LEGIBILITY NOTICE

A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.

LA-UR -86-1027 Received by OSTI CONF-860366--3

APR 0 7 1986

TITLE: GLOBAL COORDINATES AND EXACT ABERRATION CALCULATIONS APPLIED TO PHYSICAL OPTICS MODELING OF COMPLEX OPTICAL SYSTEMS

LA-UR--86-1027

DE86 008751

AUTHOR(S) G. Lawrence, AOR
C. Barnard, AOR
V.K. Viswanathan, P-15

SUBMITTED TO SPIE Technical Symposium East, March 31-April 4, 1986, Orlando, FL

Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

Global Coordinates and Exact Aberration Calculations Applied to Physical Optics Modeling of Complex Optical Systems

G. Lawrence and C. Barnard
Applied Optics Research

V. Viswanathan
Los Alamos National Laboratory

Abstract

Historically, wave optics computer codes have been paraxial in nature. Folded systems could be modeled by "unfolding" the optical system. Calculation of optical aberrations is, in general, left for the analyst to do with off-line codes. While such paraxial codes were adequate for the simpler systems being studied 10 years ago, current problems such as phased arrays, ring resonators, coupled resonators, and grazing incidence optics require a major advance in analytical capability. This paper describes extension of the physical optics codes GLAD and GLAD V to include a global coordinate system and exact ray aberration calculations. The global coordinate system allows components to be positioned and rotated arbitrarily. Exact aberrations are calculated for components in aligned or misaligned configurations by using ray tracing to compute optical path differences and diffraction propagation. Optical path lengths between components and beam rotations in complex mirror systems are calculated accurately so that coherent interactions in phased arrays and coupled devices may be treated correctly.

Introduction

This paper describes techniques that have recently been developed for modeling components in physical optics codes. The basis of most physical optics codes is a quasi-paraxial approach which treats folded systems by implicitly unfolding the system and which can treat only small tilts and decenters from the chief ray. The technique described here is much more powerful and allows components to be located at arbitrary positions in space and at large angle rotations. Advantages of the global coordinate system and associated functions are:

* Global positioning of components
* Arbitrary component rotations
* Accurate path length calculations
* Correct calculation of azimuthal rotations of the beam
* Exact or approximate aberrations of tilted components
* Surface polarization effects in s- and p-directions.

Systems which require this type of sophistication in the code include:

* Grazing incidence systems
* Systems with strongly tilted components
* Coupled resonators
* Complex folded resonators
* Polarization calculations

Component Location and Rotation in Global Coordinates

To describe the propagation of a beam through a complex 3-dimensional optical system, we define four coordinate

global - ray and vertex locations.
ray - complex amplitude distribution.
vertex - component rotations and shape definition.
surface - surface at chief ray intercept point

When the optical elements are to be located at arbitrary positions and with arbitrary rotations, it is necessary to define a global coordinate system on which the beam path and the optical components are specified. As the beams propagate through the optical system, the global coordinate system is updated. The location of the optical component is defined by the vertex position. The rotation is considered to be about the vertex location or may be made about a separate point.

The direction of the chief ray must also be defined on the same global coordinate system. It is also necessary to determine the azimuthal orientation of the complex amplitude distribution about the chief ray. Systems with out-of-plane components can rotate the beam distribution about the chief ray direction. The azimuthal rotation may be determined by establishing a ray matrix consisting of i-, j-, and k-vectors. The k-vector defines the chief ray direction.

The properties of the optical component are most easily defined in terms of a vertex coordinate system. For a rotationally symmetric surface, the z-axis is identical to the axis of symmetry.

A fourth coordinate system at the point of intersection of a ray allows calculation of the polarization properties. This surface coordinate system consists of s-, p-, and n-vectors. The n-vector is the surface normal vector. We choose the surface normal vector to point toward the center of curvature.

Figure 1 illustrates the four coordinate systems schematically.

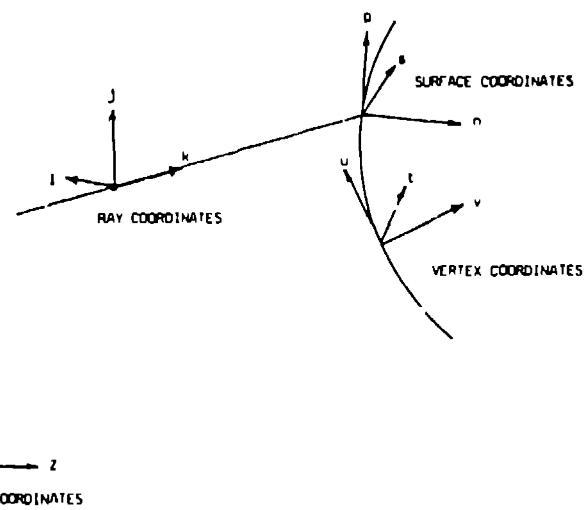

Figure 1. The global, ray, vertex, and surface coordinate systems.

The unit vectors for the systems are

$\hat{x}, \hat{y}, \hat{z}$ - global coordinate system
$\hat{i}, \hat{j}, \hat{k}$ - ray coordinate system
$\hat{t}, \hat{u}, \hat{v}$ - vertex coordinate system
$\hat{s}, \hat{p}, \hat{n}$ - surface coordinate system

The ray, vertex, and surface matrices are described in terms of unit vectors with respect to the global coordinate system.

R_{gr} - ray coordinate matrix
R_{gv} - vertex coordinate matrix
R_{gs} - surface coordinate matrix

$$R_{gr} = [(\hat{i})(\hat{j})(\hat{k})]$$
$$R_{gv} = [(\hat{t})(\hat{u})(\hat{v})] \quad (1)$$
$$R_{gs} = [(\hat{s})(\hat{p})(\hat{n})]$$

where [illegible] ( )] indicates column vectors concatenated into a matrix.

Th[illegible] of elements may be defined on the global coordinates system by specifing the vertex global coor[illegible] The vertex location and rotation may be defined absolutely or with reference to the current positi[illegible] an optical beam. The rotation of the vertex is defined by the user by Euler angles.

Relative rotation is with respect to the coordinates of the beam. The rotation angles α, β, and γ apply to x-, y-, and z-axis rotations. These rotations obey the right hand rule. A positive x-rotation is done by rotating in the direction of the fingers of the right hand when the thumb is aligned with the positive x-direction. The rotations are applied in the order x, y, and z.

Chief Ray Propagation

The propagation of a beam through an optical configuration is may be described by following the chief ray. The chief ray is defined to be the path of the center of the beam array irrespective of the actual complex amplitude distribution in the array. The movement of the chief ray is described by geometrical optics.

We begin by defining several vectors. Each is in global coordinates

- $\mathbf{r}$ - chief ray position.
- $\hat{\mathbf{k}}$ - chief ray direction.
- $\mathbf{v}$ - vertex location.
- $\mathbf{s}$ - surface intercept of chief ray.

Movement along a ray is defined as a change in the physical path length (PPL). If the PPL is q, then the vector equation is

$$\mathbf{r}_1 = \mathbf{r}_0 + q\hat{\mathbf{k}} \tag{2}$$

where $\mathbf{r}_0$ is the starting position and $\mathbf{r}_1$ is the final position.

Currently, the index of refraction is always 1 exactly. Subsequent versions may implement nonunity index of refraction.

<u>Chief Ray Surface Intercept</u>

We first find the point of closest approach to the vertex of the conic. This approach is more numerically stable than finding the distance to the vertex tangent plane, since the tangent plane may rotated so that it is far from normal to the ray.

$$q_1 = (\mathbf{v} - \mathbf{r}_0)\cdot\hat{\mathbf{k}} \tag{3}$$

The new vector position is

$$\mathbf{r}_1 = \mathbf{r}_0 + q_1\hat{\mathbf{k}} \tag{4}$$

The equation for propagation along the ray to the interception point on the surface is

$$\mathbf{r}_2 = \mathbf{r}_1 + q_2\hat{\mathbf{k}} \tag{5}$$

where $\mathbf{r}_2$ is the chief ray intercept on the surface. The incremental distance from the point of closest approach to the vertex to the surface is q_2. The solution for q_2 is described below.

The coordinates of the optical element are expressed in vertex coordinates. We will use $\hat{\mathbf{t}}$, $\hat{\mathbf{u}}$, and $\hat{\mathbf{v}}$ as vertex unit vectors and t, u, and v as vertex coordinates.

$$t = (\mathbf{v} - \mathbf{r}_1)\cdot\hat{\mathbf{t}}$$
$$u = (\mathbf{v} - \mathbf{r}_1)\cdot\hat{\mathbf{u}}$$
$$v = (\mathbf{v} - \mathbf{r}_1)\cdot\hat{\mathbf{v}}$$

The component of the ray direction vector parallel to the vertex v-axis is

$$k_v = \hat{\mathbf{k}}\cdot\hat{\mathbf{v}}$$

The surface equation in vertex coordinates is

$$f(t,u,v) = v - \frac{C}{2}[t^2 + u^2 + (1+\kappa)v^2] \tag{6}$$

where κ is the conic constant and C is the vertex curvature.

We may solve for q_2 as follows

$$\begin{aligned} Aq_2^2 - 2Bq_2 + D &= 0 \\ A &= C(1 + \kappa k_v^2) \\ B &= k_v(1 - \kappa Cv) \\ D &= -2F(t,u,v) \end{aligned} \tag{7}$$

This equation has the roots

$$q_2 = \frac{D}{\left[B \pm \sqrt{B^2 - AD}\right]} \tag{8}$$

where the sign is based on the intersection point. In general conic surfaces have two intersection points. We choose the root with the smallest positive value for q_1+q_2, unless ASI has been chosen. This result in the smallest forward propagation step. Backward propagation to a surface is not allowed, unlike ray trace codes. If neither root results in a positive q_1+q_2 sum, a ray error is issued. This implies that all of the conic surface is behind the ray. The user should consider the position of the beams at the point in the command sequence at which the vertex is defined. If ASI is selected, the root giving the largest q_1+q_2 sum is chosen. If this distance is negative, a ray failure error occurs.

By propagating to the chief ray intercept, all of the diffraction effects are accounted for in the region of the transverse distribution near the chief ray. Regions of the transverse distribution distant form the chief ray have the correct propagation distance, but the aberrations due to reflection from the optical component will, in general, occur earlier or later in the propagation step than they should.

Calculation of the Surface Coordinate System

The normal to the surface, $\hat{n}$, is calculated by taking the negative of the surface gradient.

$$\nabla f = \begin{bmatrix} \dfrac{t}{R} \\ \dfrac{u}{R} \\ 1 - (\kappa+1)\dfrac{v}{R} \end{bmatrix} \tag{9}$$

$$\hat{n} = -\frac{\nabla f}{|\nabla f|}$$

The code uses the convention that the surface normal points toward the center of curvature of the surface.

Having found $\hat{n}$ and knowing the chief ray direction, $\hat{k}$, we may define the surface coordinate system. Two tangent rays on the surface may be defined, $\hat{s}$ and $\hat{p}$. These correspond to the s- and p-directions used for polarization calculations. The vector $\hat{p}$ is in the plane of reflection and $\hat{s}$ is perpendicular to the plane of reflection, as shown in Figure 2.

$$\hat{s} = \frac{\hat{k} \times \hat{n}}{|\hat{k} \times \hat{n}|} \tag{10}$$

$$\hat{p} = \hat{n} \times \hat{s}$$

where X denotes the vector cross product.

When the beam strikes the surface at normal incidence, the cross product of $\hat{n}$ and $\hat{k}$ is zero and $\hat{s}$ is not defined. However, at normal incidence, it is not necessary to distinguish between the s- and p-vectors. In practice, when the cross product drops below a threshold value, the code assigns $\hat{s}$ and $\hat{p}$ to be identical to $\hat{t}$ and $\hat{t}$.

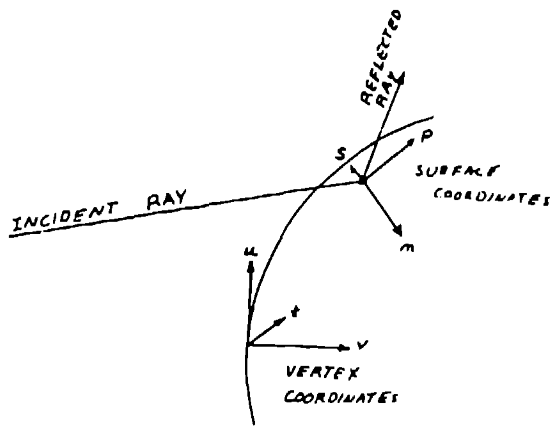

Figure 2. Illustration of surface coordinates.

Reflection

The reflection equation is

$$\hat{i}' = \hat{i} - 2(\hat{i}\cdot\hat{n})\hat{n}$$
$$\hat{j}' = \hat{j} - 2(\hat{j}\cdot\hat{n})\hat{n}$$
$$\hat{k}' = \hat{k} - 2(\hat{k}\cdot\hat{n})\hat{n} \qquad (11)$$

Equation 12 shows that reflection consists of reversing the component in the n-direction.

Calculation of Local Surface Curvature

It is necessary to calculate the local surface curvature where the chief ray strikes the surface. The local curvature is used to calculate the new phase bias to be applied to complex amplitude and also may be used for a quick, approximate calculation of the aberration, using only the astigmatism terms.

The code uses a toric phase bias of the form

$$W(x,y) = \frac{1}{2}\left[\frac{x^2}{R_x} + \frac{y^2}{R_y}\right] \qquad (12)$$

The bias phase is used to reduce the amount of phase which resides in the complex amplitude distribution. The separability of the diffraction calculations allows us to make adroit use of separable bias as described in the diffraction theory section of this manual. Spherical, cylindrical, and, in the general case, toric phase bias may be used. After reflection from a curved optical surface, this bias phase must be recalculated. This section describes the calculation of the new toric bias phase.

The surface aberration is described in terms of the sagittal and tangential radii (or s- and p-radii)

$$R_p = \frac{(1 - c^2r^2)^{3/2}}{c}$$

$$R_s = \frac{(1 - c^2r^2)^{1/2}}{c} \qquad (13)$$

By converting the initial phase bias terms to Zernike polynomials, making use of the toric mirror terms, and a modified Coddington ray trace; we may calculate the exiting toric phase bias terms. Details of this calculation are included in the GLAD and GLAD V manuals, but are too lengthy to include here.

Exact Ray Tracing

Exact ray tracing is done in the same fashion as the chief ray up to the surface intercept. The first reference surface is the plane perpendicular to the chief ray at the chief ray intercept. The second reference surface is the plane perpendicular to the exiting chief ray. For each point in the transverse distribution a new starting ray position and direction is determined. We define the vectors

$\mathbf{r}_{xy}$ - position vector for each x,y point
$\hat{\mathbf{k}}_{xy}$ - direction vector for each x,y point

The position vector is calculated to be

$$\mathbf{r}_{xy} = \mathbf{r} + x\hat{\imath} + y\hat{\jmath} \tag{14}$$

$$\hat{\mathbf{k}}_{xy} = \frac{\hat{\mathbf{k}} - (\frac{x}{R_x})\hat{\imath} - (\frac{y}{R_y})\hat{\jmath}}{|\hat{\mathbf{k}} - (\frac{x}{R_x})\hat{\imath} - (\frac{y}{R_y})\hat{\jmath}|}$$

where x,y are the local coordinates of the transverse beam distribution and R_x and R_y are the phase bias radius. In this approach, the ray slopes are determined by the phase bias radii. Exact slopes could be taken, but the errors with the simpler assumptions are very small.

The reflected direction vector for the current ray, indicated by primes on the coordinates, is

$$\hat{\mathbf{k}}'_{xy} = \hat{\mathbf{k}}_{xy} - 2(\hat{\mathbf{k}}_{xy}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}} \tag{15}$$

The vector between the chief ray intercept and the intercept of the current ray being traced is $\mathbf{r}_{xy} - \mathbf{s}$.

Consider reference planes perpendicular to the chief ray and reflected chief ray. Let δ_1 be the distance from the chief ray reference plane to the surface measured along the chief ray and δ_2 be the distance from the surface intercept to the reflected chief ray reference plane measured along the reflected chief ray. Then

$$\delta_1 = \frac{(\mathbf{r}_{xy} - \mathbf{s})\cdot\hat{\mathbf{k}}}{\hat{\mathbf{k}}_{xy}\cdot\hat{\mathbf{k}}} \tag{16}$$

$$\delta_2 = -\frac{(\mathbf{r}_{xy} - \mathbf{s})\cdot\hat{\mathbf{k}}'}{\hat{\mathbf{k}}'_{xy}\cdot\hat{\mathbf{k}}'}$$

The transverse intercepts on the reflected chief ray reference plane are found from δ_2

$$x' = (\mathbf{r}_{xy} - \mathbf{s} + \delta_2\hat{\mathbf{k}}')\cdot\hat{\imath}' \tag{17}$$
$$y' = (\mathbf{r}_{xy} - \mathbf{s} + \delta_2\hat{\mathbf{k}}')\cdot\hat{\jmath}'$$

where the unit vectors of the reflected system are found with Eq. 11.

The complex amplitude is transferred from the first reference plane to the second by

$$A(x',y') = A(x,y)e^{jk(\delta_1+\delta_2)} \tag{18}$$

In general, the point x',y' will not lie on a grid point in the new array. The value must be interpolated to the nearest neighboring grid points.

The values δ_1 and δ_2 must be extended to include the OPD from the curved reference surface to the reference plane. In the code, this is done by assuming the reference surface is an exact toric surface and calculating the path difference to the ray intercept.

Conclusion

The methods described here enable determination of properties of optical beams passing through a complex system of mirrors. The equations have been implemented successfully in a physical optics code.

References

1. C. Menchaca and D. Malacara, 'Directional curvatures in a conic surface', Appl. Opt. 23, 3258 (1 Oct 1984)."