

Giant topological Hall effect in correlated oxide thin films

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Strong electronic correlations can produce remarkable phenomena such as metal-insulator transitions and greatly enhance superconductivity, thermoelectricity, or optical non-linearity. In correlated systems, spatially varying *charge* textures also amplify magnetoelectric effects or electroresistance in mesostructures. However, how spatially varying *spin* textures may influence electron transport in the presence of correlations remains unclear. Here we demonstrate a very large topological Hall effect (THE) in thin films of a lightly electron-doped charge-transfer insulator, (Ca, Ce)MnO₃. Magnetic force microscopy reveals the presence of magnetic bubbles, whose density vs. magnetic field peaks near the THE maximum. The THE critically depends on carrier concentration and diverges at low doping, near the metal-insulator transition. We discuss the strong amplification of the THE by correlation effects and give perspectives for its non-volatile control by electric fields.

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29 The role of topology in condensed matter is the subject of a strong research effort that has led to the
30 discovery of distinct electronic phases and novel physical phenomena in the last decade¹. In
31 *reciprocal* space, strong spin-orbit coupling may produce topological insulators with exotic band
32 structures². Electronic correlations strongly enrich the diversity of possible topological phases in such
33 quantum materials¹, and various novel states including topological Mott insulators, axion insulators
34 or quantum spin liquids have been predicted, mainly in transition metal oxides³. In *real* space,
35 topological defects in a spin lattice such as skyrmions (particle-like spin configurations characterised
36 by an integer topological charge $Q \neq 0$)^{4,5} give rise to emerging phenomena such as the topological
37 Hall effect (THE)^{6,7}. However, while several oxides also exhibit topological spin textures^{5,8,9}, how these
38 couple to the electronic correlations (ubiquitous in these materials) to influence electron transport
39 has not been addressed up to now.

40 In this paper, we explore the interplay between strong correlations and micromagnetic
41 configurations in perpendicularly magnetized epitaxial thin films of a lightly doped charge-transfer
42 insulator¹⁰, CaMnO₃. Undoped CaMnO₃ is a Pbnm perovskite in which magnetic order is dominated
43 by first-neighbour antiferromagnetic super-exchange interactions, with an additional small
44 ferromagnetic component along z ($\mathbf{G}_x\mathbf{A}_y\mathbf{F}_z$ order¹¹; x, y and z are parallel to the axes of the
45 orthorhombic unit cell). A sketch of this spin structure is shown in Fig. 1a. This small moment is about
46 0.04 μ_B/Mn , corresponding to a canting angle of about 1° and the Néel temperature of bulk CaMnO₃
47 is 120 K.

48 Upon doping by just a few percent of Ce⁴⁺ at the Ca site, the material transitions to a metallic state¹²
49 and the weak moment increases up to $M_S \approx 0.7 \mu_B/\text{Mn}$, signalling the onset of ferromagnetic double
50 exchange associated with carrier delocalisation. In thin films grown on YAlO₃, the transition from an
51 insulating to a metallic state occurs at Ce concentrations as low as ~1-2% (Ref.¹³). In addition, on this
52 substrate imposing a compressive strain of ~1%, magnetoelastic anisotropy induces an easy
53 magnetization axis perpendicular to the film plane¹³. In such a perpendicular magnet where the
54 condition $\frac{2K_u}{\mu_0 M_S^2} \gg 1$ is easily met (K_u is the magnetic anisotropy and M_S the magnetization at high
55 field), magnetization should reverse through the formation of magnetic rod-like domains often
56 referred to as bubbles. Such bubble-like domains may exhibit topological properties⁶ akin to those of
57 skyrmions albeit without the need for bulk or interfacial inversion symmetry breaking usually
58 required for skyrmion formation. In fact, topological bubbles⁶ with diameters in the 100-300 nm
59 range have been observed in several bulk *centrosymmetric* manganites^{5,8,9}. Although we find no
60 direct evidence of topological spin structures in our samples, the unique coexistence of metallicity
61 and perpendicular magnetic anisotropy in a doped charge-transfer insulator thus makes (Ca,

62 Ce)MnO₃ (CCMO) a well-suited system to probe how topological magnetic configurations may
63 influence electron transport in the presence of correlations.

64 CCMO epitaxial thin films with nominal Ce concentration $x=0, 1\%, 2\%, 4\%$ and 5% were grown by
65 pulsed laser deposition on (001)-oriented YAlO₃ single crystal substrates. Fig. 1b presents a typical X-
66 ray diffraction $2\theta-\omega$ scan for a 4% sample, with Laue fringes attesting of the structural coherence of
67 the film. The inset shows that the out-of-plane parameter (collected on fully strained films) increases
68 with doping, as expected in a solid solution. The Ce doping modifies the Mn valence from Mn⁴⁺ in
69 pure CaMnO₃ and introduces Mn³⁺, as revealed by a shift towards low energies of electron energy
70 loss spectroscopy and X-ray absorption spectroscopy spectra measured at the Mn L₃ edge (see Fig. 1c
71 and Ref. ¹⁴). Accordingly, the carrier concentration also varies linearly with Ce doping, (Fig. 1c, left
72 axis).

73 Fig. 1e shows the temperature (T) dependence of the longitudinal resistivity ρ for different doping
74 levels. While the pure CaMnO₃ sample exhibits a high resistivity and a thermally activated behaviour
75 – in line with its expected charge-transfer insulating state – Ce doping yields a decrease of the
76 resistivity by several orders of magnitude and a room-temperature metallic response already for 1%
77 Ce (CCMO1). At 4% (CCMO4) and 5% (CCMO5) the resistivity shows a cusp at $\sim 110\text{K}$ that signals the
78 transition to the weak-ferromagnetic state. A signature of this transition is also present in the
79 CCMO1 and CCMO2 data and translates into a dip in the temperature dependence of $d^2\log(\rho)/dT^2$
80 (cf. Fig. 1d), indicating that these samples also possess a weak-ferromagnetic behaviour. The inset
81 presents the doping dependence of the low temperature resistivity (open circles).

82 The weak-ferromagnetic character of the films is evident from the temperature dependence of the
83 magnetization M (and of the Kerr ellipticity for $x=4\%$) displayed in Fig. 1g. For all doping levels, the T_c
84 is close to 110 K but M increases with doping, as in the bulk¹⁵ (see inset). Note that these data were
85 measured with a modest out-of-plane field of 0.1 T, so that the shape of the M vs T curves is
86 consistent with a perpendicular magnetic anisotropy. This is also illustrated in Fig. 1f that shows the
87 Kerr ellipticity vs out of plane magnetic field for a CCMO4 sample; a remanence of virtually 100% is
88 observed. Finally, the centrosymmetric nature of the films was confirmed by the absence of a signal
89 in second harmonic generation.

90 To probe the influence of the micromagnetic structure on the transport response, we have measured
91 the Hall effect as a function of temperature. The results for CCMO4 are summarized in Fig. 2. Fig. 2a
92 presents the transverse (Hall) resistivity at different temperatures. At 130 K the Hall effect is linear,
93 consistent with n-type transport. The carrier density is $n \approx 1.7 \cdot 10^{21} \text{ cm}^{-3}$, close to the nominal value of
94 $1.55 \cdot 10^{21} \text{ cm}^{-3}$ for 4% Ce, assuming that each Ce ion brings two electrons. As temperature decreases

95 below T_c (see the 95 K data), a hysteretic component develops, corresponding to the anomalous Hall
 96 effect (AHE) in a perpendicularly magnetised sample. As temperature decreases further, a third
 97 component appears in the form of a peak centred in the 1 T range. Its peculiar shape is reminiscent
 98 of the topological Hall effect (THE) observed in skyrmion systems^{7,16-18}. In these samples, the Hall
 99 effect thus comprises three components, i.e.

$$\rho_{Hall} = R_0H + R_S M + \rho_{THE} \quad (1)$$

100 To obtain the AHE and the THE from the Hall data, we first subtract a high field slope R_0 (ordinary Hall
 101 effect). To extract the AHE (second term in Eq.(1), i.e. $\rho_{AHE} = R_S M$) we use magneto-optical Kerr effect
 102 measurements on the same sample through the field dependence of the Kerr ellipticity (that is
 103 expected to be proportional to the magnetization). This decomposition is shown in Fig. 2b. Fig. 2c
 104 displays the extracted THE component for different temperatures, confirming that the THE is in the
 105 $\mu\Omega\cdot\text{cm}$ range at low temperature and decreases upon heating to vanish between 75 and 95 K.

106 The maximum amplitude of the THE is plotted as a function of temperature in Fig. 2e (the
 107 corresponding dependence for the AHE is displayed in Fig. 2d), and shows a nearly monotonic
 108 decrease as T is raised. Combining the THE data for all temperatures, we build the (T, H) phase
 109 diagram displayed in Fig. 2f that also presents the dependence of the coercive field (extracted from
 110 the AHE and the Kerr ellipticity) and the field at which the THE is maximum. The THE is highest in a $(T,$
 111 $H)$ pocket centred near 1-1.2 T and extending to about 80 K. Beyond this temperature, only the AHE
 112 is present, up to the T_c near 100 K.

113 As initially proposed by Bruno *et al*⁷, a THE arises when an electron moves in a medium with spatially
 114 varying spin texture, which endows it with a Berry phase. The effect of this Berry phase can be
 115 mapped onto that of an effective (emergent) perpendicular magnetic field \mathbf{b} that produces a Hall
 116 effect, just as an external magnetic field. The amplitude of \mathbf{b} depends on the topology of the spin
 117 textures through the density of the corresponding topological charge. For instance, in a skyrmion
 118 array $\langle b \rangle = \Phi_0/d_{sk}^2$ where Φ_0 is the flux quantum and d_{sk} the average distance between skyrmions
 119 carrying a topological charge $Q = 1$. In this simple model for strong ferromagnetic exchange coupling,
 120 the THE then writes

$$\rho_{THE} = \frac{P\langle b \rangle}{en} = \frac{\Phi_0 P}{d_{sk}^2 en} \quad (2)$$

121 with P the spin polarization and e the electron charge.

122 To gain more insight into the nature of the spin configurations responsible for the THE in our films,
 123 we have performed magnetic force microscopy measurements as a function of temperature and out-
 124 of-plane magnetic field. Fig. 3a-f show examples of such images taken at negative magnetic field,
 125 after saturating with a positive field of +3 T at 10 K. At -0.6 T (Fig. 3a), the magnetization is still fully
 126 aligned in the positive field direction and the MFM image shows a homogeneous red contrast. As H
 127 increases further towards large negative values, magnetization start to reverse and domains with
 128 negative magnetization appear (Fig. 3b). Near -1.2 T the image shows a large density of small bubble-
 129 like domains with positive magnetization surrounded by a negative magnetization background (Fig.
 130 3d). Their density progressively decreases as the field increases further to large negative values (cf.
 131 Fig. 3e taken at -1.4 T) and they vanish beyond -1.8 T (Fig. 3f). The bubbles visible in Fig. 3c-e have a
 132 typical size of ~100-300 nm, which is comparable to the size of topological bubbles detected in other
 133 centrosymmetric manganites^{5,8,9}. By analysing the images collected at different fields, one can
 134 extract the field dependence of the number of bubble-like domains (n_b). It is plotted in Fig. 3g (right
 135 axis) and compared with the field dependence of the THE (left axis). As already revealed by the
 136 images, n_b strongly varies with H and peaks near the same field H_T as the THE. The same analysis was
 137 done at 40 K (Fig. 3h) and 80 K (Fig. 3i). Similarly, the THE and n_b peak near the same H_T and the value
 138 of H_T decreases with temperature for both quantities. This suggests that the THE is created by the
 139 specific spin texture associated with the presence of these bubbles.

140 We now discuss in more detail the origin of the AHE and the THE in our films, starting with the AHE.
 141 In colossal magnetoresistance manganites such as $\text{La}_{0.7}\text{Sr}_{0.3}\text{MnO}_3$ or $\text{La}_{0.7}\text{Ca}_{0.3}\text{MnO}_3$ the temperature
 142 dependence of the AHE is peculiar. It is vanishingly small at low temperatures, developing only when
 143 temperature increases to about $T_C/2$ and peaking near T_C (Ref. ¹⁹⁻²¹). Two related models^{22,23} have
 144 been proposed to explain this unusual behaviour. Both consider that the AHE arises from the
 145 emergence of dynamic non-coplanar spin configurations having a topological character and
 146 describable as skyrmion strings (beginning at a skyrmion and ending at an antiskyrmion). As in the
 147 model of Bruno *et al*⁷ for the THE, itinerant electrons travelling through these spin textures acquire a
 148 Berry phase corresponding to an emergent magnetic field, generating a Hall effect, i.e. $\rho_{AHE} = \frac{P(b)}{en}$
 149 as in Eq. (2). However, the strings are mostly oriented randomly and their contributions practically
 150 cancel out. A slightly preferred orientation only arises through spin-orbit coupling. In colossal
 151 magnetoresistance manganites at low temperature the string density is vanishingly small
 152 (magnetization is saturated with all spins collinear) and increases strongly close to T_C (Ref. ²³). As a
 153 result, the AHE shows a maximum near T_C (Refs. ^{22,23}). In our data (Fig. 2d) this maximum is visible,
 154 but they also evidence a strong AHE at low temperature. Further theoretical work thus appears
 155 necessary to accurately describe the AHE in weakly ferromagnetic manganites such as CCMO and in

156 particular its large amplitude below ~ 20 K, perhaps reflecting a finite density of non-coplanar spin
157 configurations already at low temperature.

158 Let us now turn to the THE. The topological charge that gives rise to the THE often stems from non-
159 collinear spin structures created by symmetry breaking-induced Dzyaloshinskii-Moriya interaction
160 (DMI). We have thus considered possible sources of non-collinear spin structures, namely, cerium
161 doping (a heavy element that can create DMI), interface-induced DMI or global DMI (induced by a
162 non-centrosymmetric crystal structure), but have found that none of these putative structures are
163 present in CCMO. On the other hand, MFM indicates that the micromagnetic corrugation is the
164 largest in the field range where the THE occurs. One possibility could then be that the THE arises
165 from non-coplanar spin configurations at domain walls, with the same mechanism of the AHE due to
166 non-coplanar spin configurations in the domains. Given the relatively high anisotropy of our CCMO
167 films, we estimate that domain walls are rather narrow, with a width d_{dw} in the 10-20 nm range, and
168 thus generate angles between spins on the order of $\theta_{dw} = 180 \times a / d_{dw} = 3-6^\circ$. This is comparable to the
169 canting angles in the saturated state so that, locally, the AHE should not be much larger at the
170 domain walls than in the domains. In view of their small volume fraction, domain walls should thus
171 not generate an extra Hall effect with an amplitude as large as that of the observed THE by this
172 mechanism.

173 While DMI and the domain walls around the bubbles cannot explain the THE, we argue below that
174 the bubbles themselves can. Magnetic bubbles are characterized by a topological charge Q that can
175 be a finite integer or zero depending on the internal structure of the domain wall enclosing the
176 bubble⁴⁻⁶. Some bubbles can have a spin structure very similar to that of skyrmions (the main
177 difference is that bubbles have an extended core, while skyrmions have a point-like core) with the
178 same topological charge of Q^6 . In other words, such bubbles have a topological character and, as
179 such, in the literature they are called “skyrmion-bubbles”^{6,24}. Thus, although we cannot directly
180 verify it, the observed bubbles in our samples might carry a topological charge just as standard
181 skyrmions and as previously observed in several bulk manganites^{5,8,9} and very recently in films²⁵. In
182 this scenario, the bubbles could contribute to the THE through a Berry phase mechanism such as that
183 proposed by Bruno *et al.*⁷ However, using Eq. (2) and the average spacing between bubbles detected
184 by MFM, we calculate a THE amplitude weaker by a factor 25-40 than the experimental value. This
185 discrepancy may reflect our inability to detect very small bubbles, although they would then have to
186 be $\sim 25-40$ times more abundant than those visible in Fig. 3 ; alternatively, we postulate that the THE
187 in CCMO is amplified by electron correlations, an effect not taken into account in the simple picture
188 of Ref. ⁷.

189 To explore this possibility, we have measured the Hall response of CCMO films with different doping
 190 levels and carrier densities. As visible in Fig. 4a-d, a THE is also observed at 1%, 2% and 5% and its
 191 amplitude varies strongly with Ce concentration. The amplitude of the THE is much higher than
 192 previously reported values in other materials, cf. Table 1. In the Supplementary Information 1 and 2,
 193 we show sets of magnetic images for 2% and 5% similar to those of Fig. 3 along with their analysis;
 194 just as for the 4% sample, MFM evidences the presence of bubbles whose density with magnetic field
 195 dependence tracks that of the THE. Within error bars, the observed bubble size range is comparable
 196 to that found for 4%.

197 **Table 1. Comparison of topological Hall effect amplitudes in various materials systems**

Material system	Maximum THE ($\mu\Omega\cdot\text{cm}$)	Ref.	Note
MnSi	0.0045	¹⁶	Skyrmion system
MnP	0.01	²⁶	Fan spin structure
MnGe	0.16	²⁷	Skyrmion system
SrRuO ₃ /SrIrO ₃	0.2	²⁸	Interface system
Fe _{0.7} Co _{0.3} Si	0.5	²⁹	Skyrmion system
EuO	6	³⁰	Centrosymmetric
Ca _{0.99} Ce _{0.01} MnO ₃	120	This work	Correlated oxide

198

199 Fig. 4e summarizes the dependence of the THE amplitude with carrier density. As n decreases, the
 200 THE diverges, increasing by three orders of magnitude upon reducing the carrier density by just a
 201 factor of ~ 4 . This is in stark contrast with the expected behaviour using Eq. (2).

202 Rather than a model for the strong coupling regime⁷, one applicable to weaker coupling is actually
 203 better suited here^{31,32} (see Supplementary Information 3). In this situation, one should consider not
 204 only the adiabatic (Berry phase) processes but also non-adiabatic processes for the electron motion
 205 in a spin texture, as described in the model of Nakazawa et al³². The Hall resistivity is then expressed
 206 as

$$\rho_{THE} \propto \frac{\langle b \rangle m^* J}{n^{5/3}} \quad (3)$$

207 where J is related to the Hund coupling between conduction electrons and magnetization, multiplied
 208 by the relative magnetization (cf. Supplementary Information 4), which is proportional to the doping
 209 level (i.e. to n), cf. Fig. 1g (inset). Unlike in the strong coupling model the THE amplitude now
 210 depends on the electron effective mass. In a doped correlated insulator such as CCMO, as the
 211 transition to the insulating state is approached from the metallic side (here upon reducing the Ce
 212 doping level), the effective mass diverges³³ which is the signature of electronic correlations³⁴. This is
 213 documented experimentally for several perovskite systems, for instance Sr-doped LaTiO₃ (Ref. ³⁵)

214 where upon approaching the Mott insulating state of LaTiO_3 , m^* increases up to $m^*/m_0 \approx 30$, or Ce-
215 doped SrMnO_3 (very similar to CCMO) in which $m^*/m_0 \approx 10$ at 1-2% Ce while the bare mass is $0.6m_0$
216 (Ref. ³⁶).

217 Coming back to Eq. (3), we can incorporate the effect of electron correlations by assuming an
218 enhancement of the effective mass. With a scaling of m^* as $1/n^\alpha$ we see that ρ_{THE} has to scale as
219 $1/n^{2/3+\alpha}$. α is usually close to 1 (Ref. ^{33,34}) but can reach 2 for low film thicknesses (Ref. ³⁷). This
220 provides ground for the strong enhancement of the topological Hall effect at low doping, due to the
221 renormalization of the effective mass by electronic correlations. In Fig. 4e we compare the
222 experimental dependence of the topological Hall resistivity with the carrier density with scaling laws
223 in the strong coupling regime⁷ ($1/n$) and in the weaker coupling regime considered here. The trend
224 with doping is well captured within the weaker coupling and strong correlations regime using $\alpha=2$.
225 However, the numerical values of the experimental THE are underestimated by the model, probably
226 because the weak-coupling formula allows other types of corrections (called vertex corrections,
227 effects beyond the single-particle mass renormalization) coming from strong electron correlation,
228 which would further enhance the calculated topological Hall resistivity. While these results suggest
229 that electronic correlations can highly amplify the topological Hall response, they thus also call for a
230 detailed theoretical treatment of this phenomenon.

231 Our work shows that magnetic perovskite oxides represent an exciting class of materials exhibiting
232 novel transport phenomena stemming from non-trivial spin textures, highly tuneable through
233 doping, strain and interface engineering²⁸. In particular, the strong sensitivity of the THE to the
234 carrier density offers interesting perspective for its non-volatile control by electric-field. In
235 heterostructures combining CaMnO_3 with ferroelectrics such as BiFeO_3 large, non-volatile
236 modulation of the linear Hall effect and longitudinal resistivity have indeed been reported (Ref. ³⁸).

237

238 **Figure captions**

239 **Figure 1. Doping-dependant structural, electronic and magnetic properties of $\text{Ca}_{1-x}\text{Ce}_x\text{MnO}_3$ thin**
240 **films.** (a) Sketch of the structural and spin structure of CCMO in the orthorhombic unit cell. In the
241 right panels, the light green arrows point towards $-y$, and the dark green ones towards $+y$. The
242 canting angles are exaggerated for clarity. (b) 2θ - ω scan near the pseudocubic (001) reflection of a
243 $x=4\%$ CCMO film on YAO(001). Inset: doping dependence of the out-of-plane lattice parameter c .
244 Error bars derive from fitting the spectra with Panalytical Data Viewer. (c) Doping dependence of the
245 carrier density (open circles, left axis) and of the energy shift of the Mn L_3 peak (relative to the

246 position at 2% doping) from electron energy loss spectroscopy (up triangles, reproduced from Ref. ¹⁴)
247 and X-ray absorption spectroscopy (down triangle). Temperature dependence of the resistivity (e)
248 and the second derivative of its logarithm (d) for different doping levels. The phase diagram in the
249 inset shows the dependence of the resistivity at low temperature. (f) Kerr ellipticity vs magnetic field
250 at 15 K for a 4% film. (g) Temperature dependence of the magnetization in a field of 1 kOe (left axis,
251 solid symbols) for different doping levels and of the Kerr ellipticity (right axis, open symbols) for a 4%
252 film. The inset shows the doping dependence of the magnetization.

253 **Figure 2. Topological Hall effect in 4% CCMO.** (a) Hall effect at different temperatures. The data are
254 shifted vertically for clarity. (b) Decomposition of the Hall effect into the AHE and the THE using Kerr
255 ellipticity data at 15 K. (c) Topological Hall effect at different temperatures. As in (a), the data are
256 shifted vertically for clarity. Temperature dependence of the anomalous Hall resistivity (d) and the
257 topological Hall resistivity (e). Error bars in (d) are the standard deviation of a fit of the high field part
258 of the AHE signal. Error bars in (e) are the standard deviation of the maximum 5 values of the THE. (f)
259 Topological Hall effect vs temperature and magnetic field. The temperature dependence of the
260 coercive field H_c and the position of the THE maximum H_T are shown as green and red symbols,
261 respectively.

262 **Figure 3. Connection between micromagnetism and topological Hall effect.** (a-f) Magnetic force
263 microscopy images at 10 K after applying a positive perpendicular field of +3T, at different negative
264 magnetic fields. (g-i) Field dependence of the topological Hall resistivity (blue line, left axis) and of
265 the number of observed bubbles in the MFM images at 10 K, 40 K and 80 K, respectively. Error bars
266 correspond to the spread in the values from several image analyses using different binarisation
267 thresholds (see SI.)

268 **Figure 4. Doping dependence of the topological Hall effect.** Topological Hall effect at 20 K for 1% (a)
269 and 15 K for 2% (b), 4% (c) and 5% doping (d). Relationship of the THE (e) with the carrier density.
270 The model is shown normalized to the experimental value at $1.03 \cdot 10^{21} \text{ cm}^{-3}$. Horizontal error bars
271 correspond to the standard deviation of the carrier density in the paramagnetic regime. Vertical
272 errors bars are defined as for Fig. 2e.

273

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294 **Data availability statement**

295 The data that support the plots within this paper and other findings of this study are available from
296 the corresponding author upon reasonable request.

297

298

299 **Methods**

300 **Fabrication and structural characterisation.** The $\text{Ca}_{1-x}\text{Ce}_x\text{MnO}_3$ ($x = 0, 1, 2, 4, 5\%$ nominal Ce
301 concentrations) 20 nm thin films were grown by pulsed laser deposition from stoichiometric targets
302 on (001) YAIO_3 substrates using a Nd:YAG laser. Commercial YAIO_3 (001) oriented substrates were
303 prepared with acetone cleaning and ultrasound in propanol, and then annealed at 1000°C in high O_2
304 pressure. The substrate temperature (T_{sub}) and oxygen pressure (P_{O_2}) during the deposition were
305 620°C and 20 Pa, respectively. Post-deposition annealing was performed at $T_{sub} \approx 580^\circ\text{C}$ and $P_{O_2} = 30$
306 kPa for 30 minutes, followed by a cool-down at the same oxygen pressure. 2θ - ω X-ray diffraction
307 scans were performed with a Panalytical Empyrean equipped with a hybrid monochromator for Cu
308 $K_{\alpha 1}$ radiation and a PIXcel3D detector. The Data Viewer software was used to fit the spectra and

309 extract the out of plane lattice parameter. The thickness of the $\text{Ca}_{1-x}\text{Ce}_x\text{MnO}_3$ thin film was measured
310 by X-ray reflectivity with a Bruker D8 DISCOVER. Hall bars for magnetotransport measurements were
311 patterned by optical lithography and argon ion etching. Electrical contacts for measurements were
312 made on platinum electrodes defined by a combination of lithography and lift-off techniques.

313 **Electrical characterization.** The magnetotransport characterization of the samples was performed in
314 a Quantum Design Physical Properties Measurement System (PPMS) Dynacool. The temperature
315 dependence of the resistivity was measured at a constant current of 5 μA during a warming run after
316 field cooling in a cryostat through a closed liquid helium loop. For Hall measurements, magnetic
317 fields were swept up to ± 9 T. To separate the Hall contribution from that of the longitudinal
318 magnetoresistance, an antisymmetrization procedure was performed by separating the positive and
319 negative field sweep branches, and interpolating the two onto the same field coordinates and then
320 antisymmetrizing using

$$\rho'_+(H) = [\rho_+(H) - \rho_-(-H)]/2$$

321
$$\rho'_-(H) = [\rho_-(H) - \rho_+(-H)]/2.$$

322 **Magnetic characterization.** Superconducting Quantum Interference Device (SQUID) magnetic
323 characterization of the samples was performed in a Quantum Design Magnetic Properties
324 Measurement System (MPMS). The temperature dependence of the magnetization was measured at
325 a constant field of 1 kOe during a warming run after field cooling in a cryostat with liquid helium. The
326 magnetization loops were obtained by sweeping the magnetic field up to ± 5.5 T.

327 **Magnetic force microscopy.** The MFM experiments were carried out in a homemade cryogenic
328 atomic force microscope (AFM) using commercial piezoresistive cantilevers (spring constant
329 $k \approx 3$ N/m, resonant frequency $f_0 \approx 42$ kHz). The homemade AFM is interfaced with a Nanonis SPM
330 Controller (SPECS) and a commercial phase-lock loop (OC4)³⁹. MFM tips were prepared by depositing
331 nominally 100 nm Co film onto bare tips using e-beam evaporation. MFM images were taken in a
332 constant height mode with the scanning plane ~ 60 nm above the sample surface. MFM signal: the
333 change of cantilever resonant frequency, is proportional to out-of-plane stray field gradient⁴⁰.
334 Electrostatic interaction was minimized by nulling the tip-surface contact potential difference. Red
335 (blue) regions in MFM images represent up (down) ferromagnetic domains, where magnetizations
336 are parallel (anti-parallel) with the positive external field. Using the ImageJ software, all images were
337 binarised at the same threshold, starting from the same vertical scale. Then we used the particle
338 analysis module of this software to count the number of bubbles and estimate their size.

339 **Magneto-Optical Kerr measurements.** Magneto-optical experiments were done using light from a
340 150 W Xe arc lamp, which was dispersed by a monochromator, collimated, and then linearly
341 polarized, by the action of a Glan-Thompson prism, which was rotated by 45° with respect to the
342 modulator axis of a photoelastic modulator (PEM). After reflection on the sample surface, the light
343 goes towards an analyzer that can be set at two different angles with respect to the PEM axis,
344 namely, 0° and 90°, to record the magneto-optical signals of s- and p-polarized light, respectively. In
345 our experiments, Kerr ellipticity (ε) was measured in polar configuration with s- and p-polarized light
346 incident at angles close to the normal to the surface. The signal is collected from the detector and
347 brought to a lock-in amplifier synchronized to the frequency Ω of the PEM retardation angle. In this
348 optical arrangement, the ellipticity is given by $\varepsilon = \frac{1}{4cJ_1(\varphi_0)} \frac{I_\Omega}{I_0}$, where I_Ω is the first harmonic of the
349 intensity recorded at the detector and I_0 is the background intensity measured with a dc-multimeter.
350 The calibration constant c is determined experimentally.

351

352

353 **Author contributions**

354 MB proposed the study and supervised it with VG. LV, AS and QZ prepared the samples and
355 performed X-ray diffraction and atomic force microscopy. LV and AS performed the magnetic
356 characterization and magnetotransport experiments and analysed the data with MB and VG. BC, GH
357 and RC performed the magneto-optical Kerr effect measurements experiments. WWa and WWu
358 performed the MFM experiments and analysed them with VG and SF. SV, RA and EW performed X-
359 ray absorption spectroscopy experiments. KN and HK developed the theoretical model, with inputs
360 from JS, AB and MB. MB wrote the manuscript. All authors discussed the data and contributed to the
361 manuscript.

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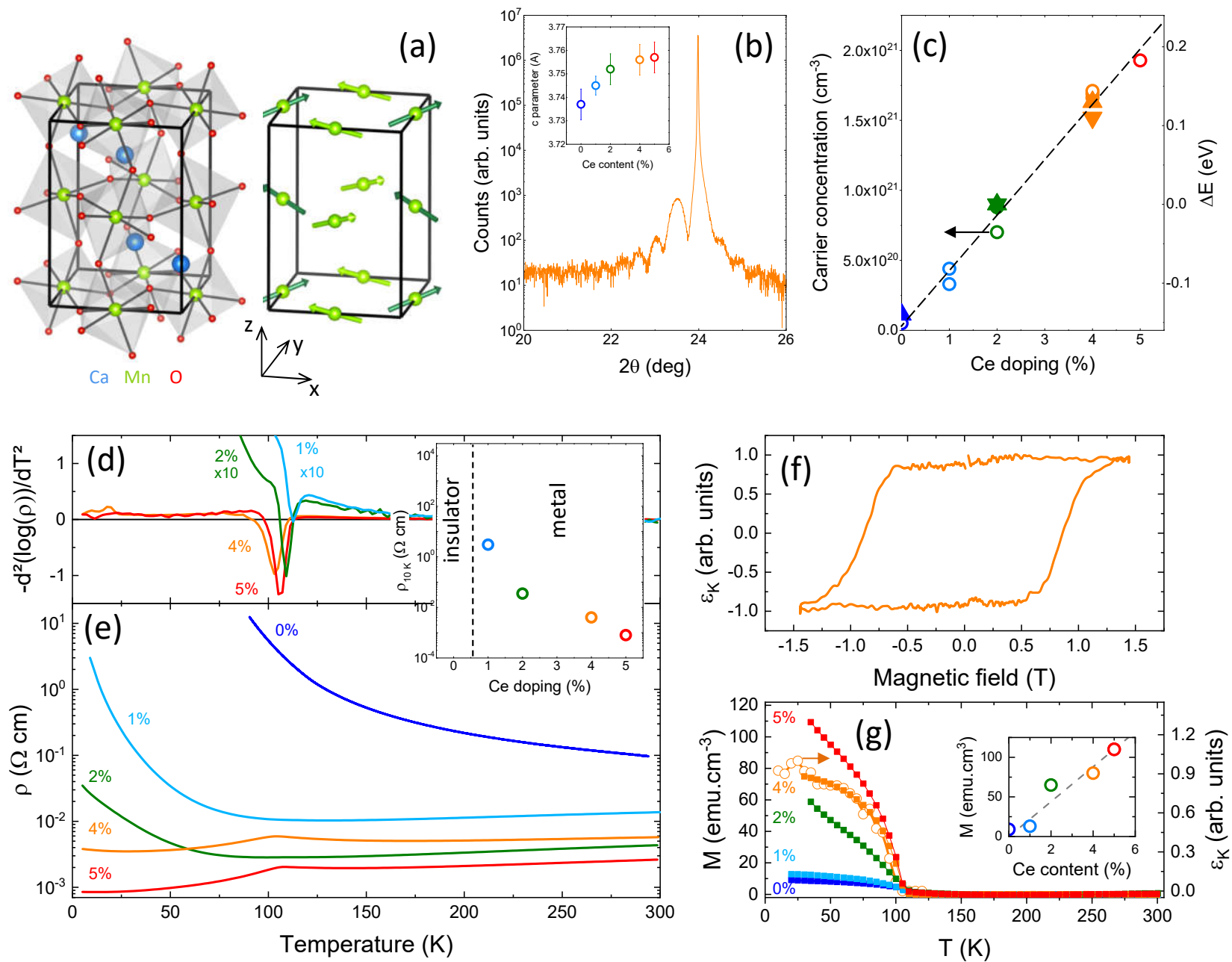


Fig. 1

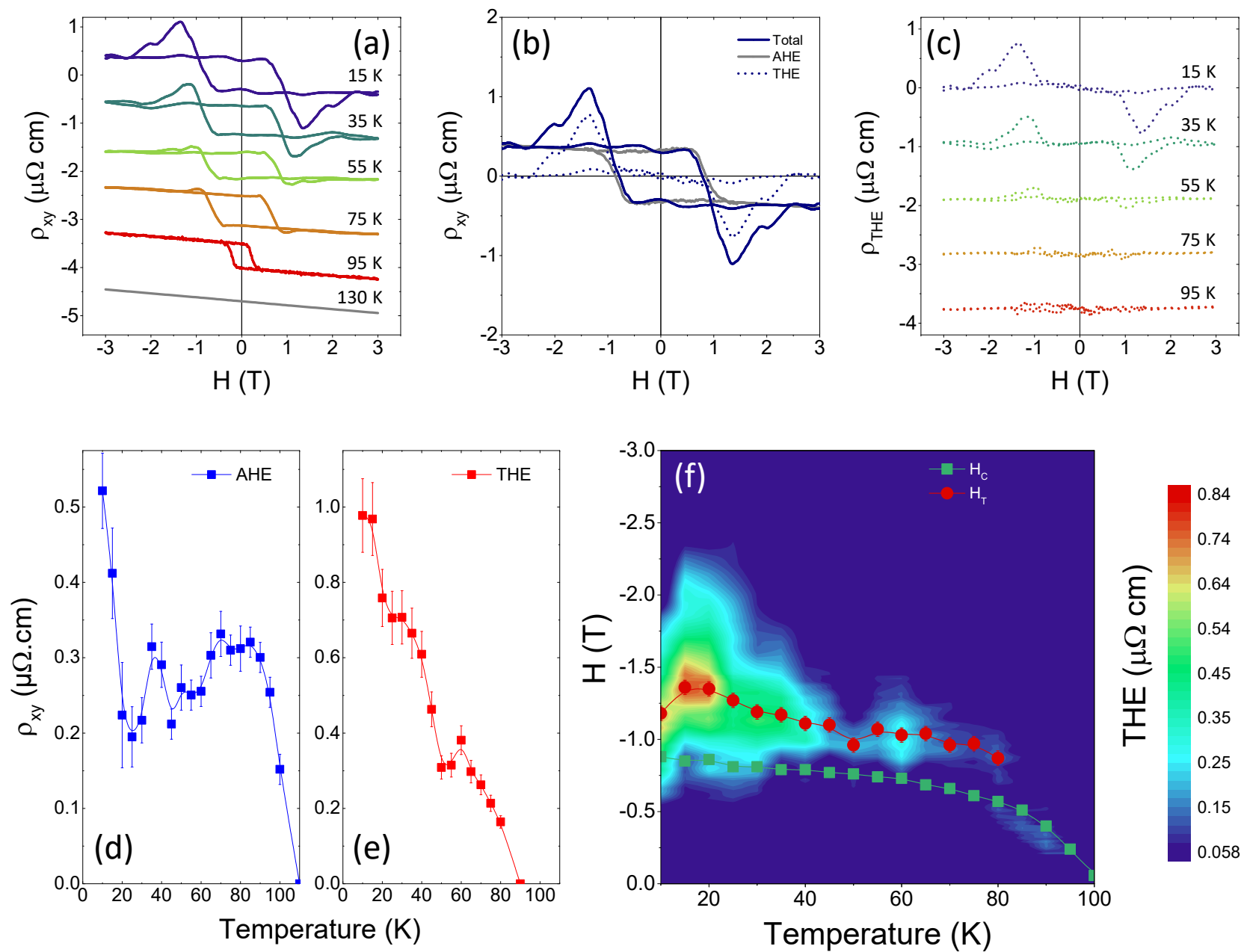


Fig. 2

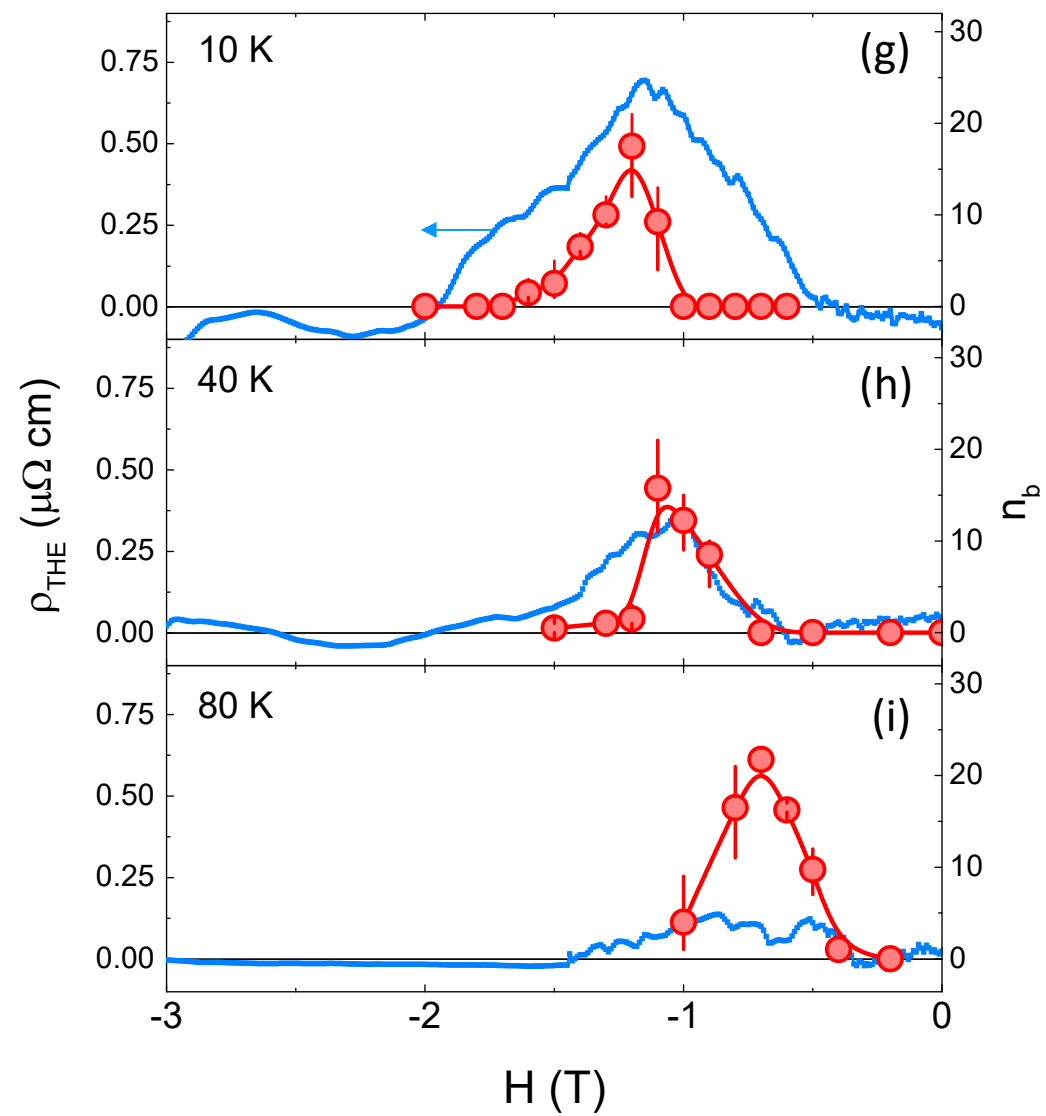
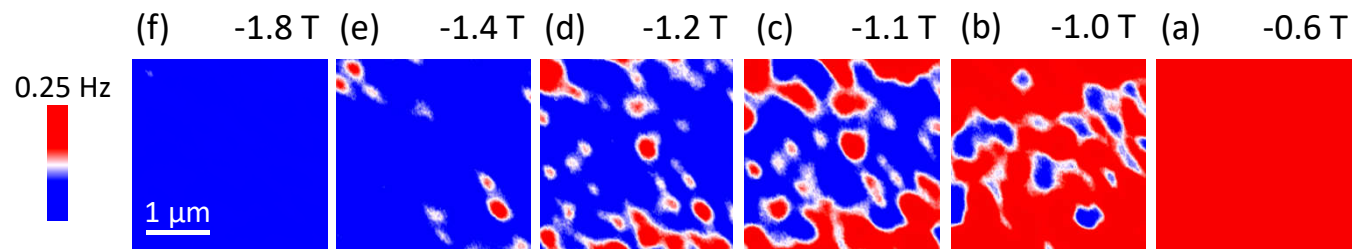


Fig. 3

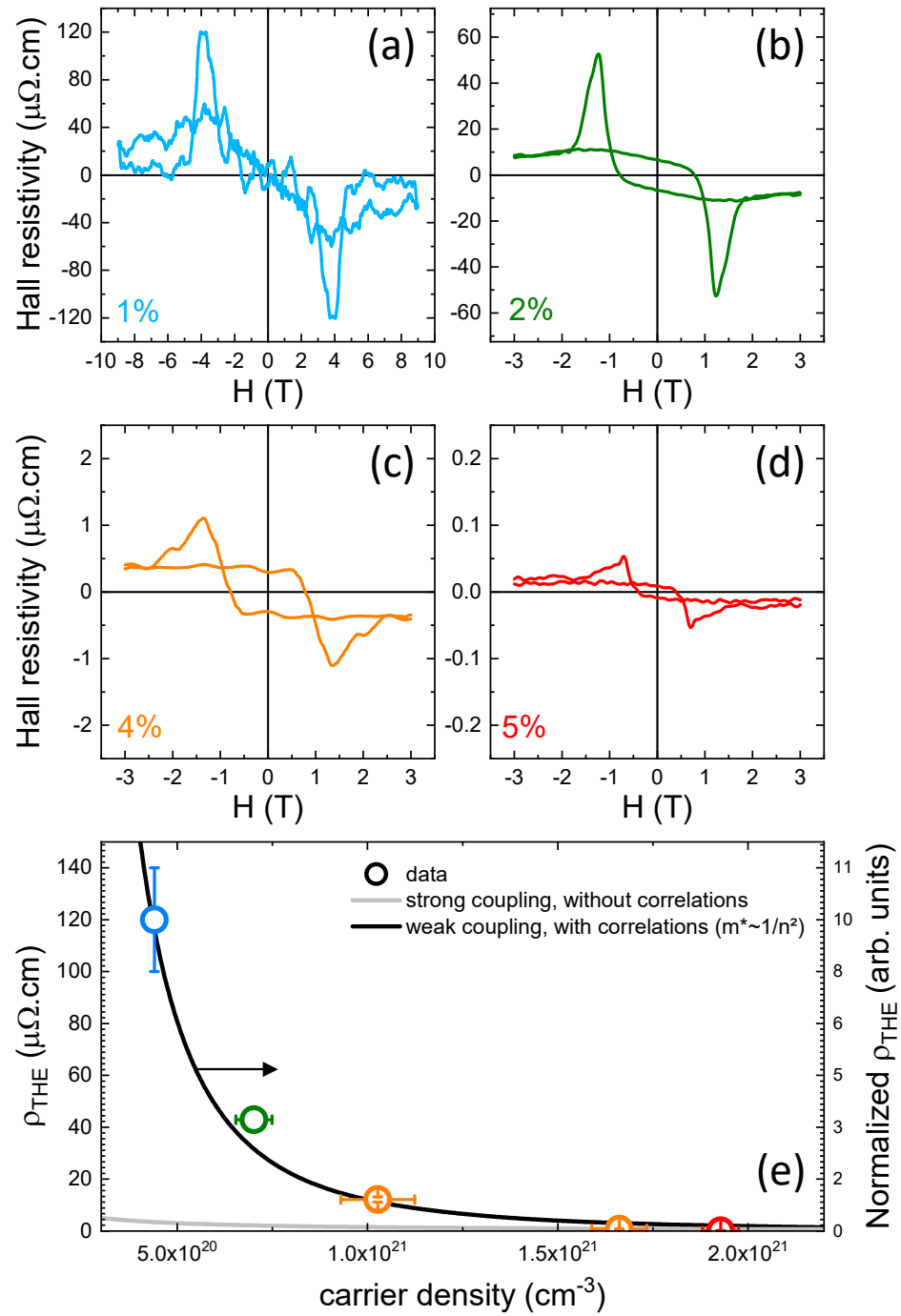


Fig. 4