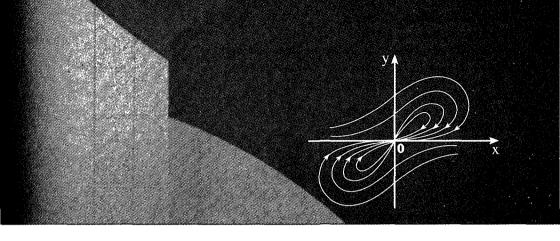
Interdisciplinary Mathematical Sciences – Vol.1



Global Attractors of Non-Autonomous Dissipative Dynamical Systems

David N Cheban

State University of Moldova, Moldova



Contents

vii

.

Preface

Notations			xxi
1.	Aut	onomous dynamical systems	1
	1.1	Some notions, notations and facts from theory of dynamical systems	1
	1.2	Limit properties of dynamical systems	8
	1.3	Center of Levinson	10
	1.4	Dissipative systems on the local compact spaces	16
	1.5	Criterions of compact dissipativity	18
	1.6	Local dissipative systems	25
	1.7	Global attractors	29
	1.8	On a Problem of J. Hale	33
	1.9	Connectedness of the Levinson's center	38
	1.10	Weak attractors and center of Levinson	43
	1.11	Asymptotic stability	47
2.	Non	-autonomous dissipative dynamical systems	53
	2.1	On the stability of Levinson's center	53
	2.2	The positively stable systems	60
	2.3	Behaviour of dissipative dynamical systems under homomorphisms .	64
	2.4	Non-autonomous dynamical systems with convergence	68
	2.5	Tests for convergence	79
	2.6	Global attractors of non-autonomous dynamical systems	89
	2.7	Global attractor of cocycles	93
	2.8	Global attractors of non-autonomous dynamical system with minimal	
		base	98
	2.9	Homogeneous dynamical systems	101

		Power-law asymptotic of homogeneous systems			
3.	Ana	lytic dissipative systems	119		
	3.1	Skew-product dynamical systems and cocycles	119		
	3.2	C-analytic systems	123		
	3.3	Converse of Lyapunov's theorem for C-analytic systems	128		
	-3.4				
	3.5	Dynamical systems in spaces of sections			
	3.6	Quasi-periodic solutions	139		
	3.7	The analogy of Cameron-Johnson's theorem	142		
	3.8	Almost periodic solutions of the weak nonlinear dissipative systems .	145		
4.	The	structure of the Levinson center of system with the condition			
	of tl	ne hyperbolicity	155		
	4.1	The chain recurrent motions	155		
	4.2	The spectral decomposition of the Levinson's center	157		
	4.3	One-dimensional systems with hyperbolic center	159		
	4.4	The dissipative cascades	164		
	4.5	The periodic dissipative systems	167		
5.	Me	thod of Lyapunov functions	171		
	5.1	Criterions of dissipativity in term of Lyapunov functions	171		
	5.2	Some criterions of dissipativity of differential equations	178		
	5.3	Theorem of Barbashin-Krasovskii for non-autonomous dynamical			
		systems	187		
	5.4	Equations with convergence	192		
	5.5	Dissipativity and convergence of some equations of 2nd and			
		3rd order	204		
	5.6	Construction of Lyapunov function for homogeneous systems \ldots .	208		
	5.7	Differentiable homogeneous systems	213		
	5.8	Global attractors of quasi-homogeneous systems	220		
6.	Diss	Dissipativity of some classes of equations 2			
	6.1	Difference equations	225		
	6.2	Equations with impulse	228		
	6.3	Convergent periodic equations with impulse	232		
	6.4	Asymptotic stability of linear functional differential equations $% \mathcal{A} = \mathcal{A} = \mathcal{A} = \mathcal{A}$	236		
	6.5	Convergence of monotone evolutionary equations $\ldots \ldots \ldots$	238		
	6.6	Global attractors of non-autonomous Lorenz systems	244		

•

.

Contents

		6.6.1	Non-autonomous Lorenz systems	245
		6.6.2	Non-autonomous dissipative dynamical systems and their	0.40
			attractors	248
		6.6.3	Almost periodic and recurrent solutions of non-autonomous	051
			Lorenz systems	
		6.6.4	Uniform averaging principle	253
		6.6.5	Global averaging principle for the non-autonomous Lorenz	950
			systems	259
7.	Upp	er sem	i-continuity of attractors	263
	7.1	Introd	$\operatorname{luction}$	263
	7.2	Maxin	nal compact invariant sets	263
	7.3	Upper	semi-continuity	266
	7.4	Conne	ectedness	272
	7.5	Applie	cations	274
		7.5.1	Quasi-homogeneous systems	274
		7.5.2	Monotone systems	276
		7.5.3	Quasi-linear systems	278
		7.5.4	Non-autonomously perturbed systems	279
		7.5.5	Non-autonomous 2D Navier-Stokes equations	280
		7.5.6	Quasi-linear functional-differential equations	284
8.	The	e relatio	onship between pullback, forward and global attractors	287
	8.1	Pullba	ack, forward and global attractors	288
	8.2		ptotic stability in α -condensing semi-dynamical systems	
	8.3	Unifo	rm pullback attractors and global attractors	299
	8.4	Exam	ples of uniform pullback attractors	300
		8.4.1	Periodic driving systems	300
		8.4.2	Pullback attractors with singleton component sets	302
		8.4.3	Distal dynamical systems	304
9.	Pu	llback	attractors of \mathbb{C} -analytic systems	307
	9.1	C-ana	lytic cocycles	309
	9.2		general facts about non-autonomous dynamical systems	
	9.3		vely uniformly stable cocycles	
	9.4		compact global pullback attractors of C-analytic cocycles with	
			act base	316
	9.5	-	uniform dissipative cocycles with noncompact base	319
	9.6		compact and local dissipative cocycles with noncompact base	323
			cations	326

.

	9.7.1	ODEs	326
	9.7.2	Caratheodory differential equations	328
	9.7.3	ODEs with impulses	329
10.	Pullback	attractors under discretization	331
	10.1 Non-a	utonomous dynamical systems and pullback attractors	333
		utonomous quasi-linear differential equation	
	10.3 Cocyc	ele property	338
	10.4 Main	result	341
		Existence of an absorbing set	341
	10.4.2	Upper semi-continuity of the pullback attractor component	
		sets	344
	10.4.3	Upper semi-continuous convergence of the discretized	
		pullback attractors	345
	10.4.4	Upper semi-continuous convergence of the discretized	
		global attractors	
	-	ton set-valued pullback attractor case	
	10.6 Apper	ndix: Proof of Lemma 10.4	353
11.	Global at	tractors of non-autonomous Navier-Stokes equations	357
	11.1 Non-a	utonomous Navier-Stokes equations	358
	11.2 Attra	ctors of non-autonomous dynamical systems	367
	11.3 Almos	st periodic and recurrent solutions of non-autonomous Navier-	
	Stoke	s equations	369
	11.4 Unifo	rm averaging for a finite interval	374
	11.5 The g	lobal averaging principle for Navier-Stokes equations	379
12.	Global at	tractors of V-monotone dynamical systems	385
	12.1 Globa	l attractors of V-monotone NDS	385
		e structure of Levinson center of V-monotone NDS	
	12.3 Almos	st periodic solutions of V-monotone systems	391
		ack attractors of V-monotone NDS	
	12.5 Appli	cations	395
		Finite-dimensional systems	395
		Caratheodory's differential equations	
		ODEs with impulse	
	12.5.4	Evolution equations with monotone operators	402
13.	Linear al	most periodic dynamical systems	407
	13.1 Bound	ded motions of linear systems	407

	13.2 Bounded solutions of linear equations	416
	13.3 Finite-dimensional systems	422
	13.4 Relationship between different types of stability	426
	13.5 Linear α -condensing systems	434
	13.6 Exponential stable systems	437
	13.7 Linear system with a minimal base	439
	13.8 Some classes of uniformly exponentially stable equations	441
	13.9 Linear periodic systems	450
	13.9.1 Exponential stable linear periodic dynamical systems	451
	13.9.2 Some classes of linear uniformly exponentially stable	
	periodic differential equations	456
14.	Triangular maps	461
	14.1 Triangular maps and non-autonomous dynamical systems	462
	14.2 Linear non-autonomous dynamical systems	463
	14.3 Quasi-linear non-autonomous dynamical systems	
	14.5 Quasi-finear non-autonomous dynamical systems	467
	14.4 Global attractors of quasi-linear triangular systems	
		469
	14.4 Global attractors of quasi-linear triangular systems	469 475
	14.4 Global attractors of quasi-linear triangular systems14.5 Almost periodic and recurrent solutions	469 475 477
Bib	14.4 Global attractors of quasi-linear triangular systems14.5 Almost periodic and recurrent solutions14.6 Pseudo recurrent solutions	469 475 477

.