Glaciohydraulic supercooling: a freeze-on mechanism to create stratified, debris-rich basal ice: II. Theory

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ABSTRACT. Simple theory supports field observations (Lawson and others, 1998) that subglacial water flow out of overdeepenings can cause accretion of layered, debris-bearing ice to the bases of glaciers. The large meltwater flux into a temperate glacier at the onset of summer melting can cause rapid water flow through expanded basal cavities or other flow paths. If that flow ascends a sufficiently steep slope out of an overdeepening, the water will supercool as the pressure-melting point rises, and basal-ice accretion will occur. Diurnal, occasional or annual fluctuations in water discharge will cause variations in accretion rate, debris content of accreted ice or subsequent diagenesis, producing layers. Under appropriate conditions, net accretion of debris-bearing basal ice will allow debris fluxes that are significant in the glacier sediment budget.

1. INTRODUCTION

It has long been evident, from theory and observation, that water flowing in conduits through or beneath a glacier will become supercooled if it ascends a sufficiently steep slope (e.g. Röthlisberger, 1968; Röthlisberger and Lang, 1987; Hooke and others, 1988). For example, ice has been observed to grow around meltwater streams emerging from beneath glaciers when air temperatures were above freezing (Röthlisberger and Lang, 1987; Strasser and others, 1992, 1996; Lawson and others, 1998). Instrumental observations have revealed supercooling in such streams and associated lakes. Frazil ice has been observed in such streams (Lawson, 1986; Strasser and others, 1992; Fleisher and others, 1993; Lawson and others, 1998), and in water rising in boreholes (Hooke and Pohjola, 1994).

Ordinarily, one would expect water to flow to and along the base of a temperate glacier (Shreve, 1972). However, basal water flowing in a channel up a sufficiently steep bed slope from an overdeepening will supercool and grow ice on the channel walls, tending to plug the channel (Röthlisberger, 1968; Röthlisberger and Lang, 1987; Hooke and Pohjola, 1994). A logical "extrapolation" of the Röthlisberger (1972) theory would allow freezing to be balanced by channel expansion caused by water pressure in excess of ice pressure, but water under super-flotation pressures would be forced out of the conduit along the ice–bed interface (personal communication from R. LeB. Hooke, 1997). Much of the water flow through overdeepenings thus is diverted from central basal channels into englacial conduits lacking the steep rise of the bedrock (Hooke and others, 1988; Hooke and Pohjola, 1994), into basal channels that run close enough to the sides of the glacier to avoid the overdeepening (Hantz and Lliboutry, 1983; Lliboutry, 1983), or into cavities or films (Röthlisberger, 1968).

Lawson and others (1998; also Lawson and Kulla, 1978; Strasser and others, 1992, 1996) present evidence that thick sequences of laminated ice are accreting to the base of Matanuska Glacier, Alaska, U.S.A., from a water system flowing out of an overdeepening. The intricate layering of the accreted ice (typically comprising layers much longer and wider than they are thick, though with occasional channel-form structures with smaller aspect ratios typically 1.5–2) suggests accretion not primarily from channels but from linked cavities, "canals", or other distributed elements of the subglacial water system.

The data of Hooke and Pohjola (1994) from the overdeepening of Storglaciären in Sweden show that at least some basal water flow does occur in a high-pressure, distributed system between the ice and till. Such flow rising from an overdeepening can produce supercooling and ice growth. The high water pressure can maintain open cavities, canals or films between the ice and its bed despite ice growing into it (e.g. Iken and others, 1983; Iken and Bindschadler, 1986; Alley, 1996). We suggest that within the range of natural glaciers, and specifically beneath Matanuska Glacier, basal freeze-on from distributed water systems occurs, entrains debris and may contribute significantly to sediment budgets.

2. THEORY

2.1. Fundamentals

We review the simplest version of the problem here; for more-complete and more-elegant treatments of the physics and thermodynamics of subglacial water flow, the reader is

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referred to Weertman (1972), Nye (1976), Hooke (1984), Röthlisberger and Lang (1987) or Lawson (1993), among other excellent sources. We restrict our calculations to temperate glaciers, those at the pressure-melting temperature throughout; a brief discussion of this mechanism beneath cold glaciers is given by Alley and others (1997). Readers who already are convinced that widespread waters flowing up a sloping bed can supercool, even after allowance for geothermal fluxes and heat of sliding, should skip directly to section 2.2. We first state the model in text, and then using equations.

The melting point of ice increases with decreasing pressure. If we assume that subglacial water is in thermal equilibrium with a temperate glacier, then the water must warm as it moves from a region of high pressure to one of lower pressure. Based on Hooke and others (1988) and Hooke and Pohjola (1994), the basal water pressure is likely to be close to the static ice-overburden pressure in an overdeepening, so water must warm as it flows up the adverse slope of an overdeepening. Possible heat sources include the viscous dissipation of the water flow (the work done to move the water, assuming no acceleration), the geothermal flux, the heat of sliding (the work done to move the ice, assuming no acceleration) and latent heat if some of the water freezes.

Depending on the air content of the water, the heat needed to warm the water as its pressure decreases equals the heat dissipated viscously by the water flow if the bed slope is about 20–70% steeper than the ice-surface slope and in the opposite direction. For a steeper bed (up to about 11 times the magnitude of the ice-surface slope, beyond which the flow direction of the water reverses), additional geothermal, sliding, or latent heat is needed to maintain the water at the pressure-melting point.

For all but the smallest water fluxes up the adverse slope of a sufficiently steep overdeepening, the geothermal heat and heat from sliding will not be enough to maintain the water at the pressure-melting point. Then the water will begin to supercool. Because the water is in intimate contact with ice, supercooling should be minimal and ice growth should occur, releasing the latent heat to maintain the water close to the melting temperature.

For a numerical statement of this, refer to the simplified, two-dimensional case shown in Figure 1. The x axis is taken as horizontal and increasing in the direction of ice and water flow. The z axis is vertical and increasing upwards. The ice surface is $z = z_s$ and the bed is $z = z_b$. We consider only those cases with $(\partial z_s)/(\partial x) \equiv \alpha_s < 0$ and $(\partial z_b)/(\partial x) \equiv \alpha_b > 0$, as shown in Figure 1. Thus, we consider flow out of an overdeepening only.

Suppose that the average water flux per unit of glacier width in a subglacial system is $Q\,\mathrm{m^3\,s^{-1}\,m^{-1}}$ (ignoring all details of how this is distributed). Suppose further that the water pressure, P_w, falls below the ice-overburden pressure, $P_i = \rho_i g h_i$, by a constant amount ΔP, in which $g = 9.8\,\mathrm{m\,s^{-2}}$ is the gravitational acceleration, $h_i \equiv z_s - z_b$ is the ice thickness, and ρ_i is the density of ice. We thus consider static loads only, and not dynamic pressure variations associated with flow over riegels or with other flow perturbations. The potential of subglacial water, Φ, then is

$$\Phi = \Phi_0 + \rho_w g z_b + \rho_i g (z_s - z_b) - \Delta P\,, \qquad (1)$$

in which Φ_0 is some arbitrary reference potential and ρ_w is the density of water. (Alternatively, we could lump ΔP into

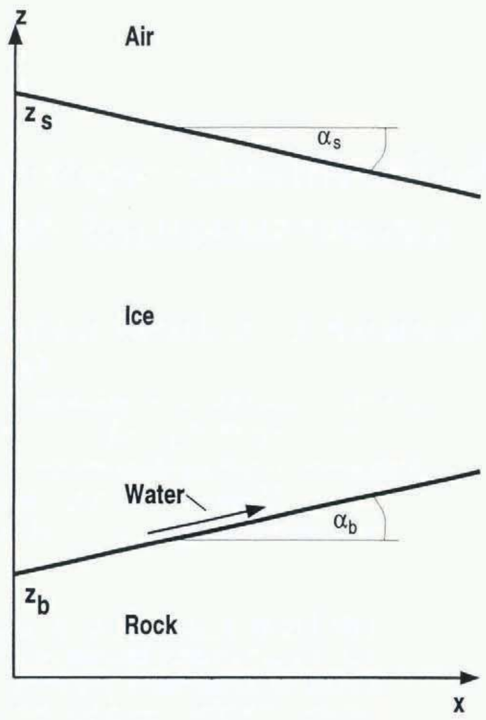

Fig. 1. Coordinate system used.

the reference potential.) Water flows down the potential gradient, which in our simplified, one-dimensional glacier is

$$\frac{\partial \Phi}{\partial x} = \rho_i g \alpha_s + (\rho_w - \rho_i) g \alpha_b\,. \qquad (2)$$

The heat generation from water flow, H_w, is the work done per unit time per unit area to move the water, given by

$$H_w = -Q \frac{\partial \Phi}{\partial x} \qquad (3)$$

in which Q again is the water flux and we have made the usual assumption that the water flow is not accelerating (e.g. Weertman, 1972; Röthlisberger and Lang, 1987). Additional heat sources are the geothermal flux, H_g, and the heat of sliding, H_s, which are discussed in more detail below.

These heats are available to warm the water and maintain it at the pressure-melting point, and to melt basal ice or, if the melt rate is negative, to allow freeze-on. The heat per unit area, H_{pmp}, needed to maintain water at the pressure-melting point with a water flux Q is

$$H_{pmp} = Q \frac{\partial P_w}{\partial x} \beta C_s \qquad (4)$$

in which C_s is the volumetric specific heat of water, $\beta \equiv (\partial T_m)/(\partial P_w)$ is the pressure dependence of the melting point T_m, and

$$\frac{\partial P_w}{\partial x} = \rho_i g (\alpha_s - \alpha_b)\,. \qquad (5)$$

Some uncertainty is attached to β. Röthlisberger (1972) and Röthlisberger and Lang (1987), among others, favored using the value for pure water, β_p, whereas Lliboutry (1983), Hooke (1991) and others have favored the larger-magnitude value for air-saturated water, β_a. We carry out calculations for both.

Balancing these heats, a layer of basal ice thickens at the freezing rate, $\dot{f}$, given by

$$\dot{f} = \frac{(H_{pmp} - H_w - H_g - H_s)\cos\alpha_b}{L} \qquad (6)$$

in which L is the volumetric heat of fusion of ice, H_g is the geothermal heat and H_s is the heat of sliding. The factor $\cos\alpha_b$ is needed in Equation (6) because $\dot{f}$ is measured normal to the sloping bed of the glacier but the other calculations are for the horizontal–vertical x–z coordinate system defined in Figure 1; however, for small α_b typical of glaciers, this factor is nearly one and can be omitted. Note that we continue to consider averages over the glacier sole. Below, we discuss the effects of spatial concentration of water flow.

Of the work done in sliding, perhaps one-third may be used to produce new mineral surface during abrasion of subglacial or englacial till or rock (Metcalf, 1979), and a much smaller amount may be used to produce defects during deformation of any new ice accreted to the glacier bed. The remainder is dissipated as heat if no accelerations occur. The work done per unit area in sliding is $\tau_b u_s$, in which u_s is the sliding velocity and τ_b is the basal drag. For simplicity, we ignore abrasion and defect creation, and write

$$H_s = \tau_b u_s \,, \tag{7}$$

which should overestimate the heat of sliding and thus underestimate the rate of basal ice accretion. Accreted ice may be melted internally by the heat of deformation; the conservative way to deal with this is to follow Robin (1955) and call u_s the surface velocity.

We now can rewrite Equation (6) by substituting for H_{pmp} from Equations (4) and (5), H_w from Equations (2) and (3), and H_s from Equation (7). This yields

$$\dot{f} = \Big\{Qg[\rho_i K\alpha_s + (\rho_w - \rho_i K)\alpha_b] - H_g - \tau_b u_s\Big\}\cos\alpha_b/L; \quad K \equiv 1 + \beta C_s \,. \tag{8}$$

Using the values of constants listed in Table 1, this becomes

$$\dot{f} = 0.103\cos\alpha_b\big[Q(5282\alpha_s + 4518\alpha_b) - H_g - 3.17\times10^{-8}\tau_b u_s\big]; \quad \beta = \beta_a \tag{9a}$$

$$\dot{f} = 0.103\cos\alpha_b\big[Q(6184\alpha_s + 3616\alpha_b) - H_g - 3.17\times10^{-8}\tau_b u_s\big]; \quad \beta = \beta_p \tag{9b}$$

in which Q is in $m^3\,s^{-1}\,m^{-1}$, H_g in $W\,m^{-2}$, τ_b in Pa, and $\dot{f}$ and u_s in $m\,a^{-1}$.

Note that the water-flow mechanism described here does not produce net accretion to the glacier as a whole. The descent of water into the overdeepening melts more ice than is frozen back on during water ascent from the overdeepening. However, much of the ice melted may be relatively clean (especially that around moulins descending through the glacier to the bed), whereas dirty basal ice is accreted (Strasser and others, 1996), so net addition of debris to the glacier is likely. Further, melting and freezing are spatially separated at various scales, so that some regions of the glacier will experience a net accretion of basal ice.

2.2. Example

If we set $H_g = H_s = 0$, an appropriate approximation for rapid water flow in channels, Equations (9) yield $\dot{f} = 0$ for $\alpha_b = -1.2\alpha_s$, $\beta = \beta_a$, and $\alpha_b = -1.7\alpha_s$, $\beta = \beta_p$, the well-known result from previous workers (e.g. Röthlisberger and Lang, 1987; Hooke, 1991). Rapid water flow in any channel with an upward slope >1.2–1.7 times the downward surface slope should cause supercooling and ice accretion in the channel. Such a channel rising out of an overdeepening would be expected to cease to function efficiently as it is partially or completely clogged with ice, so water flow in channels across overdeepenings is likely to occur englacially with a gradient that does not lead to freezing (Hooke and others, 1988; Hooke, 1991).

If we assume that the onset of summer melt causes a large, if non-steady, water flux up the adverse slope of an overdeepening, we can substitute likely values of H_g, $\tau_b u_s$, Q, α_s and α_b into Equations (9) and calculate net accretion of basal ice. Typical summertime water fluxes per unit width, averaged across the glacier width, are 10^{-2}–10^{-3} $m^3\,s^{-1}\,m^{-1}$ on Findelengletscher, Switzerland (Iken and Bindschadler, 1986), which apparently routes much of its meltwater through basal cavities at the onset of summer melting. Water fluxes approach $10^{-1}\,m^3\,s^{-1}\,m^{-1}$ on Columbia Glacier, Alaska, although the water paths are less well known there (personal communication from R. A. Walters and others, 1986). Data from Matanuska Glacier suggest that its summertime drainage may be $\geq 10^{-1}\,m^3\,s^{-1}\,m^{-1}$.

We have calculated basal freeze-on for a possible overdeepened glacier with characteristics listed in Table 1. Water flux through a basal cavity system will vary on annual and shorter time-scales; for simplicity, we assume that water flows through basal cavities from the overdeepening for a fraction of a year, t_m, and that no water flows out of the overdeepening for the rest of the year, $1 - t_m$. The geothermal flux and heat of sliding will tend to cause basal melting all year, but freeze-on from supercooling of rising water will occur only during t_m.

Our example glacier has net freeze-on at a rate of 0.113 m

Table 1. Variables used, with units. Numerical values of physical constants follow Röthlisberger and Lang (1987). Numerical values of site-specific variables used in our "standard" calculation also are given

Constant	*Meaning*	*Value or units*
C_s	Water specific heat	$4.2\times10^6\,J\,m^{-3}\,K^{-1}$
D	Overdeepening maximum depth	m
$\dot{f}$	Ice freeze-on rate	$m\,a^{-1}$
g	Gravitational acceleration	$9.8\,m\,s^{-2}$
h_r	Regelation-layer thickness	m
H_g	Geothermal flux	$0.06\,W\,m^{-2}$
H_{pmp}	Heat to keep water flux at pressure-melting point	$W\,m^{-2}$
H_w	Heat generated from water viscous dissipation	$W\,m^{-2}$
K	$1 + \beta C_s$	
L	Ice fusion heat	$3.06\times10^8\,J\,m^{-3}$
$\dot{m}$	Basal melt rate	$m\,a^{-1}$
Q	Water flux	$0.01\,m^3\,s^{-1}\,m^{-1}$
R	Debris/ice volume ratio	
t_m	Fraction of year with cavity flow	0.1
u_s	Sliding velocity	$10\,m\,a^{-1}$
x	Horizontal coordinate	m
X	Overdeepening adverse-slope length	m
z	Vertical coordinate	m
z_b	Bed elevation	m
z_s	Ice-surface elevation	m
α_b	Bed slope	0.05
α_s	Surface slope	−0.01
β_a	Air-saturated melting-point pressure dependence	$-9.8\times10^{-8}\,K\,Pa^{-1}$
β_p	Pure melting-point pressure dependence	$-7.4\times10^{-8}\,K\,Pa^{-1}$
ΔP	Water pressure–ice pressure	Pa
ρ_i	Ice density	$916\,kg\,m^{-3}$
ρ_w	Water density	$1000\,kg\,m^{-3}$
τ_b	Basal shear stress	10^5 Pa
Φ	Water poential	Pa
Φ_0	Reference water potential	Pa

a^{-1} or 0.169 m a^{-1} (depending on β) during t_m, and melting at a rate of 9.43×10^{-3} m a^{-1} during $1-t_m$. For $t_m = 0.1$, this gives net accretion of ice averaged over the base of the glacier across the overdeepening of 2.8 or 8.4 mm a^{-1}, depending on β. The sensitivity of this calculation to variations in the important parameters is shown in Figure 2.

We expect that Matanuska Glacier is among those glaciers with prominent, rapid basal accretion. However, we believe that basal accretion is important on many glaciers, albeit often at slower rates. We thus chose model parameters for our example glacier to produce somewhat slower accretion than for Matanuska Glacier, although Figure 2 shows values more appropriate for Matanuska Glacier.

We used a bed slope α_b less than appropriate for Matanuska Glacier (Lawson and others, 1998). We used only about 10% of the estimated summer water flux from beneath Matanuska Glacier (Lawson and others, 1998), based on the arguments of Hooke and others (1988) and Hooke (1991) that not all of the water flow would occur subglacially across the overdeepening. We assumed $t_m = 0.1$, a low value considering that frazil is observed in discharge from beneath Matanuska Glacier throughout the months-long summer, and that some water discharge continues year-round as shown by aufeis growth in the winter. If all of the estimated water flux of Matanuska Glacier were routed up the bed from the model overdeepening, ice would accrete at 1.69 m a^{-1} during t_m, leading to 0.16 m a^{-1} net accretion if $t_m = 0.1$. For comparison, meters of ice have accreted to the base of Matanuska Glacier over decades (accretion ≥ 0.1 m a^{-1}), as shown by the presence of atomic-bomb tritium in 4–6 m thick sections of dirty ice in which evidence of significant tectonic thickening has not been found (Strasser and others, 1996; Lawson and others, 1998).

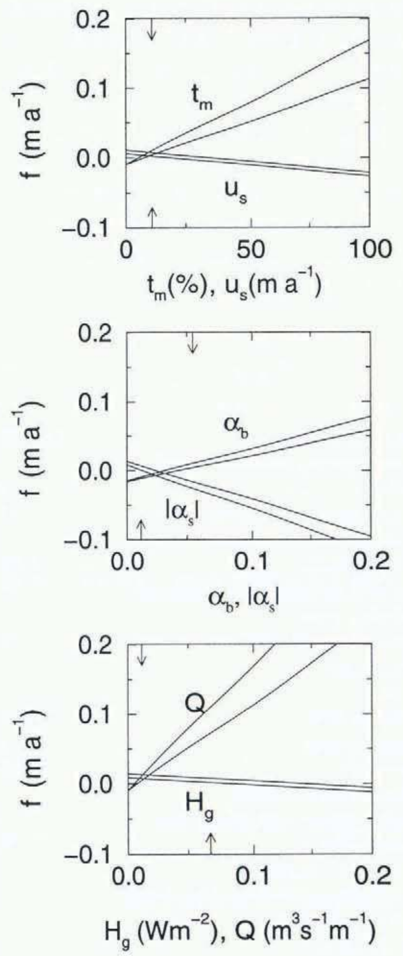

Fig. 2. Relation between average net freeze-on to the glacier sole, $\dot{f}$, and other variables. Each plot shows how varying one parameter from the "standard" calculation of Table 1 affects the amount of ice accreted during one year. In each case, the upper curve is for $\beta = \beta_a$ for air-saturated water, and the lower curve is for $\beta = \beta_p$ for pure water. Negative $\dot{f}$ values indicate melting. The value of each parameter in the standard calculation is indicated by the arrow closest to its label. The computation is especially sensitive to the cumulative water flux (Qt_m) and to the bed slope relative to the surface slope.

If an ice-contact water system ascending an overdeepening leaks downward to recharge a subglacial aquifer (cf. Boulton and Hindmarsh, 1987), some of the geothermal heat may be advected away by the groundwater flow and not reach the glacier bed. Also, if the adverse slope of the overdeepening is armored by a deforming till, some of the heat of sliding may be removed in the groundwater flow. Reduction of H_g and H_s by these mechanisms would increase ice accretion. Freeze-on also may occur slightly faster than we have calculated above, because the basal ice and its entrained debris will warm during flow from the overdeepening, absorbing some heat.

Accreted ice is likely to contain debris, as discussed below. If the debris/ice volume ratio is R with a net freeze-on rate of ice of $\dot{f}$, then the debris flux into the ice is $\dot{f}R$ per unit area, and the steady debris flux out of an overdeepening is $\dot{f}RX \approx \dot{f}RD/\alpha_b$ per unit width, where X is the distance from the deepest point to the down-glacier end of the overdeepening and D is the maximum depth. Returning to our example, suppose that $\beta = \beta_a$, giving a net basal freeze-on of about 8.4 mm a^{-1}, and suppose that the debris ratio $R = 0.1$, a low value for Matanuska Glacier basal ice (Lawson and others, 1998). Then the glacier will be entraining all of the erosion products from the area if the erosion rate is just less than 1 mm a^{-1}, a "typical" glacier erosion rate (e.g. Boulton, 1979). For a long or deep overdeepening, this may become a significant term in the sediment budget of the glacier. The order-of-magnitude larger basal accretion rate of Matanuska Glacier, together with its higher debris concentration, would allow its freeze-on to incorporate all of the erosion products from well beyond its overdeepening or else would support quite rapid erosion of the overdeepening.

Hooke (1991) demonstrated the likelihood that overdeepenings develop, in part, because sediment protects the adverse slope from erosion while the headwall is eroded rapidly. Our results are not in conflict with this model. The adverse slope likely will reduce the sediment transport capacity of streams, causing deposition equal to or in excess of removal in basal ice (see review by Alley and others, 1997).

3. APPEARANCE OF BASALLY ACCRETED ICE

The calculations above are for spatial averages across a glacier. These cannot be accurate in detail; at any time there must be flow paths carrying more water than the spatial average, separated by regions with little or no water flow in which the heat of sliding is dissipated. In the water flow paths, ice will grow more rapidly than the averages calculated above, while melting may occur in the regions between the flow paths. We now consider how the geometry of the water system would affect the appearance of the accreted ice.

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We model ice growth as occurring primarily from a distributed water system, a term we use to indicate water occupying much more of the bed than is typical for Röthlisberger channels. We assume (cf. Iken and others, 1983; Iken and Bindschadler, 1986) that the distributed water system spreads and thickens when water pressures are raised by increased water supply to the glacier, so that the drainage paths are not plugged by ice growth. Sufficient distributed flow to allow significant ice accretion is likely in linked cavities over bedrock (Walder, 1986; Kamb, 1987), with flow paths of typical dimensions 1–10 m laterally and 0.1 m vertically (Walder and Hallet, 1979). Cavities are also known to develop down-glacier of large clasts projecting from till under at least some circumstances (Boulton, 1976), although soft till is unlikely to produce cavities as large as those typical of bedrock beds. Drainage over till may be dominated by flow in broad, shallow, high-pressure, possibly braided channels in which water pressure increases with flux (the "canals" of Walder and Fowler, 1994). Transverse and vertical dimensions of these flow paths are likely to be of the same magnitudes as for bedrock-floored cavities, and longitudinal dimensions controlled by braiding might be similar to those for cavities.

Ice growth in a linked-cavity or other distributed system would probably include epitaxial growth on the roof; nucleation there would be easy, and very little supercooling would occur. However, formation of true frazil ice within the cavity or of anchor ice on or in the cavity floor is also likely. Anchor ice that floated to the top of a cavity and became incorporated in the cavity roof might add some sediment to the glacier. In situ anchor ice could be added to the glacier by cavity closure during falling water pressures. Frazil ice incorporated in the roof of a cavity, or dendritic epitaxial growth, might form a sort of meshwork that would trap water-borne sediment (Hubbard, 1991; Strasser and others, 1992; Lawson and others, 1998). Thus, we would expect accreted ice to contain debris (Lawson and others, 1998).

If most of the meltwater from Matanuska Glacier is routed through subglacial cavities, then accretion rates could be rapid enough that ice accreted during diurnal or storm-induced high-discharge periods would form a unit visible to the naked eye. Variations in debris content might occur, related to changes in suspended-sediment concentration in the water or to changes in sediment-trapping efficiency in response to evolution of the dendritic nature of the growing interface, producing layering at a sub-annual scale.

Basally accreted ice from a cavity would probably have its debris content altered, and usually increased, during regelation in a region of intimate ice–bed contact between cavities. Regelation ice is known to incorporate several per cent of debris by volume (Sugden and John, 1976, p. 61), in a layer typically ≈10 mm thick (Kamb and LaChapelle, 1964; Weertman, 1964). Regelation across the bed following accretion of dirty basal ice might increase or decrease the debris concentration of that ice, depending on details of debris entrainment and release. Net melting of ice will tend to concentrate the contained debris if clasts in contact with the bed regelate upward as the ice around them melts (Hallet, 1979). Regelation of ice into subglacial sediments between cavities also would allow debris entrainment (Iverson, 1993; Iverson and Semmens, 1995), and would produce clast-supported debris and thus quite high debris concentrations if active.

If regelation affects debris concentrations and the ice accreted during passage across a cavity is thicker than the regelation layer formed between cavities, then repeated passage of ice from cavity to inter-cavity region to cavity would form layers of ice with alternating higher and lower debris concentrations, with dimensions of the layering controlled by cavity sizes and spacings. If the accreted ice is thinner than the regelation layer, then all of the accreted ice should be altered and perhaps homogenized by regelation during passage across the inter-cavity regions.

Such processes probably are not especially important beneath Matanuska Glacier, because ice accretion appears to be so rapid that post-depositional modification would not affect much of the accreted ice; sub-annual features associated with varying accretion conditions probably dominate the appearance of the basal ice of Matanuska Glacier. Post-accretion modification would be more important if accretion were slower or occurred farther up-glacier, as is likely beneath some glaciers.

We have constructed a simple model for basal-ice accretion, to simulate how accreted ice layers might appear beneath a glacier with significant post-accretion diagenesis. We follow a length of ice for many years as it moves across a bed containing cavities of fixed length at random locations. For plotting convenience, we ignore the straining of this ice, based on the assumption that melting and freezing are rapid compared to changes in geometry caused by strain rates. During each annual cycle, ice is accreted to the glacier bed on cavity (or canal) roofs at a specified rate $\dot{f}$ for fraction of the year t_{m}, and melting occurs at a specified rate $\dot{m}$ for time $1 - t_{\mathrm{m}}$ at cavity locations and for the entire year between the cavities. We keep track of three ice types:

- glacier ice, the ice present at the start of the experiment;
- accretion ice, the ice that grew on a cavity roof and never has been in a regelation layer; and
- regelation ice, any ice that has been within distance h_{r} of the glacier bed between cavities, with h_{r}, the thickness of the regelation layer, specified.

The active regelation layer maintains a constant thickness and migrates upwards into glacier ice, accretion ice or older regelation ice at the melting rate $\dot{m}$ due to geothermal heat and heat from sliding.

Results of one simulation are shown in Figure 3, which bears at least a qualitative resemblance to basal ice of some glaciers (e.g. Lawson and others, 1998). Varying the model inputs produces differing appearances, but a wide range of simulations yields layers a few millimeters thick with lateral dimensions related to the cavity size. In the non-physical limit of cavities occupying the entire bed for t_{m} of each year and disappearing during the rest of the year, the layering becomes laterally continuous and an inversely varved sequence can be produced.

Clearly, ice accreted to a glacier will be altered by tectonic or other processes over time, producing changes in fabric, continuity of layers, and so forth (Strasser and others, 1994). Heat dissipated from deformation in temperate basal ice would be expected to produce meltwater that would escape along grain boundaries, increasing the concentration of debris in the remaining ice (Weertman, 1972; Hallet, 1979).

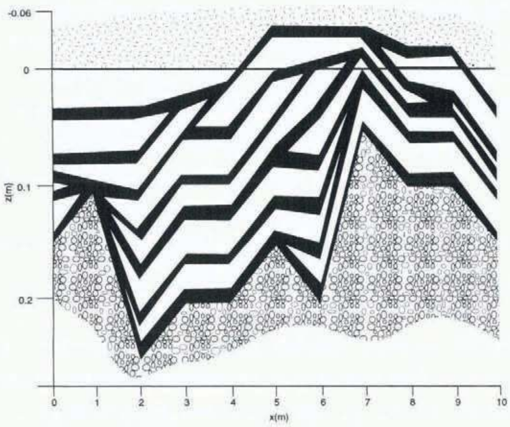

Fig. 3. Longitudinal section through basal stratigraphy generated with the model described in the text. Glacier ice (top) and bed material (bottom) are separated by regelation ice (dark) and accretion ice not affected by regelation (white). The base of the glacier began at $z = 0$ at the onset of the first melt season, and moved down (to positive numbers) with net accretion during the 10 year simulation; note that the coordinate system is fixed with respect to the original glacier ice, which in reality moves up during cavity opening. Calculations were conducted at ten equally spaced gridpoints; the eleventh gridpoint plotted is taken to be the same as the first. Quantities used include: $\dot{f} = 500$ mm a^{-1} for $t_m = 0.1$ over a 5 m long cavity placed randomly, with one cavity occurring beneath the 10 m long section each year. Melting occurs at a rate $\dot{m}$ of 10 mm a^{-1} wherever cavities are absent. The cavity portion of the model domain experiences a net addition of 41 mm a^{-1} (freeze-on of 50 mm in 0.1 a, followed by melt-off of 9 mm during the other 0.9 a), and the non-cavity portion of the model domain experiences net melting of 10 mm a^{-1}; the average accretion over the whole model domain is 31 mm a^{-1}. The regelation-layer thickness, h_r, is 10 mm.

4. OTHER MECHANISMS OF GLACIO-HYDRAULIC SUPERCOOLING

As noted by Lawson and others (1998), the complex subglacial environment may produce regions of decreasing water pressure and net freeze-on even though the average behavior over the entire glacier bed produces melting. Any pressure drop can lead to supercooling if other heat sources (geothermal, sliding, viscous dissipation in water) are not sufficiently large. For rapid water flows, viscous dissipation in the water is the dominant heat source. It alone will prevent supercooling if water pressure drops because of potential-gradient-driven flow and if the work done on the water produces heat through viscous dissipation. However, supercooling is possible if some of the pressure drop occurs because of flow up a sloping bed or because of overburden removal, or if some of the work on the water produces acceleration rather than viscous dissipation.

Situations that might produce pressure drops with supercooling include glacier calving events that remove some grounded ice (Lawson and others, 1998), or basal pressure changes related to glaciohydraulic jacking (Murray and Clarke, 1995) or ice flow across bumpy beds (Robin, 1976; Lliboutry, 1993). However, these are generally not expected to produce significant basal layers. The Lliboutry (1993) mechanism, which invokes regelation water flow through ice rather than between ice and rock, produces a steady thickness of only 0.02–0.04 m of basal ice if active. Calving is largely a grounding-line, one-time event for any piece of ice. A calving-induced pressure reduction of 10^6 Pa on a 0.1 m thick cavity would cause freeze-on of only about 0.1 mm of ice. Water-pressure reduction at one site on the glacier bed owing to stress redistribution in response to water-pressure increase nearby (Robin, 1976; Murray and Clarke, 1995) could create similar freeze-on, but the process is reversed during subsequent water-pressure increase. Ice flow from such a site might temporarily preserve a little accretion ice, but is unlikely to lead to significant net accretion. Similarly, the Robin (1976) mechanism (see also Paterson, 1981, p. 121) is one of local freeze-on in a bed that is, on average, melting, and thus is not expected to produce net accretion. Water flowing up a valley-side slope from a moulin-fed channel may grow basal ice, but preservation of this ice would require that the water not subsequently flow back down the same slope.

5. DISCUSSION AND CONCLUSIONS

The subglacial environment is complex. A variety of factors, including disequilibrium, groundwater flow, impurities, water mixing, gradients in shear stress affecting water potential (Robin and Weertman, 1973) and others, could be invoked to complicate the simple analysis here. However, it is quite clear that a large enough flow of subglacial or englacial water rising at a sufficiently steep angle will supercool and grow ice.

Overdeepenings are common in the glacial environment (Hooke, 1991). Channels carried into an overdeepening by ice flow are likely to be narrowed or clogged by rapid ice accretion. Broader areas of the bed on the adverse slope of an overdeepening may be affected by accretion from a distributed water system such as subglacial cavities or wide, low "canals" (Walder and Fowler, 1994). Some subglacial water flow has been documented across overdeepenings (Hooke and Pohjola, 1994). If such flow occurs at sufficiently high volumes for enough of the year, then our calculations show that at least some glaciers will have net basal accretion of ice from this mechanism in overdeepened regions.

Accreted ice entrains debris by various mechanisms (Strasser and others, 1996; Lawson and others, 1998). Layering defined by changes in debris content is likely, owing to time and space variations in accretion rates as well as to post-accretion changes. The typical spatial scale of variation in the water system (order of meters laterally) is likely to occur in layering of the accreted ice. A simple model based on these ideas produces basal stratigraphy that is qualitatively similar to that observed beneath some glaciers. If regelation affects accreted ice, isotopic differences between accretion ice and regelation ice may occur if there is loss or addition of water (Souchez and Lorrain, 1991), and serve to help test our model.

The general mechanisms and rates of debris entrainment by glaciers are not fully understood. This hampers our interpretation of glacial geology and glacier dynamics (cf. Beget, 1986; Alley, 1991; Hooke, 1991). The evidence that basal accretion from rising supercooled water occurs beneath at least one glacier (Strasser and others, 1992, 1996; Lawson and others, 1998), and the probability that this observation is not unique, may allow glaciological models to estimate one term in the glacial debris flux with greater accuracy than was previously possible.

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In summary, we are faced with strong observational evidence of basal ice accretion to a temperate glacier (Lawson and others, 1998). To explain this, we have been led to combine two classic results from glacier hydrology: that rising water can become supercooled, and that large inputs of surface-derived water to the glacier plumbing system cause water to spread across the bed, providing space in which water can flow up the adverse slope of an overdeepening and grow ice. For plausible values of physical variables, these mechanisms lead to predictions of net basal ice accretion beneath Matanuska Glacier and other glaciers. Significant debris entrainment by the soles of some glaciers is likely by this mechanism. Spatial and temporal variations in accretion and post-accretion modification will produce stratification in this accreted ice.

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REFERENCES

Alley, R. B. 1991. Deforming-bed origin for southern Laurentide till sheets? *J. Glaciol.*, **37**(125), 67–76.

Alley, R. B. 1996. Towards a hydrologic model for computerized ice-sheet simulations. *Hydrol. Processes*, **10**(4), 649–660.

Alley, R. B., K. M. Cuffey, E. B. Evenson, J. C. Strasser, D. E. Lawson and G. J. Larson. 1997. How glaciers entrain and transport sediment at their beds: physical constraints. *Quat. Sci. Rev.*, **16**, 1017–1038.

Beget, J. E. 1986. Modeling the influence of till rheology on the flow and profile of the Lake Michigan lobe, southern Laurentide ice sheet, U.S.A. *J. Glaciol.*, **32**(111), 235–241.

Boulton, G. S. 1976. The origin of glacially fluted surfaces — observations and theory. *J. Glaciol.*, **17**(76), 287–309.

Boulton, G. S. 1979. Processes of glacier erosion on different substrata. *J. Glaciol.*, **23**(89), 15–38.

Boulton, G. S. and R. C. A. Hindmarsh. 1987. Sediment deformation beneath glaciers: rheology and geological consequences. *J. Geophys. Res.*, **92**(B9), 9059–9082.

Fleisher, P. J., J. M. Franz and J. A. Gardner. 1993. Bathymetry and sedimentary environments in proglacial lakes at the eastern Bering Piedmont Glacier of Alaska. *J. Geol. Educ.*, **41**(3), 267–274.

Hallet, B. 1979. A theoretical model of glacial abrasion. *J. Glaciol.*, **23**(89), 39–50.

Hantz, D. and L. Lliboutry. 1983. Waterways, ice permeability at depth, and water pressures at Glacier d'Argentière, French Alps. *J. Glaciol.*, **29**(102), 227–239.

Hooke, R. LeB. 1984. On the role of mechanical energy in maintaining subglacial water conduits at atmospheric pressure. *J. Glaciol.*, **30**(105), 180–187.

Hooke, R. LeB. 1991. Positive feedbacks associated with erosion of glacial cirques and overdeepenings. *Geol. Soc. Am. Bull.*, **103**(8), 1104–1108.

Hooke, R. LeB. and V. A. Pohjola. 1994. Hydrology of a segment of a glacier situated in an overdeepening, Storglaciären, Sweden. *J. Glaciol.*, **40**(134), 140–148.

Hooke, R. LeB., S. B. Miller and J. Kohler. 1988. Character of the englacial and subglacial drainage system in the upper part of the ablation area of Storglaciären, Sweden. *J. Glaciol.*, **34**(117), 228–231.

Hubbard, B. 1991. Freezing-rate effects on the physical characteristics of basal ice formed by net adfreezing. *J. Glaciol.*, **37**(127), 339–347.

Iken, A. and R. A. Bindschadler. 1986. Combined measurements of subglacial water pressure and surface velocity of Findelengletscher, Switzerland: conclusions about drainage system and sliding mechanism. *J. Glaciol.*, **32**(110), 101–119.

Iken, A., H. Röthlisberger, A. Flotron and W. Haeberli. 1983. The uplift of Unteraargletscher at the beginning of the melt season — a consequence of water storage at the bed? *J. Glaciol.*, **29**(101), 28–47.

Iverson, N. R. 1993. Regelation of ice through debris at glacier beds: implications for sediment transport. *Geology*, **21**(6), 559–562.

Iverson, N. R. and D. J. Semmens. 1995. Intrusion of ice into porous media by regelation: a mechanism of sediment entrainment by glaciers. *J. Geophys. Res.*, **100**(B7), 10,219–10,230.

Kamb, B. 1987. Glacier surge mechanism based on linked cavity configuration of the basal water conduit system. *J. Geophys. Res.*, **92**(B9), 9083–9100.

Kamb, B. and E. LaChapelle. 1964. Direct observation of the mechanism of glacier sliding over bedrock. *J. Glaciol.*, **5**(38), 159–172.

Lawson, D. E. 1986. Observations on hydraulic and thermal conditions at the bed of Matanuska Glacier, Alaska. *Eidg. Tech. Hochschule, Zürich. Versuchsanst. Wasserbau, Hydrol. Glaziol. Mitt.* 90, 69–71.

Lawson, D. E. 1993. Glaciohydrologic and glaciohydraulic effects on runoff and sediment yield in glacierized basins. *CRREL Monogr.* 93-02.

Lawson, D. E. and J. B. Kulla. 1978. An oxygen isotope investigation of the origin of the basal zone of the Matanuska Glacier, Alaska. *J. Geol.*, **86**(6), 673–685.

Lawson, D. E., J. C. Strasser, E. B. Evenson, R. B. Alley, G. J. Larson and S. A. Arcone. 1998. Glaciohydraulic supercooling: a freeze-on mechanism to create stratified, debris-rich basal ice: I. Field evidence. *J. Glaciol.*, **44**(148), 547–562.

Lliboutry, L. 1983. Modifications to the theory of intraglacial waterways for the case of subglacial ones. *J. Glaciol.*, **29**(102), 216–226.

Lliboutry, L. 1993. Internal melting and ice accretion at the bottom of temperate glaciers. *J. Glaciol.*, **39**(131), 50–64.

Metcalf, R. C. 1979. Energy dissipation during subglacial abrasion at Nisqually Glacier, Washington, U.S.A. *J. Glaciol.*, **23**(89), 233–246.

Murray, T. and G. K. C. Clarke. 1995. Black-box modeling of the subglacial water system. *J. Geophys. Res.*, **100**(B7), 10,231–10,245.

Nye, J. F. 1976. Water flow in glaciers: jökulhlaups, tunnels and veins. *J. Glaciol.*, **17**(76), 181–207.

Paterson, W. S. B. 1981. *The physics of glaciers. Second edition.* Oxford, etc., Pergamon Press.

Robin, G. de Q. 1955. Ice movement and temperature distribution in glaciers and ice sheets. *J. Glaciol.*, **2**(18), 523–532.

Robin, G. de Q. 1976. Is the basal ice of a temperate glacier at the pressure melting point? *J. Glaciol.*, **16**(74), 183–196.

Robin, G. de Q. and J. Weertman. 1973. Cyclic surging of glaciers. *J. Glaciol.*, **12**(64), 3–18.

Röthlisberger, H. 1968. Erosive processes which are likely to accentuate or reduce the bottom relief of valley glaciers. *International Association of Scientific Hydrology Publication* 79 (*General Assembly of Bern 1967 — Snow and Ice*), 87–97.

Röthlisberger, H. 1972. Water pressure in intra- and subglacial channels. *J. Glaciol.*, **11**(62), 177–203.

Röthlisberger, H. and H. Lang. 1987. Glacial hydrology. *In* Gurnell, A. M. and M. J. Clark, *eds. Glacio-fluvial sediment transfer: an alpine perspective.* Chichester, etc., John Wiley and Sons, 207–284.

Shreve, R. L. 1972. Movement of water in glaciers. *J. Glaciol.*, **11**(62), 205–214.

Souchez, R. A. and R. D. Lorrain. 1991. *Ice composition and glacier dynamics.* New York, etc., Springer-Verlag. (Springer Series in Physical Environment 8.)

Strasser, E. B., D. E. Lawson, E. B. Evenson, J. C. Gosse and R. B. Alley. 1992. Frazil ice growth at the terminus of the Matanuska Glacier, Alaska, and its implications for sediment entrainment in glaciers and ice sheets. [Abstract.] *Geol. Soc. Am. Abstr. Programs*, **24**(3), 78.

Strasser, J. C., D. E. Lawson, E. B. Evenson and G. J. Larson. 1994. Crystallographic and mesoscale analyses of basal zone ice from the terminus of the Matanuska Glacier, Alaska: evidence for basal freeze-on in an open hydrologic system. [Abstract.] *Geol. Soc. Am. Abstr. Programs*, **26**(7), A177.

Strasser, J. C., D. E. Lawson, G. J. Larson, E. B. Evenson and R. B. Alley. 1996. Preliminary results of tritium analyses in basal ice, Matanuska Glacier, Alaska, U.S.A.: evidence for subglacial ice accretion. *Ann. Glaciol.*, **22**, 126–133.

Sugden, D. E. and B. S. John. 1976. *Glaciers and landscape: a geomorphological approach.* London, Edward Arnold.

Walder, J. S. 1986. Hydraulics of subglacial cavities. *J. Glaciol.*, **32**(112), 439–445.

Walder, J. S. and A. Fowler. 1994. Channelized subglacial drainage over a deformable bed. *J. Glaciol.*, **40**(134), 3–15.

Walder, J. and B. Hallet. 1979. Geometry of former subglacial water channels and cavities. *J. Glaciol.*, **23**(89), 335–346.

Weertman, J. 1964. The theory of glacier sliding. *J. Glaciol.*, **5**(39), 287–303.

Weertman, J. 1972. General theory of water flow at the base of a glacier or ice sheet. *Rev. Geophys. Space Phys.*, **10**(1), 287–333.

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