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Giant Resonances: A Comparison between TDHF and Fluid Dynamics in Small Amplitude Vibrations of Spherical Nuclei

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Small amplitude vibrations of spherical nuclei are considered in microscopic (RPA) and fluid-dynamical description. Assuming the concentration of transition strength into one collective state, the microscopic result can be brought into close analogy to constrained fluid-dynamical motion. The decisive difference occurs in the contribution of the microscopic kinetic energy to the collective potential energy. It is shown that extension of fluid dynamics to include dynamical distortions of the local Fermi surface is sufficient to reproduce the microscopic results. Numerical examples are given for $L=0$ and $L=2$ isoscalar modes for a Skyrme-type nucleon-nucleon force.

§ 1. Introduction

Giant resonances can be regarded as "doorways"[1] for inelastic scattering of photons, electrons and hadrons on nuclei. In that sense they do not necessarily represent stationary eigenstates of the nuclear many-body system, but rather are defined through the transition operators M_L by which they are excited from the ground state. It seems, however, that at least for the low L multipoles the transition strength tends to be concentrated in one single or very few eigenstates. This can in fact be understood in microscopic models where collective eigenstates are obtained from correlated particle-hole pairs, usually in the framework of the Random Phase Approximation (RPA).[2] To obtain an estimate of the resonance energy, however, it is not necessary to construct the model eigenstates explicitly because in case of "doorway dominance" it can be approximated by energy-weighted sum rules, once the form of the transition operator and the microscopic Hamiltonian are specified.

As is well known, the RPA equations are formally identical with the linearized, small-amplitude, Time-Dependent Hartree-Fock equations,[3] and their eigenstates can be viewed as collisionless modes propagated through the time-dependent mean field (the classical analogue being the Vlassov equation). In an infinite system, the collisionless mode is namely the zero sound. This mechanism is very different from the way modes are propagated in ordinary fluid dynamics where the

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average free path for the interacting particles is supposed to be very small in comparison with the wavelength of the mode, so that local equilibrium can be established and the modes are propagated directly through the two-body collisions. These modes correspond to the first sounds. The basic feature underlying Newtonian fluid-dynamics is therefore the locally isotropic momentum distribution, leading to an isotropic pressure within the fluid.

Such an assumption, although probably justified for calculating static properties of closed-shell nuclei,[4] seems, however, highly questionable for the discussion of dynamical phenomena at lower energies in view of the long mean free path of nucleons in nuclei which does not allow for local relaxation. This ought to hold also for the giant resonances. On the other hand, it has been observed[5] that in some cases "macroscopic" estimates for the resonance frequencies agree quite well with RPA results. It has also been observed[6] that the single-particle density derived from one single pure many-body state obeys fluid-dynamical equations if the phase factor of that state is a local single-particle operator, an assumption which does not necessarily imply locally isotropic momentum distribution.

It is the intention of the following note to make the connection between these different mechanisms more transparent by formulating them in close analogy to each other. The results for the resonance frequencies will, of course, not be new, and therefore the numerical part will be quite brief. In § 2 we use sum rules to cast the RPA-equations into a form which closely resembles the fluid-dynamical expression for the energy of a constrained motion imposed on the fluid through the excitation mechanism (§ 3). The isoscalar monopole mode is considered more closely in § 4 as an example of how the fluid-dynamical eigenmodes are related to the imposed "breathing" mode: As in the microscopic case the connection is through an energy-weighted sum rule. In § 5 we derive the explicit expressions for the frequencies of the isoscalar $L=0$ and $L=2$ resonances and show that only through distortion of the local Fermi surface the fluid-dynamical result can be brought into agreement with the microscopic expression.

For the interaction between the nucleons we shall consider an effective zero-range force of the Skyrme type[7] which for spin-saturated nuclei leads to a local potential energy density

$$v=\frac{1}{2}t_0\left(\frac{3}{4}\rho_+^2-\frac{1}{2}\left(x_0+\frac{1}{2}\right)\rho_-^2\right)+\frac{1}{16}(3t_1+5t_2)\rho_+\tau_++\frac{1}{16}(t_2-t_1)\rho_-\tau_-$$
$$+\frac{1}{64}(5t_2-9t_1)\rho_+\Delta\rho_++\frac{1}{64}(3t_1+t_2)\rho_-\Delta\rho_-+\frac{1}{16}t_3(\rho_+^3-\rho_+\rho_-^2), \qquad (1)$$

where

$$\rho_+=\rho_n+\rho_p\,, \qquad \rho_-=\rho_n-\rho_p$$

are the local total density and neutron excess and $(\hbar^2/2m)\tau_\pm$ are the corresponding kinetic energy densities.

§ 2. Sum rules and RPA

We define isoscalar and isovector transition densities by the expressions

$$\langle 0|\hat{\rho}_+(\boldsymbol{r})|\omega\rangle=\sum_{mi}(\langle m|\delta(\boldsymbol{r}_1-\boldsymbol{r})|i\rangle X_{im}+\langle i|\delta(\boldsymbol{r}_1-\boldsymbol{r})|m\rangle Y_{im}),$$

$$\langle 0|\hat{\rho}_-(\boldsymbol{r})|\omega\rangle=\sum_{mi}(\langle m|\tau_3\delta(\boldsymbol{r}_1-\boldsymbol{r})|i\rangle X_{im}+\langle i|\tau_3\delta(\boldsymbol{r}_1-\boldsymbol{r})|m\rangle Y_{im}). \tag{2}$$

Here $\hat{\rho}_+(\boldsymbol{r})$ is the operator for the local isoscalar density $\sum_{\nu=1}^{A}\delta(\boldsymbol{r}_\nu-\boldsymbol{r})$ with ground state expectation value $\rho_+(\boldsymbol{r})$; $\hat{\rho}_-(\boldsymbol{r})$ is the isovector density $\sum_{\nu=1}^{A}\tau_3(\nu)\delta(\boldsymbol{r}_\nu-\boldsymbol{r})$ with expectation value $\rho_-(\boldsymbol{r})$; $|0\rangle$ and $|\omega\rangle$ are the RPA ground- and excited (collective) states, and $|m\rangle$ and $|i\rangle$ are self-consistent single-particle and hole states. Denoting matrix elements of the single-particle density matrix by $c_{\alpha\beta}$, i.e.,

$$\rho(\boldsymbol{r},\boldsymbol{r}')=\sum_{\alpha\beta}\langle\boldsymbol{r}|\alpha\rangle c_{\alpha\beta}\langle\beta|\boldsymbol{r}'\rangle,\qquad c_{\alpha\beta}=c^*_{\beta\alpha}$$

we have

$$\frac{\partial\rho_\pm(\boldsymbol{r})}{\partial c_{mi}}=\begin{cases}\langle i|\delta(\boldsymbol{r}_1-\boldsymbol{r})|m\rangle\\ \langle i|\tau_3\delta(\boldsymbol{r}_1-\boldsymbol{r})|m\rangle\end{cases}$$

and therefore

$$\langle 0|\hat{\rho}_\pm(\boldsymbol{r})|\omega\rangle=\sum_{mi}\left(\frac{\partial\rho_\pm(\boldsymbol{r})}{\partial c_{im}}X_{im}+\frac{\partial\rho_\pm(\boldsymbol{r})}{\partial c_{mi}}Y_{im}\right). \tag{3}$$

The particle-hole and hole-particle amplitudes X_{im} and Y_{im} are determined together with the collective frequency ω through the RPA-equations

$$(\hbar\omega-\varepsilon_{ik})X_{ik}=\sum_{jl}(\langle il|\mathrm{V}|kj\rangle X_{jl}+\langle ij|\mathrm{V}|kl\rangle Y_{jl}), \tag{4a}$$

$$(-\hbar\omega-\varepsilon_{ik})Y_{ik}=\sum_{jl}(\langle il|\mathrm{V}|kj\rangle^* Y_{jl}+\langle ij|V|kl\rangle^* X_{jl}), \tag{4b}$$

where the interaction matrix elements are defined as second derivatives of the total potential energy

$$\langle il|\mathrm{V}|kj\rangle=\frac{\partial^2\int v d\boldsymbol{r}}{\partial c_{ki}\partial c_{jl}},\qquad \langle ij|\mathrm{V}|kl\rangle=\frac{\partial^2\int v d\boldsymbol{r}}{\partial c_{ki}\partial c_{lj}}. \tag{5}$$

Although the functional v of Eq. (1) depends not only on the densities ρ_+ and ρ_- but also on the kinetic energy densities τ_+ and τ_-, the latter ones can generally be considered themselves functionals of the densities ρ_+ and ρ_- and we can formally write

$$\langle il|\mathrm{V}|kj\rangle=\int\frac{\delta^2 v}{\delta\rho_+\delta\rho_+}\frac{\partial\rho_+}{\partial c_{ki}}\frac{\partial\rho_+}{\partial c_{jl}}+\int\frac{\delta^2 v}{\delta\rho_+\delta\rho_-}\left(\frac{\partial\rho_+}{\partial c_{ki}}\frac{\partial\rho_-}{\partial c_{jl}}+\frac{\partial\rho_+}{\partial c_{jl}}\frac{\partial\rho_-}{\partial c_{ki}}\right)$$
$$+\int\frac{\delta^2 v}{\delta\rho_-\delta\rho_-}\frac{\partial\rho_-}{\partial c_{ki}}\frac{\partial\rho_-}{\partial c_{jl}} \tag{6}$$

or explicitly

$$\langle il|\mathrm{V}|kj\rangle = \int\left(\frac{3}{4}t_0+\frac{3}{8}t_3\rho_+\right)\frac{\partial\rho_+}{\partial c_{ki}}\frac{\partial\rho_+}{\partial c_{jl}}$$
$$+\int\frac{1}{32}(5t_2-9t_1)\left(\frac{\partial\rho_+}{\partial c_{ki}}\Delta\frac{\partial\rho_+}{\partial c_{jl}}+\frac{\partial\rho_+}{\partial c_{jl}}\Delta\frac{\partial\rho_+}{\partial c_{ki}}\right)$$
$$+\int\left(-\frac{1}{2}t_0\left(x_0+\frac{1}{2}\right)-\frac{1}{8}t_3\rho_+\right)\frac{\partial\rho_-}{\partial c_{ki}}\frac{\partial\rho_-}{\partial c_{jl}}$$
$$+\int\frac{1}{32}(3t_1+t_2)\left(\frac{\partial\rho_-}{\partial c_{ki}}\Delta\frac{\partial\rho_-}{\partial c_{jl}}+\frac{\partial\rho_-}{\partial c_{jl}}\Delta\frac{\partial\rho_-}{\partial c_{ki}}\right)$$
$$-\int\frac{1}{8}t_3\rho_-\left(\frac{\partial\rho_+}{\partial c_{ki}}\frac{\partial\rho_-}{\partial c_{jl}}+\frac{\partial\rho_+}{\partial c_{jl}}\frac{\partial\rho_-}{\partial c_{ki}}\right)+(\tau\text{-terms}), \tag{7}$$

where the τ-terms contain functional derivatives of $\tau_\pm$ with respect to $\rho_\mp$. A similar expression holds for $\langle ij|\mathrm{V}|kl\rangle$. Multiplying the RPA-equations (4a) and (4b) by X^*_{ik} and Y^*_{ik}, respectively; inserting the matrix elements (6) and the definitions (3), we obtain after summation:

$$\hbar\omega=\sum_{ik}\varepsilon_{ik}(|X_{ik}|^2+|Y_{ik}|^2)+\int\frac{\delta^2v}{\delta\rho_+\delta\rho_+}|\langle 0|\hat\rho_+|\omega\rangle|^2d\boldsymbol{r}+\int\frac{\delta^2v}{\delta\rho_-\delta\rho_-}|\langle 0|\hat\rho_-|\omega\rangle|^2d\boldsymbol{r}$$
$$+\int\frac{\delta^2v}{\delta\rho_+\delta\rho_-}(\langle 0|\hat\rho_+|\omega\rangle\langle\omega|\hat\rho_-|0\rangle+\langle 0|\hat\rho_-|\omega\rangle\langle\omega|\hat\rho_+|0\rangle)\,d\boldsymbol{r}, \tag{8}$$

where the functional derivatives again are a formal notion which can include derivatives acting on the transition densities. On the left-hand side we have made use of the normalization condition

$$\sum_{ik}(|X_{ik}|^2-|Y_{ik}|^2)=1.$$

The first term on the right-hand side of Eq. (8) can be rewritten (making use of Eqs. (3)$\sim$(6)):

$$\sum_{ik}\varepsilon_{ik}(|X_{ik}|^2+|Y_{ik}|^2)=\sum_{ik}\frac{\hbar^2\omega^2+\varepsilon_{ik}^2}{2\hbar\omega}(|X_{ik}|^2-|Y_{ik}|^2). \tag{9}$$

Therefore, by defining a mean particle-hole energy ε through

$$\varepsilon^2=\sum_{ik}\varepsilon_{ik}^2(|X_{ik}|^2-|Y_{ik}|^2) \tag{10}$$

or by assuming the ε_{ik} to be degenerate (the effect of $\boldsymbol{l}\cdot\boldsymbol{s}$ splitting is not so important to the mean excitation energy of the giant resonance), we finally obtain the RPA-relation between the transition densities and the collective frequency:

$$\hbar^2\omega^2=\varepsilon^2+2\hbar\omega\left\{\int\frac{\delta^2v}{\delta\rho_+\delta\rho_+}|\langle 0|\hat\rho_+|\omega\rangle|^2d\boldsymbol{r}+\int\frac{\delta^2v}{\delta\rho_-\delta\rho_-}|\langle 0|\hat\rho_-|\omega\rangle|^2d\boldsymbol{r}\right.$$
$$\left.+\int\frac{\delta^2v}{\delta\rho_+\delta\rho_-}(\langle 0|\hat\rho_-|\omega\rangle\langle\omega|\hat\rho_+|0\rangle+\langle 0|\hat\rho_+|\omega\rangle\langle\omega|\hat\rho_-|0\rangle)\,d\boldsymbol{r}\right\}. \tag{11}$$

Of course, the symmetry between positive and negative frequencies is only seemingly violated by this relation. For a very collective state $|\omega\rangle$ the mean particle-hole energy ε will be practically independent of ω (in case of degenerate ε_{ik} it is trivially ω-independent). We will see in the following that in case of doorway dominance the second term on the right-hand side is also explicitly ω-independent. Therefore the relation (11) seems appropriate for the discussion of giant resonances, in cases where the sum rules are exhausted by the doorway state and thereby the transition densities are determined by the transition operators M_L. In the following discussion, we consider only the transition operators in the long wave limit. These operators are sufficient for studies of the excitation mechanisms of giant resonances. Following a suggestion of Deal and Fallieros,[8] we first consider states which exhaust the sum rules for isoscalar transition operators

$$\boldsymbol{M}_L=\sum_{\nu=1}^{A} M_L(\nu), \tag{12a}$$

$$\sum_n \hbar\omega_n\langle 0|\hat{\rho}_\pm(\boldsymbol{r})|n\rangle\langle n|\boldsymbol{M}_L|0\rangle=-\frac{\hbar^2}{2m}\boldsymbol{\nabla}\cdot(\rho_\pm(\boldsymbol{r})\boldsymbol{\nabla}M_L(\boldsymbol{r})). \tag{13}$$

If one of the states $|n\rangle$, which we shall denote by $|\omega_L\rangle$, exhausts the sum, then we have for that state

$$\hbar\omega_L\langle 0|\hat{\rho}_\pm(\boldsymbol{r})|\omega_L\rangle=-\frac{\hbar^2}{2m}\boldsymbol{\nabla}\cdot(\rho_\pm\boldsymbol{\nabla}M_L)/\langle\omega_L|\boldsymbol{M}_L|0\rangle. \tag{14}$$

The transition matrix element is determined through exhaustion of the sum

$$\sum_n \hbar\omega_n\langle 0|\boldsymbol{M}_L|n\rangle\langle n|\boldsymbol{M}_L|0\rangle$$
$$=\frac{\hbar^2}{2m}\langle 0|\sum_{\nu=1}^{A}\boldsymbol{\nabla}M_L(\nu)\cdot\boldsymbol{\nabla}M_L(\nu)|0\rangle=\frac{\hbar^2}{2m}A\langle\boldsymbol{\nabla}M_L\cdot\boldsymbol{\nabla}M_L\rangle \tag{15}$$

which then leads to

$$\hbar\omega_L|\langle 0|\hat{\rho}_\pm(\boldsymbol{r})|\omega_L\rangle|^2=\hbar^2(\boldsymbol{\nabla}\cdot(\rho_\pm(\boldsymbol{r})\boldsymbol{\nabla}M_L))^2/2mA\langle\boldsymbol{\nabla}M_L\cdot\boldsymbol{\nabla}M_L\rangle. \tag{16a}$$

This equation shows that the second term on the right-hand side of Eq. (11) is ω-independent and we finally obtain

$$\omega_L^2=(\varepsilon/\hbar)^2+(mA\langle\boldsymbol{\nabla}M_L\cdot\boldsymbol{\nabla}M_L\rangle)^{-1}\Big\{\int\frac{\delta^2 v}{\delta\rho_+\delta\rho_+}(\boldsymbol{\nabla}\cdot(\rho_+\boldsymbol{\nabla}M_L))^2 d\boldsymbol{r}$$
$$+\int\frac{\delta^2 v}{\delta\rho_-\delta\rho_-}(\boldsymbol{\nabla}\cdot(\rho_-\boldsymbol{\nabla}M_L))^2 d\boldsymbol{r}+2\int\frac{\delta^2 v}{\delta\rho_+\delta\rho_-}(\boldsymbol{\nabla}\cdot(\rho_+\boldsymbol{\nabla}M_L))(\boldsymbol{\nabla}\cdot(\rho_-\boldsymbol{\nabla}M_L))\,d\boldsymbol{r}\Big\}. \tag{17a}$$

In $N=Z$ nuclei ρ_- is approximately zero. In that case, therefore, the last two terms in Eq. (17a) drop out. A similar equation is derived by the direct calculation of two-body matrix elements in Ref. 19). For states $|\omega_L^{(T=1)}\rangle$ which exhaust the sum

rules for isovector operators

$$\boldsymbol{M}_L^{(T=1)} = \sum_{\nu=1}^{A} \tau_3(\nu)\, M_L(\nu), \tag{12b}$$

we can proceed in a similar way for that part of the potential energy which commutes with $M_L^{(T=1)}$. Then we obtain

$$\hbar\omega_L^{(T=1)} |\langle 0|\hat{\rho}_{\pm}(\boldsymbol{r})|\omega_L^{(T=1)}\rangle|^2 = \hbar^2 (\boldsymbol{\nabla}\cdot(\rho_{\mp}(\boldsymbol{r})\boldsymbol{\nabla}M_L))^2/2m \int \rho_{+} (\boldsymbol{\nabla}M_L)^2 d\boldsymbol{r}\,, \tag{16b}$$

and the RPA-frequency is given by

$$(\omega_L^{(T=1)})^2 = (\varepsilon/\hbar)^2 + (mA\langle \boldsymbol{\nabla}M_L\cdot\boldsymbol{\nabla}M_L\rangle)^{-1}\Big\{\int \frac{\delta^2 v}{\delta\rho_{+}\delta\rho_{+}} (\boldsymbol{\nabla}\cdot(\rho_{-}\boldsymbol{\nabla}M_L))^2 d\boldsymbol{r}$$
$$+ \int \frac{\delta^2 v}{\delta\rho_{-}\delta\rho_{-}} (\boldsymbol{\nabla}\cdot(\rho_{+}\boldsymbol{\nabla}M_L))^2 d\boldsymbol{r} + 2\int \frac{\delta^2 v}{\delta\rho_{+}\delta\rho_{-}} (\boldsymbol{\nabla}\cdot(\rho_{+}\boldsymbol{\nabla}M_L))\,(\boldsymbol{\nabla}\cdot(\rho_{-}\boldsymbol{\nabla}M_L))\, d\boldsymbol{r}\Big\}. \tag{17b}$$

For isovector operators $\boldsymbol{M}_L^{(T=1)}$ and $\hat{\rho}_{-}$ the velocity-dependent parts of the Skyrme force, however, contribute to the sums (13) and (15). This means that on the right-hand side of Eq. (16b) the total density ρ_{+} then has to be replaced by

$$\tilde{\rho} = \rho_{+} + \frac{2m}{\hbar^2}(t_1 + t_2)\rho_p\rho_n\,. \tag{18}$$

§ 3. Fluid dynamics

In the preceding section we have discussed the RPA-relation between frequency and transition densities in case of sum rule exhaustion. As outlined in § 1, the underlying dynamics is that of a collisionless mode propagation through the mean field. Let us now consider the fluid-dynamical Hamilton function for irrotational flow of a two-fluid system. If protons cannot be transformed into neutrons or vice versa, we have

$$\mathcal{H}(\rho_{+}, \rho_{-}, \phi_{+}, \phi_{-}) = \frac{m}{2}\int \{((\boldsymbol{\nabla}\phi_{+})^2 + (\boldsymbol{\nabla}\phi_{-})^2)\rho_{+} + 2(\boldsymbol{\nabla}\phi_{+}\cdot\boldsymbol{\nabla}\phi_{-})\rho_{-}\}\, d\boldsymbol{r} + \int u d\boldsymbol{r}\,. \tag{19}$$

The isoscalar (isovector) mass densities $m\rho_{+}$, $(m\rho_{-})$ and the velocity potentials $\phi_{+} = \frac{1}{2}(\phi_p + \phi_n)$, $(\phi_{-} = \frac{1}{2}(\phi_n - \phi_p))$ are the canonically conjugate coordinates. The potential part $\int u d\boldsymbol{r}$ is a functional of the local densities ρ_{+}, ρ_{-} and their derivatives and represents the total energy of the many-body system in the static case, i.e.,

$$\int u d\boldsymbol{r} = \langle H\rangle = \int \tau_{+} d\boldsymbol{r} + \int v d\boldsymbol{r}\,, \tag{20}$$

where the potential energy density v is given by the expression (1). It will later on be decisive how to express the kinetic energy densities $\tau_{\pm}$ as functionals of

ρ_+, ρ_- and its derivatives. For the moment, however, it will be sufficient to assume as in the preceding section that there exist functionals $\tau_\pm = \tau_\pm(\rho_+, \rho_-)$ without specifying their explicit form. Hamilton's equations then lead to

$$\dot{\rho}_+ = \nabla\cdot(\rho_+\nabla\phi_+ + \rho_-\nabla\phi_-), \qquad \dot{\rho}_- = \nabla\cdot(\rho_+\nabla\phi_- + \rho_-\nabla\phi_+), \tag{21a}$$

$$\dot{\phi}_+ = \frac{1}{2}((\nabla\phi_+)^2 + (\nabla\phi_-)^2) + \frac{1}{m}\frac{\delta u}{\delta\rho_+}, \qquad \dot{\phi}_- = (\nabla\phi_+\cdot\nabla\phi_-) + \frac{1}{m}\frac{\delta u}{\delta\rho_-}, \tag{21b}$$

the equations of continuity and Bernoulli's equations, respectively. For small oscillations around the equilibrium densities

$$\rho_\pm(\boldsymbol{r}, t) = \rho_\pm(\boldsymbol{r}) + \delta\rho_\pm(\boldsymbol{r}, t), \tag{22}$$

the linearized equations of motion are

$$\delta\dot{\rho}_+ = \nabla\cdot(\rho_+\nabla\phi_+ + \rho_-\nabla\phi_-), \qquad \delta\dot{\rho}_- = \nabla\cdot(\rho_+\nabla\phi_- + \rho_-\nabla\phi_+), \tag{23a}$$

$$\dot{\phi}_+ = \frac{1}{m}\left(\frac{\delta^2 u}{\delta\rho_+\delta\rho_+}\delta\rho_+ + \frac{\delta^2 u}{\delta\rho_+\delta\rho_-}\delta\rho_-\right), \qquad \dot{\phi}_- = \frac{1}{m}\left(\frac{\delta^2 u}{\delta\rho_-\delta\rho_+}\delta\rho_+ + \frac{\delta^2 u}{\delta\rho_-\delta\rho_-}\delta\rho_-\right), \tag{23b}$$

where again the functional derivatives are a formal notion which generally will contain derivatives acting on the density changes $\delta\rho_\pm$.

In case of the Giant Resonances we are not really interested in the eigenmodes of the system of Eqs. (23), but rather in a constrained motion of the fluid, imposed on the system through the excitation mechanism. Bertsch[9] has pointed out the connection between the transition operators M_L and the velocity potential ϕ. Thus, by imposing a constraint on the motion by choosing

for isoscalar modes
$$\phi_+(\boldsymbol{r}, t) = \dot{\alpha}(t) M_L(\boldsymbol{r}), \qquad \phi_-(\boldsymbol{r}, t) \equiv 0, \tag{24a}$$

for isovector modes
$$\phi_+(\boldsymbol{r}, t) \equiv 0, \qquad \phi_-(\boldsymbol{r}, t) = \dot{\alpha}(t) M_L(\boldsymbol{r}), \tag{24b}$$

the density changes are determined through Eq. (23a)

for isoscalar modes
$$\delta\rho_\pm(\boldsymbol{r}, t) = \alpha(t)\delta\rho_\pm(\boldsymbol{r}) = \alpha(t)\nabla\cdot(\rho_\pm(\boldsymbol{r})\nabla M_L), \tag{25a}$$

for isovector modes
$$\delta\rho_\pm(\boldsymbol{r}, t) = \alpha(t)\delta\rho_\pm(\boldsymbol{r}) = \alpha(t)\nabla\cdot(\rho_\mp(\boldsymbol{r})\nabla M_L), \tag{25b}$$

with $\alpha(t)$ as a time-dependent small amplitude. The frequency of this constrained motion is then determined by the form of the Hamilton function (19). Namely, inserting the expressions (24) and (25) into Eq. (19) we obtain

$$\mathcal{H} = \frac{m}{2}\dot{\alpha}(t)^2\int(\nabla M_L\cdot\nabla M_L)\rho_+ d\boldsymbol{r} + \int u(\rho_+, \rho_-)\, d\boldsymbol{r} + \frac{\alpha(t)^2}{2}\left\{\int\frac{\delta^2 u}{\delta\rho_+\delta\rho_+}\delta\rho_+{}^2 d\boldsymbol{r} + 2\int\frac{\delta^2 u}{\delta\rho_+\delta\rho_-}\delta\rho_+\delta\rho_- d\boldsymbol{r} + \int\frac{\delta^2 u}{\delta\rho_-\delta\rho_-}\delta\rho_-{}^2 d\boldsymbol{r}\right\}, \tag{26}$$

where we have made use of the fact that the static densities ρ_+ and ρ_- satisfy $(\delta u/\delta\rho_+)\rho_+\equiv 0$ and $(\delta u/\delta\rho_-)\rho_-\equiv 0$. The expression (26) is time-independent for harmonic motion in α with the frequency for the isoscalar mode (25a) given by

$$\omega_L{}^2 = B_L{}^{-1}\cdot C_L = \left(m\int \rho_+ (\boldsymbol{\nabla} M_L\cdot\boldsymbol{\nabla} M_L)\,d\boldsymbol{r}\right)^{-1} \times\left\{\int \frac{\delta^2 u}{\delta\rho_+\delta\rho_+}(\boldsymbol{\nabla}\cdot(\rho_+\boldsymbol{\nabla} M_L))^2 d\boldsymbol{r} + 2\int \frac{\delta^2 u}{\delta\rho_+\delta\rho_-}(\boldsymbol{\nabla}\cdot(\rho_+\boldsymbol{\nabla} M_L))(\boldsymbol{\nabla}\cdot(\rho_-\boldsymbol{\nabla} M_L)\,d\boldsymbol{r} + \int \frac{\delta^2 u}{\delta\rho_-\delta\rho_-}(\boldsymbol{\nabla}\cdot(\rho_-\boldsymbol{\nabla} M_L))^2 d\boldsymbol{r}\right\}, \tag{27a}$$

and for the isovector mode (25b)

$$(\omega_L{}^{(T=1)})^2 = B_L{}^{-1}\cdot C_L{}^{(T=1)} = \left(m\int \rho_+ (\boldsymbol{\nabla} M_L\cdot\boldsymbol{\nabla} M_L)\,d\boldsymbol{r}\right)^{-1} \times\left\{\int \frac{\delta^2 u}{\delta\rho_+\delta\rho_+}(\boldsymbol{\nabla}\cdot(\rho_-\boldsymbol{\nabla} M_L))^2 d\boldsymbol{r} + 2\int \frac{\delta^2 u}{\delta\rho_+\delta\rho_-}(\boldsymbol{\nabla}\cdot(\rho_+\boldsymbol{\nabla} M_L))(\boldsymbol{\nabla}\cdot(\rho_-\boldsymbol{\nabla} M_L))\,d\boldsymbol{r} + \int \frac{\delta^2 u}{\delta\rho_-\delta\rho_-}(\boldsymbol{\nabla}\cdot(\rho_+\boldsymbol{\nabla} M_L))^2 d\boldsymbol{r}\right\}. \tag{27b}$$

Clearly, with respect to the inertia parameters B_L and the contributions of the *potential* energy $\int v d\boldsymbol{r}$ to the force constants C_L these results are identical to relations (17) which originated from the RPA. For the contributions of the kinetic energy $\int \tau_\pm d\boldsymbol{r}$ to the force constants C_L, however, there is a decisive difference. While in the RPA they appear through the self-consistent particle-hole energies ε in the form

$$\left(\frac{\varepsilon}{\hbar}\right)^2 mA\langle\boldsymbol{\nabla} M_L\cdot\boldsymbol{\nabla} M_L\rangle,$$

the fluid dynamical result contains them through terms of the form

$$\frac{\hbar^2}{2m}\int \frac{\delta^2\tau_\pm}{\delta\rho^2}(\boldsymbol{\nabla}\cdot(\rho\boldsymbol{\nabla} M_L))^2 d\boldsymbol{r}\,.$$

The latter quantity is evidently determined by the functional form of $\tau_+(\rho_\pm,\rho_-)$ which in turn depends on our assumptions about the local momentum distribution.

§ 4. The breathing mode

It is instructive to consider the case of a monopole mode in more detail. For the sake of simplicity we shall do this for a system consisting of only one kind of particles with density $\rho_0(\boldsymbol{r})$ in the ground state. We define the "breathing mode" through its velocity potential $\phi_0=\dot{\alpha}\boldsymbol{r}^2/2$ which leads to the density change

$$\delta\rho_0 = \alpha\left(3\rho_0 + r\frac{\partial}{\partial r}\rho_0\right) \tag{28}$$

and with Eq. (25a) to the frequency

$$\omega_0^2 = \int \left(\frac{\partial^2 u}{\partial \rho^2}\right)_{\rho_0} \left(3\rho_0 + r\frac{\partial}{\partial r}\rho_0\right)^2 r^2 dr / mA\langle r^2\rangle = C_0/B_0 . \tag{29}$$

As is well known, Eq. (28) is identical with the first-order term in an expansion of the Werntz-Überall (WÜ) model[10]

$$\rho(\boldsymbol{r}_1 t) = (1+\alpha(t))^2 \rho_0((1+\alpha(t))r) = \rho_0 + \delta\rho(r, t)$$
$$= \rho_0 + \alpha\boldsymbol{\nabla}\cdot\left(\rho_0\boldsymbol{\nabla}\frac{r^2}{2}\right) + \alpha^2\boldsymbol{\nabla}\cdot\left(\boldsymbol{\nabla}\left(\rho_0\frac{r^2}{2}\right)\right) + \cdots .$$

If the static density $\rho_0(r)$ satisfies the "equilibrium" condition

$$\left(\frac{\delta}{\delta\rho}\int u d\boldsymbol{r}\right)_{\rho_0} = \left(\frac{\delta u}{\delta\rho}\right)_{\rho_0} = \lambda , \tag{30}$$

then also the WÜ model leads to the frequency (29). If, however, the equilibrium condition is fulfilled only with respect to a change of α, i.e.,

$$\left(\frac{\partial}{\partial\alpha}\int u d\boldsymbol{r}\right)_{\alpha=0} = \left(\int \frac{\delta u}{\delta\rho}\frac{\partial\rho}{\partial\alpha} d\boldsymbol{r}\right)_{\alpha=0} = 0 , \tag{31}$$

then the WÜ model leads to a quite different result for the force constant C_0:

$$C_0 = \left(\frac{\partial^2}{\partial\alpha^2}\int u d\boldsymbol{r}\right)_{\alpha=0} = \int \left(\frac{\delta^2 u}{\delta\rho^2}\right)_{\rho_0}\left(\frac{\partial}{\partial\alpha}\rho\right)^2 d\boldsymbol{r} + \int \left(\frac{\delta u}{\delta\rho}\right)_{\rho_0}\left(\frac{\partial^2}{\partial\alpha^2}\rho\right) d\boldsymbol{r} . \tag{32}$$

(The inertia parameter B_0 is, of course, unchanged because $\mathcal{H}$ is quadratic in ϕ and therefore the first order is sufficient to determine B_0.) A good example is provided by a functional u which contains only volume terms. Then the equilibrium condition (30) cannot be satisfied for a density ρ_0 which vanishes outside a finite space region, and one can require only the "saturation" condition (31) to be fulfilled. In this case the functional derivatives $\delta/\delta\rho$ can be replaced by ordinary derivatives $\partial/\partial\rho$ and we obtain from Eq. (32)

$$C_0 = 4\pi\int \left(\frac{\partial^2 u}{\partial\rho^2}\right)_{\rho_0}\left(3\rho_0 + r\frac{\partial}{\partial r}\rho_0\right)^2 r^2 dr + 4\pi\int \left(\frac{\partial u}{\partial\rho}\right)_{\rho_0}\left(6\rho_0 + 6r\frac{\partial}{\partial r}\rho_0 + r^2\frac{\partial^2}{\partial r^2}\rho_0\right) r^2 dr .$$

Together with the condition (31) all the terms containing derivatives of ρ_0 cancel to give the final result

$$\omega_0^2 = \int \left(\frac{\partial^2 u}{\partial\rho^2}\right)_{\rho_0} 9\rho_0^2 d\boldsymbol{r} / mA\langle r^2\rangle .$$

In case of a square-well density

$$\rho_0 = \kappa\theta(R-r), \qquad \kappa = \frac{3A}{4\pi R^3},$$

this reduces to

$$\omega_0^2=\frac{K}{m\langle r^2\rangle}, \qquad K=9\kappa\left(\frac{\partial^2 u}{\partial\rho^2}\right)_\kappa. \tag{33}$$

Here, evidently, only the first term in $\delta\rho_0$ (Eq. (28)) can contribute and we obtain the "nuclear matter" compressibility K.

The square well also provides an example how the fluid-dynamical eigenfrequencies are related to the breathing mode: In this case the eigenmodes are given by

$$\delta\rho_n(r,t)=\alpha(t)\kappa\left((-)^{n+1}j_0(k_nr)\theta(R-r)-\frac{R}{n^2\pi^2}\delta(R-r)\right), \tag{34}$$

with the wave numbers k_n determined by the boundary condition

$$j_0(k_nR)=0, \quad \text{i.e.,} \quad k_nR=n\pi, \quad n=1,2,3,\cdots. \tag{35}$$

The corresponding velocity potential is

$$\phi_n(r,t)=(-)^n\dot{\alpha}(t)\frac{1}{k_n^2}j_0(k_nr).$$

The continuity equation then is satisfied everywhere, and from Bernoulli's equation for $r<R$*) we obtain the eigenfrequencies

$$\omega_n^2=\frac{\kappa}{m}\left(\frac{\partial^2 u}{\partial\rho^2}\right)_\kappa k_n^2.$$

The breathing mode from Eq. (28)

$$\delta\rho_0=\alpha(t)\kappa(3\theta(R-r)-R\delta(R-r))$$

can be expanded in terms of the eigenmodes (34)

$$\delta\rho_0(r,t)=6\sum_{n=1}^{\infty}\delta\rho_n(r,t)$$

and for the "transition strength" $S_n=|\int r^2\delta\rho_n d\boldsymbol{r}|$ one obtains

$$S_n=\alpha_n\kappa\cdot 24\pi R^5/(n\pi)^4,$$

where $\alpha_n=\sqrt{\hbar/2B_n\omega_n}$ and the mass parameter B_n is obtained as $m\kappa(2R^5/n^4\pi^3)$. This then leads to a "sum rule" relating the breathing-mode frequency ω_0 (33) to the hydrodynamical eigenfrequencies ω_n:

$$\omega_0^2=\sum_{n=1}^{\infty}\omega_n^3|S_n|^2/\sum_{n=1}^{\infty}\omega_n|S_n|^2. \tag{36}$$

It is just this averaging of the eigenfrequencies ω_n^2 with the "strength function" $\omega_n|S_n|^2\propto 1/n^4$ which replaces the square-well redius R occuring in the boundary condition (35) by the mean square radius occuring in Eq. (33):

*) For free motion the gradient of the pressure at the surface ($r=R$) must vanish. Therefore the surface cannot contribute to the time derivative of the current and it is sufficient to satisfy Bernoulli's equation inside the square well.

$$\frac{\kappa}{m}\left(\frac{\partial^2 u}{\partial \rho^2}\right)_\kappa \frac{\pi^2}{R^2}\frac{\sum 1/n^2}{\sum 1/n^4}=\frac{\kappa}{m}\left(\frac{\partial^2 u}{\partial \rho^2}\right)_\kappa \cdot \frac{9}{3R^2/5}=\omega_0{}^2.$$

Clearly, in fluid dynamics the sum rule (36) is not exhausted by one state, and the frequency of the (constrained) breathing mode ω_0 lies slightly above the lowest "normal sound" mode ω_1 ($\omega_0=1.23\omega_1$), which also sometimes is used for comparison with microscopic results.[11]

§ 5. Explicit expressions for the resonance energies

Let us now introduce a definite form for the kinetic energy density functionals[12]

$$\tau_\pm = C_{\mathrm{TF}}(\rho_n{}^{5/3}\pm\rho_p{}^{5/3})+C_{\mathrm{ST}}\left(\frac{(\nabla\rho_n)^2}{\rho_n}\pm\frac{(\nabla\rho_p)^2}{\rho_p}\right)+\frac{1}{3}(\Delta\rho_n\pm\Delta\rho_p) \tag{37}$$

with constants C_{TF} and C_{ST}, where TF refers to the volume kinetic energy in Thomas Fermi approximation, ST refers to a surface correction term for the kinetic energy. The assumption underlying the volume part of this form is a spherical local Fermi surface. The total static energy from Eq. (20) can then generally be written as

$$\int u d\boldsymbol{r}=\langle\mathrm{TF}\rangle+\langle\mathrm{ST}\rangle+\sum_\sigma\langle\sigma\rangle+\langle\mathrm{S}\rangle+\langle\mathrm{C}\rangle, \tag{38}$$

where TF represents volume terms of the kinetic energy, ST the surface correction, $\langle\sigma\rangle$ volume terms of the potential energy of the form $\int\rho_p{}^{\sigma_p}d\boldsymbol{r}$, $\int\rho_n{}^{\sigma_n}d\boldsymbol{r}$, $\int\rho_p{}^{\sigma_p}\rho_n{}^{\sigma_n}d\boldsymbol{r}$, $\langle\mathrm{S}\rangle$ surface terms of the form $\int(\nabla\rho_p)^2d\boldsymbol{r}$, $\int(\nabla\rho_n)^2d\boldsymbol{r}$, $\int\nabla\rho_p\cdot\nabla\rho_n d\boldsymbol{r}$, $\int(\nabla\rho_n)^2\rho_p/\rho_n d\boldsymbol{r}$, $\int(\nabla\rho_p)^2\rho_n/\rho_p d\boldsymbol{r}$ (the latter resulting from the $\rho\cdot\tau$ terms of v in Eq. (1)), and finally the direct Coulomb energy $\langle\mathrm{C}\rangle$. The exchange part of the Coulomb energy in Slater approximation is of the $\langle\sigma\rangle$ form with $\sigma=4/3$. Variation of the energy functional (38) with respect to ρ_p and ρ_n (or ρ_+, ρ_-) under subsidiary condition of fixed proton and neutron numbers Z and N leads to the coupled system of integro-differential equations[13]

$$\frac{\delta u}{\delta\rho_p}=\lambda_p,\qquad \frac{\delta u}{\delta\rho_n}=\lambda_n \quad\text{or}\quad \frac{\delta u}{\delta\rho_+}=\frac{1}{2}(\lambda_n+\lambda_p),\qquad \frac{\delta u}{\delta\rho_-}=\frac{1}{2}(\lambda_n-\lambda_p) \tag{39}$$

with λ_p and λ_n as Lagrange parameters.

It is readily verified that the expressions

$$I_1=\int\left(\frac{\delta u}{\delta\rho_p}\right)\rho_p d\boldsymbol{r}+\int\left(\frac{\delta u}{\delta\rho_n}\right)\rho_n d\boldsymbol{r}=\int\left(\frac{\delta u}{\delta\rho_+}\right)\rho_+ d\boldsymbol{r}+\int\left(\frac{\delta u}{\delta\rho_-}\right)\rho_- d\boldsymbol{r} \tag{40a}$$

and

$$I_2=\int\left(\frac{\delta u}{\delta\rho_p}\right)(\boldsymbol{r}\cdot\nabla)\rho_p d\boldsymbol{r}+\int\left(\frac{\delta u}{\delta\rho_n}\right)(\boldsymbol{r}\cdot\nabla)\rho_n d\boldsymbol{r} \tag{40b}$$

can both be expressed again completely in terms of the expectation values contained in Eq. (38). Namely, we have

$$I_1=\frac{5}{3}\langle \mathrm{TF}\rangle+\langle \mathrm{ST}\rangle+\sum_{\sigma_p}\sigma_p\langle\sigma_p\rangle+\sum_{\sigma_n}\sigma_n\langle\sigma_n\rangle+\sum_{\sigma_n\sigma_p}(\sigma_p+\sigma_n)\langle\sigma_p\sigma_n\rangle$$
$$+2\langle \mathrm{S}\rangle+2\langle \mathrm{C}\rangle=\lambda_p Z+\lambda_n N\,, \tag{41a}$$
$$I_2=-3\langle \mathrm{TF}\rangle-\langle \mathrm{ST}\rangle-3\sum_{\sigma}\langle\sigma\rangle-\langle \mathrm{S}\rangle-5\langle \mathrm{C}\rangle$$
$$=-3\lambda_p Z-3\lambda_n N\,. \tag{41b}$$

These equations are satisfied by solutions of the system (39). Especially, we have $3I_1+I_2=0$, i.e.,

$$2(\langle \mathrm{TF}\rangle+\langle \mathrm{ST}\rangle)+3\{\sum_{\sigma_p}(\sigma_p-1)\langle\sigma_p\rangle+\sum_{\sigma_n}(\sigma_n-1)\langle\sigma_n\rangle$$
$$+\sum_{\sigma_n\sigma_p}(\sigma_p+\sigma_n-1)\langle\sigma_p\sigma_n\rangle\}+5\langle \mathrm{S}\rangle+\langle \mathrm{C}\rangle=0\,.$$

This equation is, of course, just the usual "saturation" condition.

For a given change $\delta\rho_L=\boldsymbol{\nabla}\cdot(\rho\boldsymbol{\nabla}M_L)$ in the density ρ (we omit the indices $+$, $-$ for the moment) the force constants C_L of Eqs. (27) are given as sums of the ground state expectation values $\langle \mathrm{TF}\rangle$, $\langle \mathrm{ST}\rangle$, $\langle\sigma\rangle$, $\langle \mathrm{S}\rangle$ and $\langle \mathrm{C}\rangle$ in the case of the isoscalar monopole and quadrupole modes.

In the isoscalar $L=0$ case the simplest way to obtain this result is to consider density changes created through a parameter α such that

$$\delta\rho_\pm(\boldsymbol{r})=\alpha\left(\frac{\partial\rho_\pm}{\partial\alpha}\right)_{\rho_0}=\alpha\boldsymbol{\nabla}\cdot\left(\rho_\pm\boldsymbol{\nabla}\frac{r^2}{2}\right)=\alpha(3\rho_\pm+(\boldsymbol{r}\cdot\boldsymbol{\nabla})\rho_\pm)\,, \tag{42}$$

where $\rho_\pm(\boldsymbol{r})$ are the static equilibrium densities satisfying Eqs. (39). Then we have

$$C_0=\frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha}\int u(\rho_+\rho_-)\,d\boldsymbol{r}\,.$$

Comparing Eq. (42) with the definitions of I_1 and I_2 in Eqs. (40) leads to

$$C_0=4\langle \mathrm{TF}\rangle+4\langle \mathrm{ST}\rangle+9\sum_{\sigma}(\sigma-1)^2\langle\sigma\rangle+25\langle \mathrm{S}\rangle+\langle \mathrm{C}\rangle \tag{43}$$

and

$$\omega_0^2=C_0/mAr_{rms}^2\,.$$

This expression agrees with the result of Zamick,[14] derived from square well and harmonic oscillator determinants and with the sum-rule result of Martorell et al.[15] It is legitimate to compare it to the sum m_3 of Ref. 15) because we have assumed that the sum is exhausted by the breating mode.

In the isoscalar quadrupole ($L=2$) case we have

$$\delta\rho_{\pm}(\boldsymbol{r})=\alpha_2\boldsymbol{\nabla}\cdot\left(\rho_{\pm}\boldsymbol{\nabla}\frac{r^2}{2}s_M(Y_{2M}+Y_{2M}^*)\right),\qquad s_M=\begin{cases}1 & \text{for}\quad M=0,\\ \sqrt{2} & \\ 1 & \text{for}\quad M=2.\end{cases}\tag{44}$$

For spherically symmetric static densities $\rho_{\pm}(s)$ the force constant C_2 from Eq. (27a) can be rewritten in the form

$$C_2=\frac{1}{4\pi}\left\{\left[24\sum_{\sigma}\langle\sigma\rangle+28\langle\mathrm{S}\rangle+28\langle\mathrm{ST}\rangle+24\langle\mathrm{TF}\rangle+36\langle\mathrm{C}\rangle\right]\right.$$
$$\left.-2\left[\int\frac{\delta u}{\delta\rho_+}r^2\frac{\partial^2}{\partial r^2}\rho_+d\boldsymbol{r}+\int\frac{\delta u}{\delta\rho_-}r^2\frac{\partial^2}{\partial r^2}\rho_-d\boldsymbol{r}\right]\right\}.$$

By virtue of Eqs. (39) the second term inside the curly bracket is equal to $-24(\lambda_nN+\lambda_pZ)$, so we have

$$C_2=\frac{1}{4\pi}\{24(\sum_{\sigma}\langle\sigma\rangle+\langle\mathrm{TF}\rangle-\lambda_nN-\lambda_pZ)+28(\langle\mathrm{S}\rangle+\langle\mathrm{ST}\rangle)+36\langle\mathrm{C}\rangle\}.$$

The relation (41b) serves to eliminate all pure volume terms:

$$C_2=\frac{1}{4\pi}\{20(\langle\mathrm{S}\rangle+\langle\mathrm{ST}\rangle)-4\langle\mathrm{C}\rangle\}.\tag{45}$$

Together with Eq. (23a) the $L=2$ frequency then is given by

$$\omega_2^2=C_2/\left(5m\int\rho_+r^4dr\right)=\frac{4}{mA\langle r^2\rangle}(\langle\mathrm{ST}\rangle+\langle\mathrm{S}\rangle-\langle\mathrm{C}\rangle/5).\tag{46}$$

Similar to the liquid-drop model[16] this result contains only the surface contributions to the total energy (apart from the Coulomb term which agrees with the liquid drop result). The factor in front of the surface energy is in agreement with the microscopic sum-rule result which is in the notation of Ref. 15) (without Coulomb term)

$$\omega_2^2=4(\langle T\rangle+\langle V_{qm}\rangle)/mA\langle r^2\rangle.\tag{47}$$

Here $\langle V_{qm}\rangle$ represents energy contributions from the quadratic momentum dependence of the effective two-nucleon force, i.e., surface terms of the potential energy and volume terms originating in the $\rho\tau$-terms of Eq. (1). But obviously, as a consequence of our assumption about the functional form of $\tau_{\pm}$, both the volume terms of the kinetic energy ($\sigma=5/3$) and the volume part of the $\rho\tau$-terms ($\sigma=8/3$) contained in the result (47) are absent from our result (46), while surface terms are correctly reproduced. The volume terms, however, are decisive for the experimentally observed $A^{-1/3}$-dependence of ω_2, as is also evident from the term containing the single-particle energy ε in Eq. (15). This failure can be directly traced to the assumption of a spherically symmetric local Fermi surface.

In the TDHF propagation there is no mechanism (e.g., through two-body collisions) to restore the spherical symmetry of the Fermi surface once it is distort-

ed. In case of the $L=2$ modes it is, however, easy to incorporate the distortion into a fluid-dynamical formalism (i.e., a formalism which does only consider densities, without going back to wave functions). In the $L=2$ case the Fermi surface at point $\boldsymbol{r}$ is an ellipsoid in momentum space giving by

$$p_F(\boldsymbol{r},\vartheta,\varphi)=p_F(\boldsymbol{r})N(\alpha_2)\,(1+\alpha_2 s_M(Y_{2M}(\vartheta,\varphi)+Y_{2M}^*(\vartheta,\varphi))), \tag{48}$$

where $N(\alpha_2)$ is a normalization factor chosen to conserve the local density $\rho(\boldsymbol{r})$. Then we have (up to $O(\alpha_2^2)$):

$$\rho(\boldsymbol{r})=\frac{16\pi}{(2\pi\hbar)^3}\frac{p_F{}^3(\boldsymbol{r})}{3}=\frac{4}{(2\pi\hbar)^3}\int d\Omega_p\int_0^{p_F(\boldsymbol{r},\vartheta,\varphi)}p^2dp=\rho(\boldsymbol{r})\left(1+\frac{3}{2\pi}\alpha_2^2\right)N(\alpha_2)^3,$$

i.e.,

$$N(\alpha_2)=1-\alpha_2^2/2\pi\,.$$

The volume part τ_{TF} of the kinetic energy density is now given by

$$\tau_{\mathrm{TF}}(\boldsymbol{r},\alpha_2)=\tau_{\mathrm{TF}}(\boldsymbol{r},\alpha_2=0)\left(1+\frac{5}{2\pi}\alpha_2^2\right). \tag{49}$$

For the $L=2$ oscillation around the spherical equilibrium density ρ_0

$$\rho(\boldsymbol{r})=\rho_0(\boldsymbol{r})+\alpha_2\delta\rho(\boldsymbol{r})=\rho_0+\alpha_2 r\rho_0' s_M(Y_{2M}+Y_{2M}^*)$$

the amplitude α_2 is the following functional of $\rho(\boldsymbol{r})$:

$$\alpha_2(\rho)=-\frac{2\pi}{3A}\int\rho(\boldsymbol{r})s_M(Y_{2M}+Y_{2M}^*)\,d\boldsymbol{r}\,. \tag{50}$$

The second-order change of the volume part of the kinetic energy

$$\langle\mathrm{TF}\rangle=\frac{\hbar^2}{2m}\int\tau_{\mathrm{TF}}d\boldsymbol{r}\cdot\left(1+\frac{5}{2\pi}(\alpha_2(\rho))^2\right)$$

evaluated at $\rho=\rho_0$ is therefore given by

$$(\delta^2\langle\mathrm{TF}\rangle)_{\rho_0}=\frac{\hbar^2}{2m}\left(\int\left(\frac{\delta^2\tau_{\mathrm{TF}}}{\delta\rho^2}\right)_{\rho_0}\delta\rho^2d\boldsymbol{r}+\int\tau_{\mathrm{TF}}(\rho_0)\,d\boldsymbol{r}\cdot\frac{5}{2\pi}2(\delta\alpha_2)^2\right).$$

From Eq. (50) we have $\delta\alpha_2=-2\pi/3A\cdot\int\delta\rho s_M(Y_{2M}+Y_{2M}^*)\,d\boldsymbol{r}=1$ which leads to

$$(\delta^2\langle\mathrm{TF}\rangle)_{\rho_0}=\frac{\hbar^2}{2m}\int\left(\frac{\delta^2\tau_{\mathrm{TF}}}{\delta\rho^2}\right)_{\rho_0}\delta\rho^2d\boldsymbol{r}+\frac{10}{2\pi}\langle\mathrm{TF}\rangle_{\rho_0}\,.$$

This result shows that inclusion of the distortion of the local Fermi sphere leads to an additional term of $(20/4\pi)\langle\mathrm{TF}\rangle$ (and a corresponding volume term ($\sigma=8/3$) originating from the $\rho\tau$ part of Eq. (1)) in the fluid-dynamical result for the force constant C_2 in Eq. (45). These are exactly the terms needed to obtain agreement with the microscopic result (47) for the frequency ω_2:

$$(\omega_2)^2=\frac{4}{mA\langle r^2\rangle}\left(\langle\mathrm{TF}\rangle+\langle\mathrm{ST}\rangle+\left\langle\sigma=\frac{8}{3}\right\rangle+\langle\mathrm{S}\rangle-\langle\mathrm{C}\rangle/5\right). \tag{51}$$

§ 6. Numerical results

For the Skyrme-force parameters given in Ref. 18) leading to an effective mass equal to the nucleon mass, $m^*=m$, we compare for $L=0$ and $L=2$ isoscalar modes the numbers resulting from fluid-dynamical and microscopic methods. The force parameters are:[18] $t_0=-1089.0$ MeV fm^3, $t_1=251.11$ MeV fm^5, $t_2=-150.66$ MeV fm^5, $t_3=17270$ MeV fm^6, $x_0=0.583$. For the kinetic energy density (37) we use the constants[12] $c_{\mathrm{TF}}=3/5\,(3\pi^2)^{2/3}$, $c_{\mathrm{ST}}=1/36$. As discussed in Ref. 13), the density distributions obtained from the integro-differential Eqs. (39) agree reasonably well with Hartree-Fock calculations (which we take from Ref. 19)), although the small value of c_{ST} leads to a suppression of the exponential tail. As an example we present the density profiles for ^{208}Pb in Fig. 1. Table I lists the different contributions in Eq. (38) to the total energy for solutions of Eqs. (39), the $\sigma=8/3$ and $\sigma=4/3$ terms representing the volume part of $\rho\tau$ and Coulomb exchange energy in Slater approximation, respectively. The total binding energies E_B are surprisingly close to experimental values $E_B(\mathrm{exp})$, while the last column gives the HF-results.

Table II contains the frequencies $\hbar\omega_L$ according to Eqs. (43) and (51) calculated with the expectation values of Table I. As discussed in the last section, the ratios of energy-weighted sums m_3/m_1 (in notation of Ref. 15)) are formally

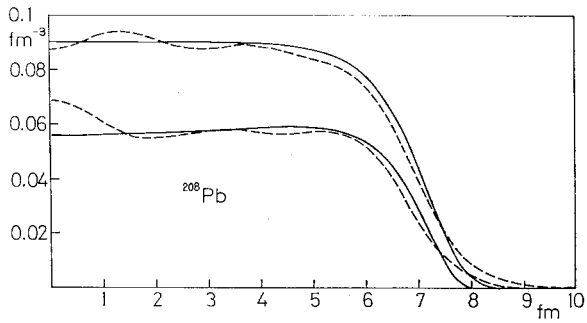

Fig. 1. The proton and neutron densities resulting from Eqs. (39) (full lines) and from Hartree-Fock calculations[19] (dashed lines) for the ($m^*=m$) Skyrme force of Ref. 18) (for ^{208}Pb).

Table I. Ground-state expectation values for the different contributions to the total binding enery E_B, calculated for solutions of Eqs. (39). Except for the mean square radius (fm^2) all numbers are given in units of MeV. The last two columns contain experimental and Hartree Fock results (from Ref. 19)) for the binding energies.

	⟨TF⟩	⟨ST⟩	⟨σ=2⟩	⟨σ=3⟩	⟨σ=8/3⟩	⟨σ=4/3⟩	⟨S⟩	⟨C⟩	⟨r^2⟩	$-\lambda_n$	$-\lambda_p$	$-E_B$	$-E_B$ (exp)	$-E_B$ (HF)
^{40}Ca	672.5	29.3	−1782.4	579.6	0	−7.9	87.1	82.0	10.95	12	5	339.8	342.1	330.8
^{90}Zr	1603.3	49.3	−4269	1446.3	−4.9	−15.6	151.8	254.6	17.89	8.5	6.5	784.2	783.9	736.0
^{208}Pb	3917.1	97.9	−10205	3551	−40.6	−31.8	268.2	823.1	30.36	5.4	4.9	1620	1636.5	1551.9

Table II. Resonance energies for $L=0$ and $L=2$ modes. $\hbar\omega$ values are calculated according to Eqs. (43) and (51). $(m_3/m_1)^{1/2}$ are formally identical expressions[15] calculated with HF wave functions. These and the RPA results are taken from Ref. 19).

	Monopole				Quadrupole			
	ω_0	$\omega_0 \cdot A^{1/3}$	$(m_3/m_1)^{1/2}$	RPA	ω_2	$\omega_2 \cdot A^{1/3}$	$(m_3/m_1)^{1/2}$	RPA
^{40}Ca	30.6	104.7	28.4	26.5	17.1	58.5	16.0	16.3
^{90}Zr	25.0	111.9	23.9	22.9	13.4	60.2	13.1	15.0
^{208}Pb	19.6	116.2	18.4	17.1	10.4	61.4	10.0	12.0

identical with Eqs. (43) and (51); their evaluation with HF wave functions leads to the results given in the m_3/m_1 columns of Table II. The values listed under "RPA" are also taken from Ref. 19), making use of the mean single-particle energies ε (cf. Eqs. (15)) and including contributions from unbound states. Detailed discussion of this method is given in Ref. 19).

§ 7. Conclusion

For simple cases of small amplitude vibrations in finite nuclei we have compared two very different dynamical descriptions: The microscopic TDHF (or equivalently RPA) method and the macroscopic fluid-dynamical equations. By "macroscopic" we mean that the local density and velocity potential are the only dynamical quantities to occur in the formulation. A close connection between both formulations is established through the assumption that the transition strength for a given multipole operator M_L is concentrated on one collective eigenstate of the nuclear system. This assumption leads to (microscopic) expressions for the resonsnce energies which closely resemble the frequency of a constrained fluid-dynamical motion where the velocity potential is restricted to the considered multipole field M_L. For the breathing mode of a square-well density we have shown that this frequency is connected to the free fluid-dynamical eigenmodes through a sum rule which, however, is not exhausted by the breathing mode.

The only and basic difference between the microscopic and macroscopic results occurs for the contributions of the kinetic energy of the static system to the restoring force for the vibration. Microscopically given by an average single-particle energy, it is macroscopically determined through the second derivative of the kinetic energy density with respect to the local density. Therefore the fluid-dynamical result is sensitively dependent on the functional form of the kinetic energy which in turn is determined through the local momentum distribution. We have shown that local isotropic momentum distribution is not sufficient to describe surface oscillations correctly, but that it is essential to allow dynamical distortions of the local Fermi surface. This leads to a simple example for a non-Newtonian fluid with a non-isotropic pressure term. The effect of this Fermi

surface distortion on the resonance frequency is drastic and the good agreement with experimentally observed $L=2$ resonances[20] ($\hbar\omega_2 = 63 \cdot A^{-1/3}$ MeV) indicates that giant resonances are indeed propagated essentially through the mean field, without local equilibrium being established as in ordinary sound. With proper choice of the kinetic energy functional the fluid-dynamical formalism is, however, sufficient to reproduce the microscopic results.

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