall performance***

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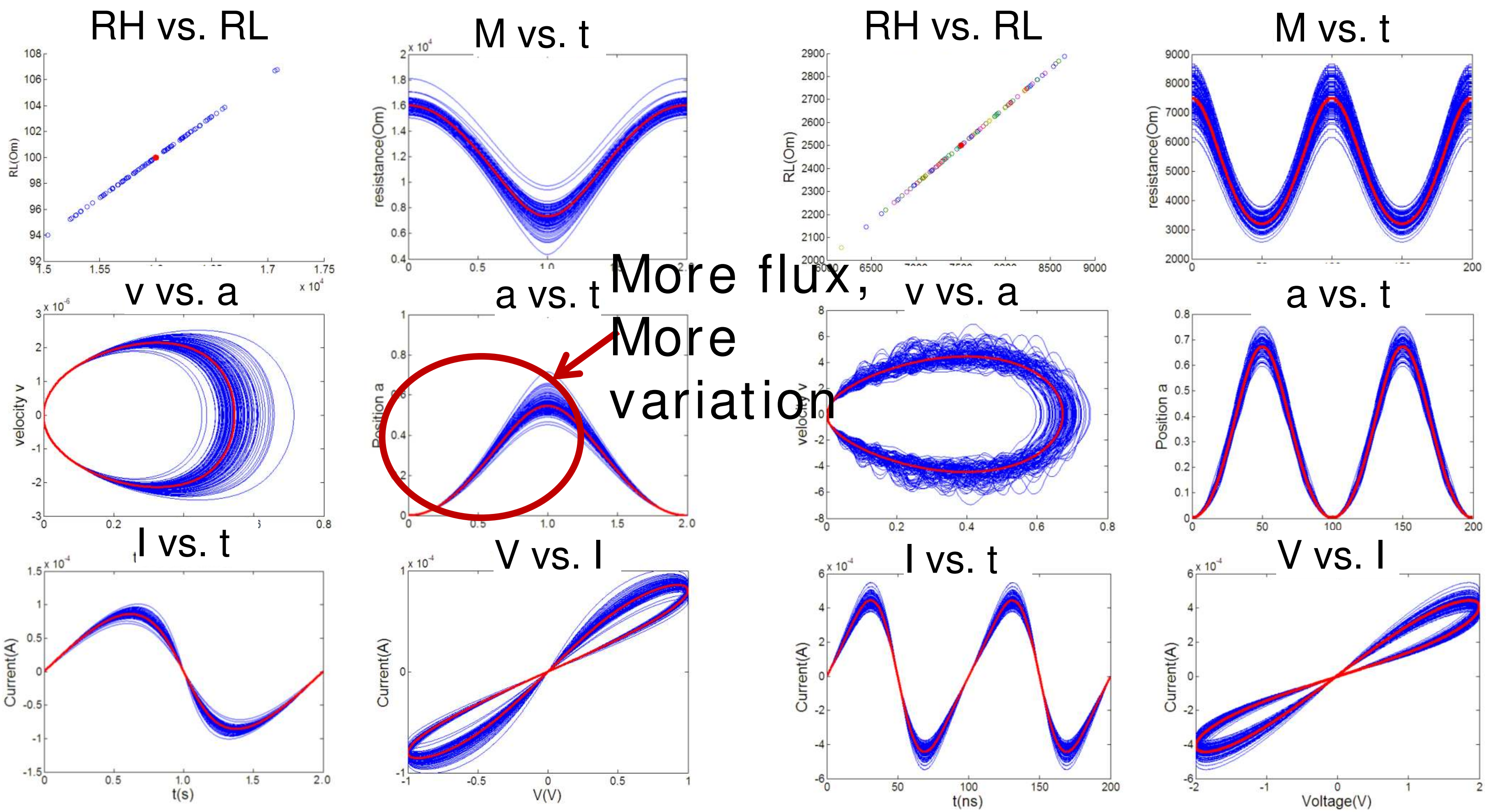
PERFORMANCE ANALYSIS

TiO_2 Thin-film / Spintronic



PERFORMANCE ANALYSIS

- Main source of process variation
  - TiO_2 memristor: TF
  - Spintronic memristor: LER
- Variation estimation of M:
  - TiO_2 memristor: -36.5% to 24.1%
  - Spintronic memristor: -16.3% to 21.1%
- Signal type does not affects the variation

  Flux does

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- Model simplification
- Statistical analysis
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SUMMARY

- We evaluate the impact of geometry variations quantitatively:
  - TiO2 thin-film and spintronic memristors;
  - Electrical properties of memristors; and
  - Static and memristive parameters.
- A simple LER sample generation algorithm is proposed to speed up the related Monte-Carlo simulations.
- This device modeling methodology can be expended to other materials.
- The process-variation analysis will benefit memristor-based design.
- Model download:
  http://eeweb.poly.edu/hli/index_files/memristors.htm

Q & A?

Supplementary slides

MODEL EQUATIONS

- Thin-film memristor

$$M(a) = a \cdot R_L + (1-a) \cdot R_H$$

$$\frac{da(t)}{dt} = \mu_v \cdot \frac{R_L}{h^2} \cdot \frac{V(t)}{M(a)}$$

- For spintronic memristor

$$M(a) = \frac{R_H \cdot R_L}{R_H \cdot a + R_L \cdot (1-a)}$$

$$\frac{da(t)}{dt} = \frac{\Gamma_v}{l} \cdot J_{eff}(t), J_{eff} = \begin{cases} J(t) = \dfrac{V(t)}{M(a) \cdot l \cdot z}, & J(t) \geq J_{cr} \\ 0 & , J(t) < J_{cr} \end{cases}$$

RESULTS

TABLE IV. 3σ MIN./MAX. OF TIO2 MEMRISTOR PARAMETERS.

Sinusoidal Voltage	LER only		Thickness only		Overall	
	-3σ	$+3\sigma$	-3σ	$+3\sigma$	-3σ	$+3\sigma$
R_H & R_L	-5.4%	4.1%	-5.5%	4.8%	-6.4%	7.3%
$M(\alpha)$	-5.4%	4.1%	-37.1%	20.8%	-36.5%	24.1%
$\alpha(t)$	0.0%	0.0%	-13.3%	27.5%	-14.7%	27.4%
$v(\alpha)$	0.0%	0.0%	-9.3%	15.6%	-10.4%	16.9%
$i(\alpha)$	-4.7%	5.7%	-9.3%	15.7%	-10.7%	17.2%
Power	-4.7%	5.7%	-8.8%	14.1%	-10.1%	15.6%

Square wave Voltage	LER only		Thickness only		Overall	
	-3σ	$+3\sigma$	-3σ	$+3\sigma$	-3σ	$+3\sigma$
R_H & R_L	-5.3%	3.7%	-6.2%	5.2%	-6.6%	6.9%
$M(\alpha)$	-5.3%	3.7%	-17.8%	13.2%	-15.4%	14.4%
$\alpha(t)$	0.0%	0.0%	-12.1%	16.6%	-13.0%	15.6%
$v(\alpha)$	0.0%	0.0%	-11.6%	17.7%	-12.5%	16.7%
$i(\alpha)$	-4.0%	5.2%	-11.7%	17.7%	-12.6%	17.6%
Power	-4.0%	5.2%	-7.7%	9.8%	-8.5%	10.1%

RESULTS

TABLE V. 3σ MIN./MAX. OF SPINTRONIC MEMRISTOR PARAMETERS.

Sinusoidal Voltage	LER only		Thickness only		Overall	
	-3σ	$+3\sigma$	-3σ	$+3\sigma$	-3σ	$+3\sigma$
R_H & R_L	-15.3%	22.9%	-6.1%	5.8%	-16.4%	20.9%
$M(\alpha)$	-15.1%	23.3%	-11.0%	11.0%	-16.3%	21.1%
$\alpha(t)$	-9.7%	8.1%	-8.4%	9.5%	-11.8%	8.1%
$v(\alpha)$	-10.7%	22.1%	-9.1%	9.9%	-21.5%	22.5%
$i(\alpha)$	-18.5%	18.5%	-8.9%	10.1%	-17.7%	17.8%
Power	-18.4%	18.6%	-8.3%	9.4%	-17.8%	17.8%

Square wave Voltage	LER only		Thickness only		Overall	
	-3σ	$+3\sigma$	-3σ	$+3\sigma$	-3σ	$+3\sigma$
R_H & R_L	-15.8%	22.0%	-5.3%	5.7%	-15.9%	24.2%
$M(\alpha)$	-15.6%	21.8%	-8.5%	9.7%	-17.0%	25.5%
$\alpha(t)$	-13.1%	13.8%	-7.5%	7.7%	-17.2%	16.2%
$v(\alpha)$	-16.5%	20.7%	-10.0%	8.3%	-20.1%	25.2%
$i(\alpha)$	-19.5%	17.1%	-9.0%	9.3%	-22.1%	20.5%
Power	-19.4%	17.1%	-7.6%	7.7%	-20.9%	19.6%

MODEL SIMPLIFICATION

- Compute state of each filament
  Combine them to get the overall performance
  - Spintronic memristor:
    - each filament is in either R_{iL} or R_{iH} states (under geometric variation)
    - the whole device can be regarded as serial connection of all the filaments
  - TiO_2 thin-film memristor:
    - each filament is a smaller memristor
    - TiO_2 memristor is the parallel connection of them

LER SIMULATION ALGORITHM

- We use this main function to mimic LER :

$$\Delta L = L_{\max} \cdot \sin(f \cdot x) + L_{normal} \cdot p$$

ΔL: LER noise per nm
$L_{\max}$: weight of low frequency noise
L_{normal}: weight of normal distribution
f: Frequency for low frequency noise
x: random number of uniform distribution (0.5-1.5)
p: random number of standard normal distribution